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THE

PHYSICAL REVIEW.

THE PRINCIPLE OF PROJECTIVE COVARIANCE.

BY ISRAEL MAIZLISH.

SYNOPSIS.

The Principle of Projective Covariance.—The principle here presented asserts that the laws of physics are covariant with respect to the following transformation: $x_c = G(\sigma, r)x$, $y_c = G(\sigma, r)y$, $z_c = G(\sigma, r)z$, $t_c = G(\sigma, r)t$, where x, y, z, t are the Minkowski variables, r is the distance of a point in S_4 from the origin of a system of coördinates, and σ is an arbitrary constant.

It is shown that if Ψ is a physical quantity depending only upon a single variable ξ, the relation between Ψ and ξ is $\Psi = K\xi^{\alpha/\beta}$, where $\Psi_c = G^\alpha(\sigma, r)\Psi$, $\xi_c = G^\beta(\sigma, r)\xi$, and $K =$ constant. Examples are given of the application of the principle.

Comparison with the Principle of Similitude.—While there are similarities between the present work and Dr. Tolman's, the Principle of Projective Covariance does not in any way depend on the Principle of Similitude. It is pointed out that the Principle of Similitude, however, may be regarded as a very special interpretation of the Principle here presented. Concepts like "miniature universe" are shown to be extraneous, and, by taking a special form of the Principle of Projective Covariance, it is shown that the latter may be so interpreted as to give us also an extended universe.

1. While endeavoring to discover a simple method of solving certain problems in mathematical physics, especially the one relating to black-body radiation, it occurred to the writer that there might exist a transformation with respect to which the laws of physics were covariant. A subsequent investigation showed that, as far as it has been carried, this is the case.

2. Considering a space of four dimensions (S_4) we will now make the following

Postulate: The laws of physics are covariant with respect to the following transformation:

$$\begin{aligned} x_c &= G(\sigma, r)x, \\ y_c &= G(\sigma, r)y, \\ z_c &= G(\sigma, r)z, \\ t_c &= G(\sigma, r)t, \end{aligned} \tag{C}$$

where x, y, z, and t are the Minkowski variables, and $G(\sigma, r)$ is an arbitrary[1] function of the distance, r, and an arbitrary constant[2] σ.

This Postulate, for the sake of brevity, will be called the *Principle of Projective Covariance.*[3]

SOME TRANSFORMATION-EQUATIONS.

3. We will now proceed to derive a number of equations connecting certain physical quantities with those obtained by the application of the Principle of Projective Covariance.

4. *Distance:* Let $P(x, y, z, t)$ be a point in S_4 whose distance from the origin of a system of coördinates is r. Then,[4] if $P_c(x_c, y_c, z_c, t_c)$, and r_c be the corresponding quantities when the transformation (C) is applied, it is readily seen that

$$r_c = G(\sigma, r)r. \tag{1}$$

5. *Velocity:* To derive the transformation-equation for velocity we note that, by definition, the dimensions of velocity are

$$[v] = [L][T^{-1}].$$

By the Principle of Projective Covariance,

$$[v_c] = [L_c][T_c^{-1}].$$

But

$$l_c = G(\sigma, r)l,$$

and

$$t_c = G(\sigma, r)t.$$

Therefore,

$$v_c = v. \tag{2}$$

6. *Acceleration:* The dimensions of acceleration are

$$[a] = [L][T^{-2}].$$

Since

$$l_c = G(\sigma, r)l,$$

and

$$t_c = G(\sigma, r)t,$$

---

[1] It is possible to give the function $G(\sigma, r)$ such a form as to make it zero, infinite, or indeterminate for a certain value of r. The transformation would then be meaningless. There is, however, no particular reason for imputing such a form to $G(\sigma, r)$ rather than one of a multiplicity of legitimate functions.

[2] It is convenient to include σ in the function $G(\sigma, r)$. We shall presently consider a special form of $G(\sigma, r)$ in order to manifest conspicuously the differences between the present work and Dr. Tomlan's.

[3] The word *projective* is used here to mean non-metrical. The title is suggested by the special form of $G(\sigma, r)$ considered in paragraph 29. There, it may be added, the word has its usual significance.

[4] The subscript $_c$ will be used to signify that the quantity in question has been obtained by the application of the Principle of Projective Covariance.

it is seen that[1]

$$a_c = G^{-1}(\sigma, r)a. \tag{3}$$

7. *Force:* Universal or proportionality constants will obviously be unaffected by the Principle of Projective Covariance. Further, we may regard the number of Faraday tubes having their ends on a body as a measure of the charge on that body. The unit of electricity is, accordingly, the Faraday tube. Let us now consider a conductor C. The number of Faraday tubes having their ends on C is equal to the charge, q, on C. The only way in which our transformation-equations can affect the number of these tubes is to alter their "density" upon C, either by a uniform compression or expansion, or, in general, by a distortion of C. But the ***total number*** of Faraday tubes will remain unaltered.[2] The charge, q, will therefore be the same as it was originally. It is thus evident that q is invariant under the above transformation; that is,

$$q_c = q. \tag{4}$$

Now, Coulomb's Law asserts that the force between two electric charges, q and q', is directly proportional to their product, and inversely to the square of the distance between them; that is,

$$F \propto \frac{q \cdot q'}{r^2}. \tag{a}$$

By the Principle of Projective Covariance we also have

$$F_c \propto \frac{q_c \cdot q_c'}{r_c^2}. \tag{b}$$

Divide (b) by (a). The result is:

$$F_c = \left[\frac{q_c \cdot q_c'}{q \cdot q'} \cdot \frac{r^2}{r_c^2}\right] F,$$

or, by virtue of (1) and (4),

$$F_c = G^{-2}(\sigma, r)F. \tag{5}$$

8. *Mass:* In general, however, we have

$$F = ms'' = (\text{mass})(\text{acceleration});$$

i.e.,

$$F = ma. \tag{f}$$

By the Principle of Projective Covariance,

$$F_c = m_c a_c.$$

---

[1] $G^n(\sigma, r)$ or simply G^n will be written for $[G(\sigma, r)]^n$.

[2] The consideration that the Faraday tubes may be modified by different "ethers" is trivial, as, in this paper, we are not concerned with model universes.

In consideration of (5) and (3) this equation may be written

$$G^{-2}(\sigma, r)F = m_c G^{-1}(\sigma, r)a.$$

Substituting ma for F, this reduces to

$$G^{-2}(\sigma, r)ma = m_c a \cdot G^{-1}(\sigma, r).$$

Hence,

$$m_c = G^{-1}(\sigma, r)m. \tag{6}$$

Temperature: To derive the transformation-equation for absolute temperature, we may proceed as follows:

9. According to the Boltzmann-Planck entropy-law, we may write

$$\Delta S = K \log \frac{W_2}{W_1},$$

where ΔS is the increase in entropy resulting from the passage of a body from state 1 to state 2. The ratio W_2/W_1 is easily seen to be a pure number, and does not contain σ. For, assuming with Gibbs[7] that

$$\log W = \frac{\Psi - \epsilon}{\theta},$$

a simple substitution shows that the dimension of W_2/W_1 is zero, and that σ does not appear in this ratio. Hence, on applying the transformation (C), we obtain

$$\Delta S_c = \Delta S.$$

But now,

$$\Delta S = \int^{*} \frac{dQ}{T} = \Delta S_c = \int^{*} \frac{dQ_c}{T_c}.$$

Hence, it follows that

$$\frac{dQ}{T} = \frac{dQ_c}{T_c}.$$

10. In this equation dQ has the dimensions of energy. If we let $E = mv^2/2$, and $E_c = m_c v_c^2/2$, it is readily seen that, in consideration of (2) and (6),

$$E_c = G^{-1}(\sigma, r)E. \tag{7}$$

The equation

$$\frac{dQ}{T} = \frac{dQ_c}{T_c}$$

now becomes

$$\frac{dQ}{T} = \frac{G^{-1}(\sigma, r) \cdot dQ}{T_c}.$$

Hence,

$$T_c = G^{-1}(\sigma, r)T. \tag{8}$$

---

[7] Gibbs, Statistical Mechanics.

11. The same result may be arrived at by a somewhat different procedure. Thus, according to Stefan's Law,

$$\varphi = \text{energy density} = \text{energy/volume},$$
$$= kT^4,$$

where k is a constant. By the Principle of Projective Covariance, and equations (1) and (7), it is seen that

$$\varphi_c = kT_c^4 = \frac{\varphi}{G^4(\sigma, r)} = \frac{kT^4}{G^4(\sigma, r)}.$$

Therefore,

$$T_c = G^{-1}(\sigma, r)T \tag{8}$$

as above.

TABLE OF TRANSFORMATIONS.

12. For the sake of future reference it will be convenient to tabulate the results already obtained.

Name of Quantity.	**Transformation-Equation.**
Length	$l_c = G(\sigma, r)l.$
Time	$t_c = G(\sigma, r)t.$
Velocity	$v_c = v.$
Acceleration	$a_c = G^{-1}(\sigma, r)a.$
Mass	$m_c = G^{-1}(\sigma, r)m.$
Force	$F_c = G^{-2}(\sigma, r)F.$
Pressure	$P_c = G^{-4}(\sigma, r)P.$
Density	$d_c = G^{-4}(\sigma, r)d.$
Volume	$V_c = G^3(\sigma, r)V.$
Energy	$E_c = G^{-1}(\sigma, r)E.$
Energy-density	$\varphi = G^{-4}(\sigma, r)\varphi.$
Temperature	$T_c = G^{-1}(\sigma, r)T.$
Frequency	$\nu_c = G^{-1}(\sigma, r)\nu.$

APPLICATION OF THE PRINCIPLE.

13. In order to obtain unique solutions of physical problems by means of the Principle of Projective Covariance, it has thus far been found necessary to restrict ourselves to the case where we have one independent variable (ξ), and one dependent variable (Ψ). It is for this case that a unique relation between Ψ and ξ has been found.

14. Let us suppose that some quantity Ψ is expressed as a function of some other quantity ξ; that is, that we have

$$\Psi = \varphi(\xi).$$

By the Principle of Projective Covariance we also have

$$\Psi_c = \varphi(\xi_c).$$

The transformation-equations for Ψ and ξ can easily be obtained. Let, then,

$$\Psi_c = G^a(\sigma, r)\Psi,$$

and

$$\xi_c = G^{\beta}(\sigma, r)\xi, \qquad (\alpha \neq 0, \quad \beta \neq 0), \tag{9}$$

where α and β are numerical constants. Then,

$$\Psi_c = G^{\alpha}(\sigma, r)\Psi = \varphi(\xi_c) = \varphi(G^{\beta}\xi);$$

and, therefore,

$$\Psi = \varphi(\xi) = G^{-\alpha}(\sigma, r)\varphi[G^{\beta}(\sigma, r)\xi]. \tag{10}$$

15. It will now be shown that the solution of the functional equation (10) is

$$\Psi = K\xi^{\alpha/\beta},$$

where K is a constant.

16. To this end, let

$$G^{-\alpha}(\sigma, r)\xi^{-\alpha/\beta}\varphi[G^{\beta}(\sigma, r)\xi] = \eta[G^{\beta}(\sigma, r)\xi]. \tag{11}$$

Next, combine this equation with equation (10). The result is

$$\xi^{-\alpha/\beta}\varphi(\xi) = \eta[G^{\beta}(\sigma, r)\xi]. \tag{12}$$

In eq. (12) replace ξ by $G^{\beta}(\sigma, r)\xi$. We shall then have

$$[G^{\beta}(\sigma, r)\xi]^{-\alpha/\beta}\varphi[G^{\beta}(\sigma, r)\xi] = \eta[G^{2\beta}(\sigma, r)\xi];$$

that is,

$$\eta[G^{\beta}(\sigma, r)\xi] = \eta[G^{2\beta}(\sigma, r)\xi]. \tag{13}$$

Now, by replacing ξ by $G^{-\beta}(\sigma, r)\xi$, equation (13) becomes

$$\eta[\xi] = \eta[G^{\beta}(\sigma, r)\xi]. \tag{14}$$

By virtue of equation (13) we also have

$$\eta[\xi] = \eta[G^{2\beta}(\sigma, r)\xi],$$

and, by mathematical induction,

$$\eta[\xi] = \eta[G^{n\beta}(\sigma, r)\xi]. \tag{15}$$

In equation (15) if $G^{n\beta}(\sigma, r) > 1$, let

$$\xi = G^{-n\beta}\zeta.$$

By this transformation, (15) will now become

$$\eta[\zeta] = \eta[G^{-n\beta}(\sigma, r)\zeta]. \tag{16}$$

From physical considerations it is clear that $\eta(0)$ exists. Hence, it follows, from (15) and (16), that the only solution of (14) is

$$\eta[\xi] = \eta[G^{\beta}(\sigma, r)\xi] = \text{const.} = K.$$

Therefore,

$$\xi^{-\alpha/\beta}\varphi(\xi) = \eta[G^{\beta}(\sigma, r)\xi] = K;$$

that is, since $\Psi = \varphi(\xi)$,

$$\Psi = K\xi^{\alpha/\beta}. \tag{A}$$

17. It follows that whenever we are certain that Ψ is a function of ξ only, all that is needed to get the relation between Ψ and ξ is to find α and β, and substitute the value of α/β in formula (A).

18. *Example 1:* As our first illustration, consider the photo-electric effect, and assume with Einstein that

$$\Psi = \Delta E = E - E_0 = \varphi(\nu),$$

where $\varphi(\nu)$ is an unknown function of the frequency, ν, of the incident light. By referring to the Table of Transformations it will be seen that

$$\alpha = -1, \qquad \beta = -1, \qquad \text{and} \qquad \therefore \ \frac{\alpha}{\beta} = +1.$$

Hence, by formula (A),

$$\Delta E = h\nu,$$

where h is a constant (in this case, Planck's).

19. *Example 2:* It is known that the wave-length (λ_m) for which the spectral energy is a maximum is a function of the absolute temperature. Let us find the function. Assume that

$$\Psi = \lambda_m = \varphi(T).$$

By referring to the Table of Transformations it will be seen that

$$\frac{\alpha}{\beta} = -1.$$

Therefore, by formula (A),

$$\lambda_m = bT^{-1},$$

or,

$$\lambda_m T = b,$$

where b is a constant.

20. *Example 3:* As another illustration, consider the mean kinetic energy (E_m) of the molecules of a perfect gas. Let us assume that E_m is a function of the absolute temperature only. We thus have

$$\Psi = E_m = \varphi(T).$$

From the Table of Transformations it is seen that

$$\alpha = -1, \qquad \text{and} \qquad \beta = -1.$$

Hence, $\alpha/\beta = +1$, and, by formula (A),

$$E_m = nT,$$

where n is a constant.

21. It may here be remarked that the solutions of the four classical functional equations, namely,

$$\begin{aligned} \varphi(u + v) &= \varphi(u) + \varphi(v), \qquad (1) \\ \varphi(u + v) &= \varphi(u)\varphi(v), \\ \varphi(uv) &= \varphi(u) + \varphi(v), \end{aligned}$$

and

$$\varphi(uv) = \varphi(u)\varphi(v),$$

have been obtained by means of formula (A). The solutions of the last three equations were made to depend on the solution of equation (1), the latter being readily obtained by the application of the Principle of Projective Covariance.

22. The Principle of Projective Covariance will help us to determine unknown functions which depend upon more than one variable. Consider, for example, the frequency, ν, of a thin, flexible string of density d, total mass m, radius r, and length l, when stretched by a force of F dynes. Let us assume that

$$\nu = \varphi(l, m, F). \tag{1}$$

By the Principle of Projective Covariance we also have

$$\nu_c = \varphi(l_c, m_c, F_c). \tag{2}$$

Referring to the Table of Transformations it is seen that

$$\nu_c = G^{-1}(\sigma, r)\nu, \qquad l_c = G(\sigma, r)l,$$

$$m_c = G^{-1}(\sigma, r)m, \qquad \text{and} \qquad F_c = G^{-2}(\sigma, r)F.$$

Therefore,

$$\nu_c = G^{-1}(\sigma, r)\nu = \varphi(l_c, m_c, F_c)$$
$$= \varphi(Gl, G^{-1}m, G^{-2}F)$$

and

$$\nu = \varphi(l, m, F) = G\varphi(Gl, G^{-1}m, G^{-2}F). \tag{3}$$

It is evident that one solution of (3) is

$$\nu = k\sqrt{\frac{F}{ml}},$$

where k is a constant. The general solution of (3), however, would be of the form

$$\nu = kl^{\alpha}m^{\beta}F^{\gamma},$$

where α, β, and γ are constants staisfying the following condition-equation:

$$\alpha - \beta - 2\gamma = -1. \tag{4}$$

23. To determine α, β, and γ uniquely, we would make two simple experiments, using the same material in both. In the first experiment we would vary F and keep m and l constant. In the second, we would vary m (that is, r) while keeping F and l fixed. The results of these

experiments[1] would be that

$$\gamma = \tfrac{1}{2}, \qquad \text{and} \qquad \beta = -\tfrac{1}{2}.$$

Using this information and the condition-equation (4), we get immediately,

$$\alpha = -\tfrac{1}{2}.$$

Hence,

$$\nu = k\sqrt{\frac{F}{ml}},$$

or, writing $m = \pi r^2 d \cdot l$,

$$\nu = k\sqrt{\frac{F}{\pi r^2 d l^2}}.$$

THE FUNDAMENTAL PROBLEM.

24. It should be observed that no direct method of getting a unique solution of a problem such as the one just given has yet been found. The Principle of Projective Covariance would, of course, be of very great service if it enabled us to solve uniquely the general equations

$$\begin{aligned} \Psi &= \varphi(\xi, \eta, \cdots, \omega), \\ \Psi_c &= \varphi(\xi_c, \eta_c, \cdots, \omega_c), \end{aligned} \qquad (Q)$$

where

$$\Psi_c = G^\gamma(\sigma, r)\Psi, \qquad \xi_c = G^\alpha(\sigma, r)\xi,$$

$$\eta_c = G^\beta(\sigma, r)\eta, \qquad \cdots, \qquad \textit{and} \qquad \omega_c = G^\delta(\sigma, r)\omega.$$

The solution of (Q) has not, however, been obtained. In default thereof, the Principle of Projective Covariance may still be used in verifying known functions (physical equations), and predicting possible forms of unknown functions.

25. In the case of two independent variables ξ and η, and if the reasonable assumption be made that Ψ is expressed as

$$\Psi = K\xi^n\eta^m,$$

the solution has been obtained in the form

$$\Psi = K\xi^n\eta^m, \qquad (D)$$

with

$$n = (\gamma + \beta\epsilon)/(\alpha + \beta), \qquad m = (\gamma - \alpha\epsilon)/(\alpha + \beta),$$

$$\Psi_c = G^\gamma(\sigma, r)\Psi \qquad \xi_c = G^\alpha(\sigma, r)\xi,$$

$$\eta_c = G^\beta(\sigma, r)\eta, \qquad \text{and} \qquad \epsilon = n - m.$$

---

[1] These experiments are, of course, unnecessary, if we wish to combine with the Principle of Projective Covariance the theory of dimensions. In that case, α, β, and γ would be obtained by combining *two* equations from the theory of dimensions with the condition-equation given by the Principle of Projective Covariance.

26. Although the attempt to express ϵ in terms of α, β, and γ has not been successful, it is worth pointing out that formula (D), [$\alpha + \beta \neq 0$, $\gamma - \alpha\epsilon \neq 0$, and $\gamma + \beta\epsilon \neq 0$], gives the correct result in every case.

27. At the time of writing it does not appear possible to solve (Q) with the help of the Principle of Projective Covariance only. Just how far the work can be extended, and what additional assumptions, in particular cases, will be needed, cannot be definitely stated without further investigation. It is hoped that there will be an opportunity, in the future, to consider anew some of the problems which originally led to the Principle here presented.

A Special Form of the Function $G(\sigma, r)$.

28. Since $G(\sigma, r)$ is arbitrary,[1] evidently $G(\sigma, r) = \sigma^a r^b$, where a and b are any constants, is a possible form of $G(\sigma, r)$. Now, upon placing a equal to 1, and b equal to zero, we get

$$G(\sigma, r) = \sigma.$$

29. The transformation with respect to which the laws of physics are covariant now takes the following special form:[2]

$$\begin{aligned} x_c &= \sigma x, \\ y_c &= \sigma y, \\ z_c &= \sigma z, \\ t_c &= \sigma t, \end{aligned} \qquad (PT)$$

where σ is an arbitrary constant. From what has preceded, it is obvious that the transformation for length is

$$l_c = \sigma l.$$

The transformation-equations for the other quantities considered above are the same as those already given except that σ replaces $G(\sigma, r)$.

Comparison between the Present Work and Dr. Tolman's.

30. For the sake of comparison, the present work and Dr. Tolman's are presented in tabular form.

The Present Work.	*Dr. Tolman's Work.*
The Principle of Projective Covariance:	The Principle of Similitude:
The laws of physics are covariant with respect to the following transformation: $x_c = G(\sigma, r)x$, $y_c = G(\sigma, r)y$, $z_c = G(\sigma, r)z$, $t_c = G(\sigma, r)t$.	The fundamental entities of which the physical universe is constructed are of such a nature that from them a miniature universe could be constructed exactly similar in every respect to the present universe.

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[1] See footnote 1.

[2] In this special form, the transformation is projective. Hence, the title *Projective Covariance.*

Transformations for fundamental physical quantities:

$$l_c = G(\sigma, r)l,$$
$$t_c = G(\sigma, r)t,$$
$$m_c = G^{-1}(\sigma, r)m,$$
$$q_c = q,$$
$$S_c = S.$$

Working formula for two variables:

$$\Psi = K\xi^{\alpha/\beta},$$

(see (A) above).

Relations between fundamental quantities of the miniature universe (denoted by primed letters) and those of the present universe:

$$m' = m/x, \qquad l' = xl,$$
$$q' = q, \qquad t' = xt, \qquad S' = S.$$

Working formula:

Not given by Tolman.

(See, however, foot-note 11.)

31. The reader will note that there are similarities between the present work and Dr. Tolman's: the method of development in some instances being almost identical in both papers. Nevertheless, in spite of these coincidences, the Principle of Projective Covariance is entirely independent of the Principle of Similitude. On the other hand, the Principle of Similitude may be regarded as a SPECIAL INTERPRETATION of the Principle of Projective Covariance.

32. The Principle of Similitude is a special case not only of the general Principle of Covariance, (C), but also of the special form of it considered in paragraph 29. For, since σ is entirely arbitrary, the transformation may be regarded either as a compression or an expansion. Hence, in addition to the "*miniature universe*" of the Principle of Similitude, the Principle of Projective Covariance (that is, the special form of it as given in paragraph 29), by a suitable interpretation, could also be made to yield what may be called an EXTENDED UNIVERSE.

33. The conclusion drawn from the preceding remarks is that concepts like "miniature universe" are extraneous. That this is the case, may be strikingly shown by considerations like the following:

34. Let us assume for the moment that the general function $G(\sigma, r)$ is of the form

$$G(\sigma, r) = n^{\sigma - r}, \tag{a}$$

where n is any positive number greater than 1. The transformation for length is

$$r_c = G(\sigma, r)r. \tag{b}$$

Now, in general, if $\sigma > r > 1$, it is evident from (a) and (b) that

$$r_c > r;$$

and if $r > \sigma > 1$, we shall have

$$r_c < r.$$

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[11] Formula (A) can be shown to follow from the Principle of Similitude. This was done by the writer in a paper read before the Iowa Academy of Science, April, 1920.

Similar considerations hold for $\sigma > r < 1$, and $r > \sigma < 1$. The Principle of Projective Covariance holds whether $r_c > r$ or $r_c < r$. But, if we are contemplating a model universe, it would manifestly have to be of such a nature that, for certain values, a length would experience an expansion, while for other values, it would suffer a contraction. Hence, in general, a geometrical figure would experience a distortion; and, in certain cases, may not have any resemblance whatever to the original figure (that is, to the figure as seen by an observer of the present universe). Under these circumstances, then, the model universe would not be "*exactly similar in every respect to the present universe.*" [1]

35. In justice to Dr. Tolman it should be emphasized that none of his work is invalidated by the Principle here presented. It appears, however, that the Principle of Similitude is merely a very special case of the general Principle of Projective Covariance.

36. When one considers the concepts of space and time as reflected in the works of Einstein and Minkowski, and if one interprets the present Principle to mean that physical equations are "homogeneous in Minkowski variables," the simplicity of the Principle will be quite evident.

37. The chief advantages of the present work consist in (1) avoiding extraneous concepts (*e.g.*, "miniature universe") and the attendant unprofitable attempts at visualization; (2) not making use of Einstein's postulate of the constancy of the velocity of light; and (3) the derivation of a working formula.

I am grateful to Professor E. B. Wilson, of the Massachusetts Institute of Technology, for having read part of the manuscript.

Reed College,
Portland, Oregon,
Dec., 1920.

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[1] R. C. Tolman, Phys. Rev., Aug., 1914.

THE LAWS OF ABSORPTION OF X-RAYS.[1]

BY F. K. RICHTMYER.

SYNOPSIS.

Measurement of Absorption of X-rays.—The use of a filtered beam of rays can at best give only approximate values of the coefficients, for such a beam is not homogeneous. It is pointed out that logarithmic absorption is no proof of homogeneity. In these experiments homogeneous radiation was secured by using a Bragg spectrometer and the effects of secondary radiation were avoided by placing the absorbing material between the two slits which limited the beam. By taking these and other precautions a precision of 1–2 per cent. was obtained. For wave-lengths below 0.15 Å. it was found better to place the thin analyzing crystal of NaCl nearly at right angles to the beam and to use the radiation reflected from atom planes perpendicular to the ones usually used.

X-ray Absorption by Ag, Al, Cu, Mo, Pb and H_2O for Wave-lengths about 0.13 to 0.7 Å.—The linear relation, established by Hull and Rice, between the *mass-absorption coefficient* and λ^3, $\mu/\rho = F\lambda^3 + \sigma/\rho$, holds accurately for these substances until, as λ is increased, the characteristic K limit is approached, when the values of μ/ρ are increasingly too small. The linear relation also holds above the K limit, F having a much smaller value but σ/ρ, the *mass-scattering coefficient* being about the same. The *atomic absorption coefficient* for wave-lengths below K is a linear function of N^4, where N is the atomic number, in agreement with Owen's results. The *general law of X-ray absorption* is therefore as follows: $\mu_a = kN^4\lambda^3 + \sigma_a$ where k is a universal constant equal to 2.29×10^{-27} below the k limit and to 0.33×10^{-27} above that limit. The experimental curves give no evidence for the existence of the "*J*" *radiation* reported by certain observers for Ag, Al, Cu and H_2O. The absorption for water was measured down to 0.093 A but shows no indication of an approach to the absorption found for the hard γ-rays of RaC.

GENERAL STATEMENTS ON ABSORPTION MEASUREMENTS.

An accurate knowledge of the laws of X-ray absorption by various media is of obvious importance in connection with the problems of atomic structure and of the nature of radiant energy. One may classify all measurements of X-ray absorption coefficients according as the investigator (1) has used a strictly homogeneous beam obtained by reflection from a crystal grating or (2) has attempted to get a more or less homogeneous beam either by employing the characteristic radiation of some target or by filtering out the longer waves from a gross beam by a filter such as aluminum and then assuming that the resultant filtered beam is practically homogeneous. In the latter case the mean wave-length of the beam is given. Thus Auren,[1] in studying the absorption coefficients of a large number of compounds, from which to compute the coefficients of the components, filters his radiation (voltage applied to tube not stated) through 1.12, 2.05, 5.00 and 10.00 mm. of aluminum respectively. He gets the mean wave-length of each filtered beam by measuring the absorption coefficient of copper for the beams and compares this coefficient with data from Barkla for the variation of the absorption coefficient of copper as a function of wave-length. The mean wave-length of the filtered beams are .38, .36, .34, and .30 Ångströms, respectively. He concludes, from absorption measurements by use of these wave-lengths, that "although the atomic absorption coefficient in a great number of elements may be proportional to a power of the atomic number (yet) . . . in certain elements there are very marked discontinuities."

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[1] The work herein described was done at the Research Laboratory of the General Electric Co. The writer wishes hereby to make grateful acknowledgment, particularly to Dr. A. W. Hull, for the many courtesies extended and for the laboratory facilities made available.

In an earlier paper[2] Auren concludes that the atomic absorption coefficient is not proportional to the fourth power of the atomic number of the absorber, contrary to conclusions reached by Bragg and Pierce,[3] and confirmed later by Owen[4] for the absorption of the α line of Pd obtained by reflection from a crystal.

The use of a filtered beam for absorption measurements may give approximate results so long as one of the characteristic absorption limits of the absorber does not fall within the spectral region covered by the filtered beam; *but at best the results can be only approximations to the true values of the absorption coefficients.*

Another error should be pointed out in absorption measurements involving filtered radiation. Assuming the general absorption law for homogeneous radiation

$$I = I_0 e^{-\mu x}$$

one gets the linear equation between log I and x:

$$\log I = -\mu x + \log I_0.$$

Conversely it has been assumed[5] that a strictly homogeneous radiation has been obtained by filtering when a plot between log I and x is a straight line. This conclusion is not necessarily justified. The linear relation simply means that μ has the same value for all components of the radiation, which may be homogeneous or heterogeneous; or in other words that, for the radiation remaining after filtering, μ does not change with λ. As is shown by data contained herein and also by Barkla and White[1] there is comparatively little change in the mass absorption coefficient, μ/ρ, for water below $\lambda = .2$ Å. In the case of Al the change in μ/ρ from $\lambda = .15$ Å. down to the shortest X-ray wave-lengths yet obtained is not more than 20 per cent. This is the obvious explanation of the linear relation between log I and x after filtering through Al as obtained by Rutherford, Barnes, and Richardson, rather than the one given by these writers, namely that there is no decrease in wave-length in X-radiation by use of voltages above 125,000.

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[1] Phil. Mag., 37, p. 165 (1919).

[2] Phil. Mag., 33, p. 471 (1917).

[3] Phil. Mag., 28, p. 626 (1914).

[4] Proc. Roy. Soc., 94, p. 510 (1919).

[5] *E.g.*, see Rutherford, Barnes, and Richardson, Phil. Mag., 30, p. 339 (1915).

On the contrary absorption measurements in which a strictly homogeneous beam has been obtained by reflection from a crystal grating seem to lead to perfectly definite laws. Thus, Hull and Rice,[2] conclude that the mass absorption coefficient, μ/ρ, is given as a function of the wave-length, λ, by an equation of the form

$$\frac{\mu}{\rho} = F\lambda^3 + \frac{\sigma}{\rho}, \tag{1}$$

where σ/ρ is the so-called mass-scattering coefficient, and F is a constant. These investigators verified this equation for aluminum, copper, and lead from .147 Å. to .392 Å. They assumed σ/ρ to be constant and equal to .12. In this equation the term $F\lambda^3$ is due to fluorescent radiation. It is frequently written τ/ρ, and is then called the mass-fluorescent absorption coefficient. Owen[5] shows that τ/ρ is proportional to the cube of the atomic number of the absorber, *i.e.*,

$$\frac{\tau}{\rho} = CN^3 \tag{2}$$

at wave-length $\lambda = .589$ Å. But it should be remarked that Owen's results can hold only for those substances whose K absorption limit is on the long wave-length side of $\lambda = .589$ Å. A similar law may possibly hold for the group of elements whose K absorption limit is on the short wave-length side of $\lambda = .589$ Å., but with a different value of C.

Purpose of the Investigation.

In view of the simplicity of the laws expressed by (1) and (2) and of the corresponding importance which they thereby assume, it seemed desirable to extend these measurements over as great a range of wave-lengths as possible, getting measurements of the highest attainable precision, in order to answer the following questions.[3]

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[1] Phil. Mag., 34, p. 270 (1915).

[2] Phys. Rev., VIII., p. 326 (1916).

[3] Preliminary reports of the results of this investigation have been given in papers presented to the American Physical Society. See Richtmyer and Grant, Phys. Rev., and

I. Do equations (1) and (2) accurately hold over a wide range of wave-lengths, especially if the measurements be extended to the long wave-length side of the K absorption limit—the latter of course with change in value of the constant F?

II. Is the term σ/ρ constant for all substances?

III. Is there, as reported by Barkla and White,[1] Williams,[2] Owen[3] and Dauvillier,[4] a discontinuity in the absorption curve, indicating a "J" radiation, similar to the discontinuities that mark the K and L radiations?

APPARATUS AND METHOD.

The apparatus, patterned after the well-known Bragg X-ray spectrometer, needs no special description. The X-ray tube used was of the Coolidge type, the filament of which was operated by a well-insulated storage battery. For the short wave-length work was used a tube of the radiation type, specially exhausted. For the longer wave-lengths a water-cooled tube was employed, thus giving sufficient energy for accurate measurements in the neighborhood of 1.0 Å. Voltage was applied direct to the tube, without rectification, from the secondary of a high tension transformer, the primary of which was connected to a specially controlled 60 cycle A.C. dynamo. Under normal conditions the variations in voltage, over long periods was never greater than 0.1 per cent. The radiation type tube was operated at as high as 110 kv. r.m.s.

The target was approximately 45 cm. from the NaCl crystal. In line with target and crystal, and between the two, were two micrometer slits with lead jaws limiting the X-ray beam to a few tenths of a millimeter in width. The absorbing material to be studied was placed between these two slits, thus avoiding the troublesome secondary radiation when the absorber is placed next to the ionization chamber.

Above .15 Å. the crystal was used in the ordinary way, *i.e.*, the X-ray beam was reflected from the surface planes of a thin crystal. At shorter wave-lengths the glancing angle becomes rather small, of the order of 1 degree, and considerable trouble was experienced from stray secondary radiation entering the chamber directly. It was found that this was in part avoided if a crystal about three millimeters thick were employed,

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Richtmyer, PHYS. REV. The writer wishes to express his indebtedness to Dr. Kerr Grant of Adelaide University, South Australia, who collaborated in the early part of this investigation and thereby made many of these measurements possible.

[1] Phil. Mag., 34, p. 270 (1917).

[2] Proc. Roy. Soc., 94, p. 567 (1919).

[3] Proc. Roy. Soc., 94, p. 339 (1919).

[4] Ann. de ph., XIV., p. 49 (1920).

and placed as shown in Fig. 1, thus using the atom planes at right angles to the major surfaces of the crystal. For these wave-lengths the crystal is sufficiently transparent, but cuts off the fluorescent radiation of longer wave-lengths coming from slit edges, crystal mounting, etc.

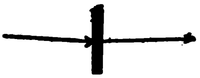

Fig. 1.

The ionization chamber was of brass, lead covered, approximately 60 cm. long by 10 cm. diameter. Rays were admitted through a thin mica window attached with sealing wax. A micrometer slit, with lead jaws immediately in front of the chamber, protected it from stray radiation. The collecting rod was supported by sulphur insulation, with suitable guard rings. Methyl bromide vapor was found to be most satisfactory for filling the chamber. This substance boils at 4° C. and is rather troublesome to handle but it did not attack the insulation as did some of the other vapors used. The collecting rod was charged to a potential of 200 volts by dry cells.

The ionization current was measured by observing the rate of increase of potential of the insulated system by means of an electroscope of the Bumstead type, the plates of which were charged to $+225$ and -225 volts respectively. The sensitivity was of the order of 200 divisions per volt. The rate of drift of the electroscope leaf was observed by a stop watch.

In order to eliminate observational and instrumental errors as far as possible the following precautions were observed.

1. The intensity of the X-ray beam was so adjusted (by current through filament or by applied voltage) that the rate of drift of the electroscope leaf was of the order of one or two divisions per second, some 60 divisions being covered for each reading. Errors in time measurements were of the order of 0.2 per cent. for each reading. The mean of several readings was taken for each observation. Correction was made for natural leak which was usually less than one per cent. of the ionization current measured.

2. The thickness of the absorber was so chosen that the intensity of the transmitted beam was seldom less than 40 per cent. or more than 65 per cent. of the incident beam.

3. When making an observation at any angle θ, and wave-length λ, the voltage applied to the tube was so adjusted that there was not present in the X-ray beam second order energy of wave-length $\lambda/2$. This was determined by the equation

$$\lambda_{\max} = \frac{12.3}{V},$$

where $\lambda_{\max}$ is the wave-length in Ångströms corresponding to the maximum frequency produced by V kilovolts. For example, if one is working at 0.4 Å. there should not be present in the incident beam any radiation of second order of 0.2 Å. Or

$$V = \frac{12.3}{.2} = 61.5 \text{ kilovolts}$$

is the (peak) voltage just necessary to produce radiation of 0.2 Å. Consequently a voltage somewhat less than this is to be employed. To guard against errors in estimating the maximum permissible voltage, due to distortion of wave form, a check was frequently made by observing whether, with the maximum permissible voltage as estimated, there was any radiation at $\theta/2$.[1]

The method of making observations was briefly as follows: At any given (chamber) angle 2θ, the time required for the gold leaf, after breaking earth connection, to drift from division A to division B, say 60 divisions apart, was observed, several readings being taken. The absorbing material was then put in place and several more readings taken. The absorber was removed and the first set repeated. The reciprocals of the mean average time with and without the absorber give quantities proportional to the respective ionization current. The absorption constant, μ, was then determined from the usual law

$$I = I_0 e^{-\mu x}.$$

From knowledge of the density of the absorber and its thickness, or of the mass per square centimeter of absorbing material in the path of the X-ray beam, the mass absorption coefficient μ/ρ was determined.

The following table will make the observations and method of computation clear.

Determination of μ for Molybdenum.

30.3 kilovolts; 1.70 milliamperes.

Absorber: Molybdenum No. 1 + No. 2; $\rho x = .0358$ gms. per sq. cm. Time to charge from division 20 to division 100:

	Absorber out	Absorber in	Absorber out
	32.0 sec.	64.8 sec.	31.8 sec.
	32.2	64.2	31.6
	31.8	64.2	31.6
Mean time		64.4	31.85
1/T (proportional to current)		.0314	.0155
Corrected for leakage		.0311	.0152

$$\frac{\mu}{\rho} = \frac{1}{\rho X} \log_e \frac{I_0}{I} = \frac{2.30 \times .311}{.0358} = 20.0 \text{ gms.}$$

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[1] Assuming $\theta = \sin\theta$ in the equation $n\lambda = 2d \sin\theta$.

The center of the direct beam was located on the divided circle by ionization measurements and the wave-length corresponding to any *chamber* angle 2θ was computed from the well known equation

$$\mu\lambda = 2d \sin \theta,$$

where d was taken as 2.814×10^{-8} Å. The calibration was always checked by setting on the K_α lines of tungsten.

TABLE I.

λ	μ/ρ for					
	H_2O	Al	Cu	Mo	Ag	Pb
.09 Å	.165					
.120	.175					
.135	.179	.193	.59	1.51		
.184	.199	.239	1.25	3.27	4.26	3.20
.209	.207	.276	1.74	4.55	5.94	5.11
.234	.220	.328	2.31	6.15	9.50	7.00
.258	.234	.393	3.00	8.10	12.5	9.35
.283	.249	.469	3.87	10.75	15.6	12.0
.308	.266	.567	4.87	13.5	19.5	15.5
.332	.284	.670	6.02	16.7	23.6	19.3
.356	.306	.795	7.29	19.8	28.5	23.7
.368	.319	.861	8.10	21.7	31.0	26.3
.381	.335	.940	8.85	23.7	34.0	29.0
.393	.348	1.017	9.64	25.6	37.0	31.9
.405	.358	1.095	10.2	27.5	39.2	34.5
.417	.376	1.174	11.3	29.5	43.7	37.9
.430	.394	1.285	12.3	31.5	47.8	41.4
.453		1.477	13.4	37.3	56.0	47.0
.479		1.706	16.6	42.5		55.0
.503		1.95	19.0	49.4	11.5	60.8
.551		2.49	24.9	63.8	15.1	73.0
.600		3.18	31.6	80.7	19.6	
.650		4.05	39.5	15.1	24.3	
.715		5.32	52.8	19.9		
.743		5.83		22.6		
.798				27.2		
.847				32.3		
.895				36.0		
.943				43.4		

Data and Results.

The following substances have been studied: water, aluminum, copper, molybdenum, silver, and lead, over wave-lengths ranging from .093 Å. to .95 Å. Owing partly to experimental difficulties, however, not all of these substaces have been studied over the entire range. Thus measurements on lead were not made at wave-lengths greater than .550 Å. because of the difficulty of making homogeneous lead foil of the order of .001 in. thick, to say nothing of the difficulty of measuring, to 1 per cent., the thickness after it is made. And in passing, it may be remarked that the securing of homogeneous absorbing films and of measuring their mass per square centimeter, constituted one of the serious difficulties of this investigation.

Table I. and Figs. 2 to 8 show the values of μ/ρ as a function of wave-length for the substances studied. In the table, the data given for aluminum are actual observed points. Because of the fact that measurements were not in general made at the same wave-length settings for all the substances, some of the values given for the substances other than aluminum were read from curves plotted through observed points. Figures 2–8 however are plotted from actual observations, and show μ/ρ plotted as a function of both λ and λ^3. With few exceptions the values of μ/ρ are accurate to one per cent., as may be judged from the smoothness of the curves.

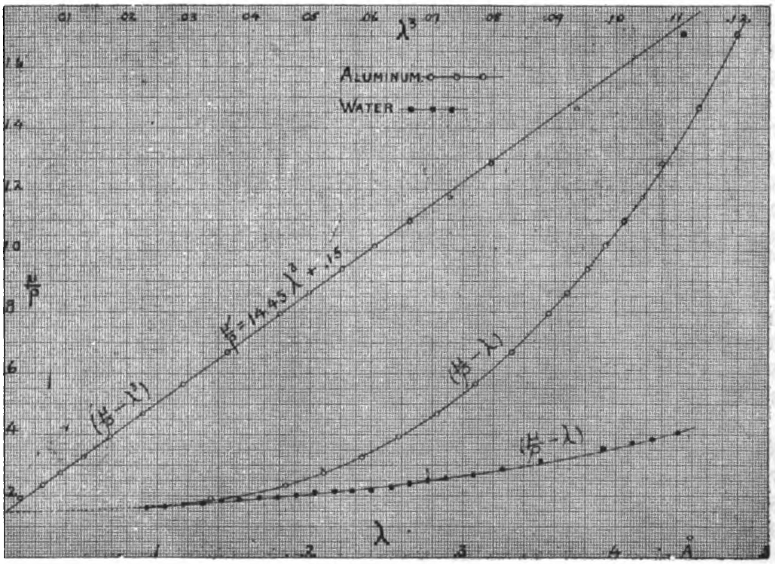

Fig. 2.

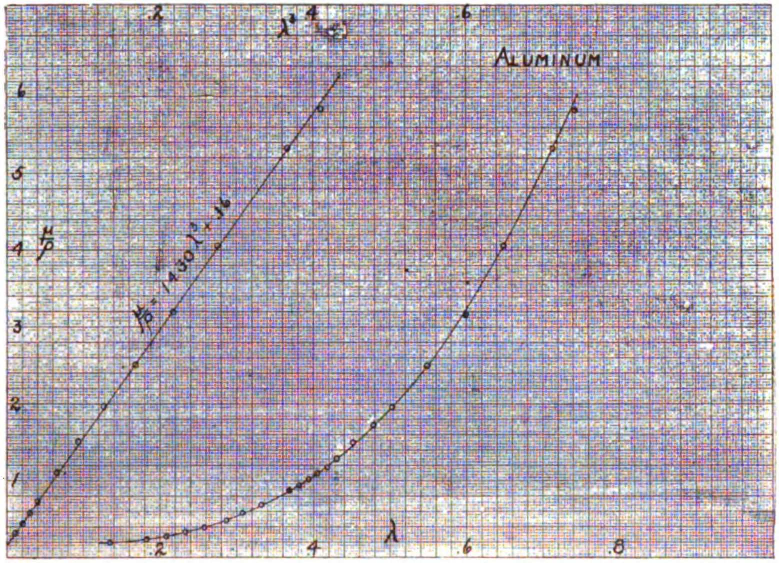
.2
λ³ .4
.6
ALUMINUM
6
5
4
μ/ρ
3
2
1
μ/ρ = 14.30 λ³ + .16
λ
.2
.4
.6
.8

Fig. 3.

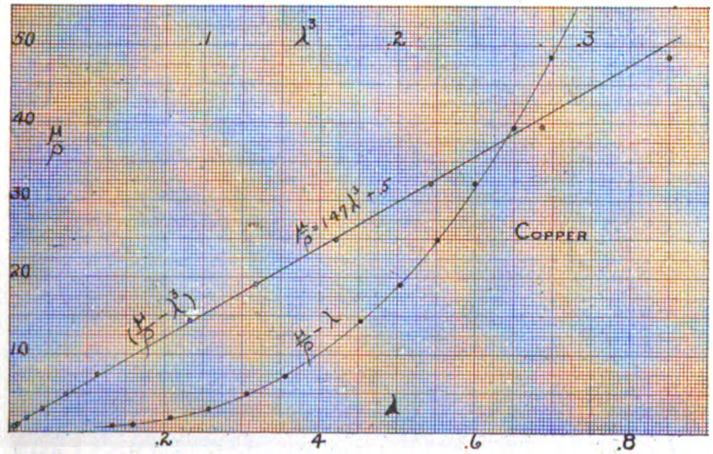
50
.1
λ³
.2
.3
40
μ/ρ
30
μ/ρ = 147 λ³ + .5
COPPER
20
(μ/ρ − λ³)
μ/ρ − λ
10
λ
.2
.4
.6
.8

Fig. 4.

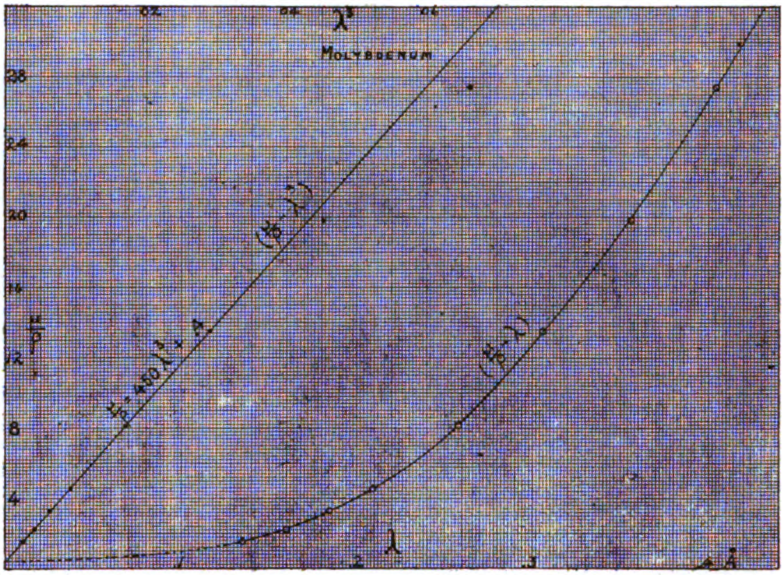
.02
.04
λ^3
.06
MOLYBDENUM
28
24
20
16
12
8
4
$\frac{\mu}{\rho}$
$\frac{\mu}{\rho} = 450\lambda^3 + A$
$(\frac{\mu}{\rho} - \lambda^3)$
$(\frac{\mu}{\rho} - \lambda)$
λ
.1
.2
.3
Å

Fig. 5.

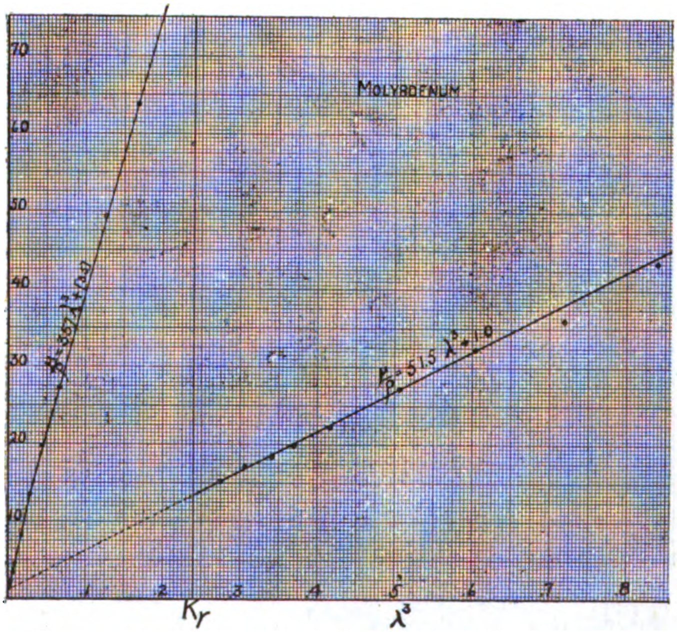
70
60
50
40
30
20
10
MOLYBDENUM
$\frac{\mu}{\rho} = 357\lambda^3 + (3.3)$
$\frac{\mu}{\rho} = 51.5\lambda^3 + 1.0$
.1
.2
.3
.4
.5
.6
.7
.8
K_γ
λ^3

Fig. 6.

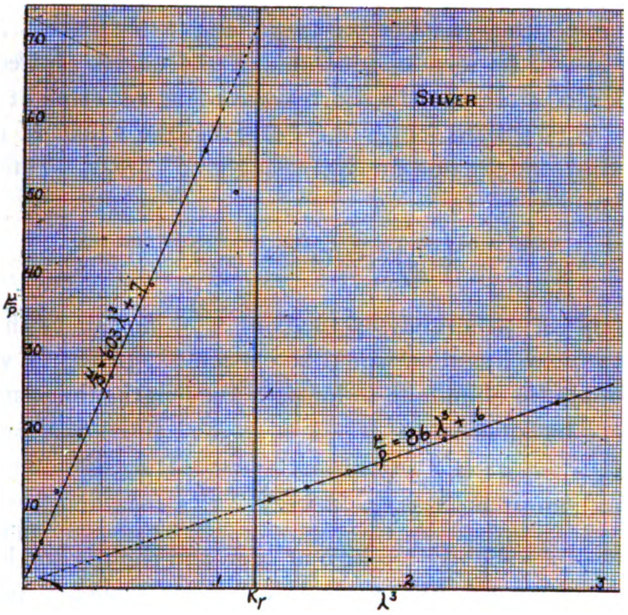
SILVER
70
60
50
40
30
20
10
$\frac{\mu}{\rho}$
$\frac{\mu}{\rho} = 603\lambda^3 + 7$
$\frac{\mu}{\rho} = 86\lambda^3 + 6$
.1
K_r
λ^3
.2
.3

Fig. 7.

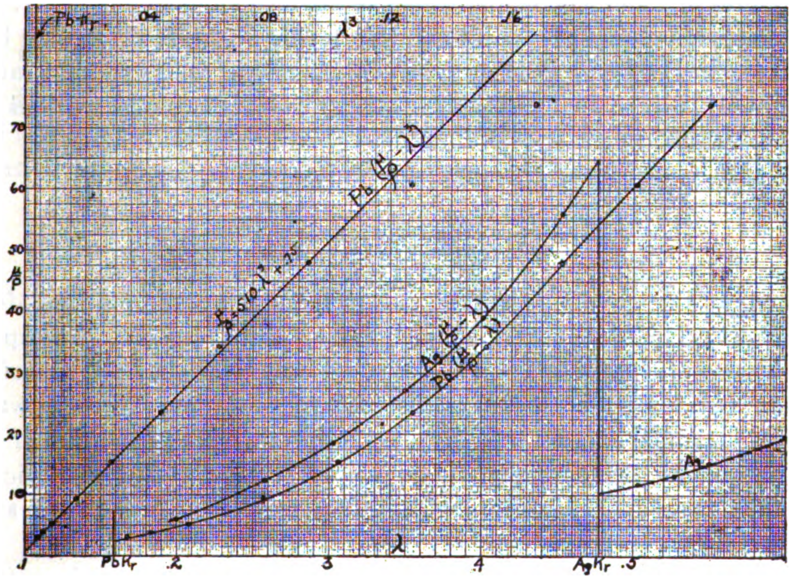
Pb K_r
.04
.08
λ^3
.12
.16
70
60
50
40
30
20
10
$\frac{\mu}{\rho}$
$\frac{\mu}{\rho} = 510\lambda^3 + 7.5$
Pb $(\frac{\mu}{\rho} - \lambda)$
Ag $(\frac{\mu}{\rho} - \lambda)$
Pt $(\frac{\mu}{\rho} - \lambda)$
Ag
λ
.1
PbK_r
.2
.3
.4
AgK_r
.5

Fig. 8.

CONCLUSIONS.

I. *Variation of μ/ρ with Wave-length and with Atomic Number of Absorber.*—It will be seen in general that the curves plotted with μ/ρ as a function of λ^3 are, at the shorter wave-lengths, straight lines; but that as one goes toward longer wave-lengths, the values of μ/ρ as predicted by this straight line are increasingly too great. Thus in Fig. 2 the equation

$$\left(\frac{\mu}{\rho}\right)_{\mathrm{Al}} = 14.45\lambda^3 + .15$$

represents μ/ρ very accurately up to about .4 Å., but above that value of λ values of μ/ρ are less than would be expected. In Fig. 3, which is an extension of μ/ρ for aluminum to longer wave-lengths the same tendency is noticed. The equation

$$\left(\frac{\mu}{\rho}\right)_{\mathrm{Al}} = 14.30\lambda^3 + 16$$

gives values of μ/ρ slightly too high at .4 Å. and slightly too low at .7 Å.

Reference to copper, Fig. 4, shows the same tendency to concavity toward the λ^3 axis. This concavity is still more marked in the case of substances of higher atomic number, as is shown by Figs. 5 and 7 for molybdenum and silver respectively.

Figures 6, 7 and 8 show the values of μ/ρ for molybdenum, silver, and lead respectively on the long wave-length side of the K absorption line. Here again the linear relation between μ/ρ and λ^3 seems to hold rather accurately, except that in the case of lead the last two observations fall very much below the straight line.

It may be concluded therefore that the general equation

$$\frac{\mu}{\rho} = F\lambda^3 + \frac{\sigma}{\rho} \qquad (1)$$

represents the law of absorption, provided one is not approaching, from the short wave-length side, a characteristic absorption band. To express the law accurately there must be deducted from the right hand side of (1) a term involving some unknown function of λ/λ_K where λ_K is the wave-length corresponding to the K absorption limit. Keeping in mind this limitation the following equations give the mass-absorption coefficient for the several substances studied, and the range over which the equation holds.

In order to ascertain how μ/ρ varies with the atomic number of the absorber it is necessary to discuss separately the two terms $F\lambda^3$ and σ/ρ. Since it is the atom as a whole which acts as a unit in absorption it is

TABLE II.

On Short Wave-length Side of the Respective K Absorption Limit.

Aluminum.....	$\mu/\rho = 14.45\lambda^3 + .15$	.1Å to .4Å
	$\mu/\rho = 14.30\lambda^3 + .16$	.4 to .7
Copper........	$\mu/\rho = 147\lambda^3 + .5$	.1 to .6
Molybdenum ..	$\mu/\rho = 450\lambda^3 + .4$	.1 to 35
Silver[15]........	$\mu/\rho = 603\lambda^3 + .7$	.1 to .4

On Long Wave-length Side of the K Absorption Limit.

Molybdenum ..	$\mu/\rho = 51.5\lambda^3 + 1$	
Silver[1]	$\mu/\rho = 86\lambda^3 + .6$	
Lead..........	$\mu/\rho = 510\lambda^3 + .75$	

λ is expressed in Ångström units.

first necessary to reduce values of μ/ρ, in which absorption is given for a layer of material thick enough to contain 1 gram per sq. cm., to the atomic absorption coefficient, μ_a, for a layer containing one atom per sq. cm. This is done by dividing both sides of equation (1) by n/A, the number of atoms per gram, where n is Avogadro's constant 6.06×10^{23}, and A is the atomic weight. This gives

$$\mu_a = \frac{\mu/\rho}{n/A} = \frac{A}{n} \cdot F\lambda^3 + \frac{A}{n} \cdot \frac{\sigma}{\rho}, \tag{3}$$

$$= F_a\lambda^3 + \sigma_a \tag{4}$$

where $F_a = F(A/n)$, and $\sigma_a[= (\sigma/\rho) \cdot (A/n)]$ is the atomic scattering coefficient.

Figure 9, plotted from the last two columns of Table III. shows the relation between F_a and the atomic number of the absorber. The slope

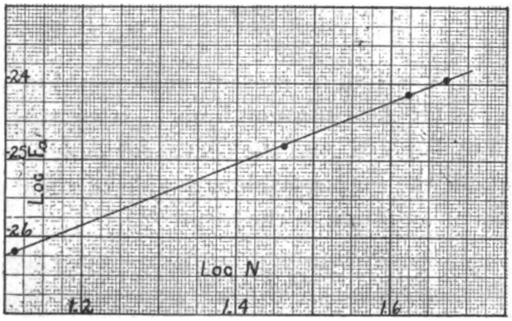

Fig. 9.

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[1] Data on Silver is approximate, since measurements were not made as carefully as on other substances.

TABLE III.

Substance.	At. Number.	F.	$F_a = FA/n$.	Log N.	Log F_a.
Al	13	14.45	64.5×10^{-23}	1.114	−27.810
Cu	29	147	1,550	1.462	−25.190
Mo	42	450	7,130	1.623	−25.854
Ag	47	650[16]	11,600	1.672	−24.053

of the curve is 4.00. We may accordingly write

$$F_a = kN^4. \tag{5}$$

The mean value of k for these four elements is 2.29×10^{-27}. Putting this value of F_a in equation (4) gives

$$\mu_a = 2.29 \times 10^{-27} N^4 \lambda^3 + \sigma_a \tag{6}$$

as the value of the atomic absorption coefficient as a function of atomic number and wave-length, *provided* the values of λ are well removed (*i.e.*, on the short wave-length side) from the K characteristic absorption limit. Although of numerical significance only, it may be observed that this value of k is about one third of Planck's constant h. So that for keeping the equation in mind we may write, approximately

$$\mu_a = \frac{h}{3} N^4 \lambda^3 + \sigma_a. \tag{6'}$$

Equation (5) agrees with the conclusions of Owen,[5] who, however, worked at only one wave-length.

It must be kept in mind, as pointed out above, that (6) does not represent the value of μ_a as the absorption limit is approached. To make the equation complete we should have to write

$$\mu_a = kN^4 \lambda^3 + \sigma_a - f\left(\frac{\lambda}{\lambda_K}\right), \tag{7}$$

where $f(\lambda/\lambda_K)$ is a function, at present unknown, but which increases, perhaps rapidly, as the K absorption limit λ_K is approached.

An inspection of Table II. and the curves for molybdenum, silver, and lead shows that on the long wave-length side of their characteristic K absorption limits an equation of the form

$$\frac{\mu}{\rho} = F'\lambda^3 + \frac{\sigma}{\rho} \tag{8}$$

expresses the relation between μ/ρ and λ. Data is not sufficiently accurate (on account of the very thin absorbing sheets necessary) to deter-

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[1] Taken from the short wave-length data for Ag.

mine how F_a' depends on atomic number. But a plot, similar to that for F_a shows that

$$F_a' = k'N^{4.4}.$$

All that can be said is that the exponent of N is at least of the same order as that in the equation for F_a.

II. *The Value of the Mass Scattering Coefficient.*—From Fig. 2 it is seen that the value of the mass scattering coefficient of aluminum, σ/ρ, is .15. Data for the remaining elements is not sufficiently accurate to determine values of σ/ρ except to say that there seems to be a tendency for an increase in σ/ρ with increasing atomic number of absorber. Much more data at very short wave-lengths is necessary to determine how σ/ρ varies with N.

But it is to be noted, that, so far as data goes, the value of σ/ρ as determined from data on silver and molybdenum is to a first approximation the same on the long wave-length side of the K absorption limit as on the short wave-length side, for both curves cut the σ/ρ axis at approximately the same point.

The importance of this is shown in Fig. 10, in which the value of μ/ρ is shown diagrammatically as a function of λ^3. Allow X-rays of wave-length λ_1 to fall on a *thin* sheet of the absorber whose characteristic K limit is λ_K. The vertical line ad is proportional to the total energy removed from the transmitted beam. Of this energy an amount proportional to ab is re-radiated as K fluorescent radiation; an amount proportional to bc, as L fluorescent radiation; and an amount proportional to cd is scattered without change of wave-length. Subject therefore to the correction term mentioned in equation (7) the mass absorption coefficient for a given substance is given by

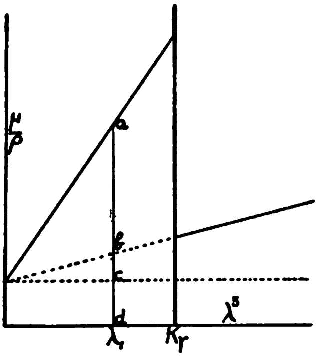

Fig. 10.

$$\frac{\mu}{\rho} = K\lambda^3\Big]_0^K + L\lambda^3\Big]_0^L + \frac{\sigma}{\rho}. \tag{9}$$

The term $K\lambda^3]_0^K$ is to be used from $\lambda = 0$ to the K absorption limit and determines the K fluorescent radiation; the term $L\lambda^3]_0^L$ is to be used up to the first L absorption limit. L itself is probably made up of several terms.

In spite of the correction term mentioned in equation (7) these laws of X-ray absorption are exceedingly simple as compared with the corresponding laws in the ultra-violet and visible region of the spectrum. It may not be too much to hope that a gradual extension of these laws to longer and longer wave-lengths may lead to some clue as to absorption of visible radiation.

III. *Is there Evidence of a Discontinuity in Absorption Curves Indicative of a Characteristic "J" Radiation?*—Table IV. gives the location of the discontinuities in the curves for μ/ρ as observed by various investigators.

TABLE IV.

N	**Element.**	**Barkla and White.**[10]	**Williams.**[11]	**Owen.**[12]	**Dauvillier.**[13]
6......	C	.42 Å.	—	—	—
7......	O	.39	—	—	—
13......	Al	.37	.49 Å.	—	.358
14......	Si	—	—	.493	—
29......	Cu	—	.448	—	—
35......	Br	—	—	—	.227
47......	Ag	—	—	—	.178

Barkla and White's data is shown in Fig. 11.[17] They assume that the absorption of water is due mainly to oxygen; and of paraffin mainly to carbon. In the present paper no data is shown for carbon, but reference to Fig. 2 will show that there is no discontinuity in water at .39 Å.

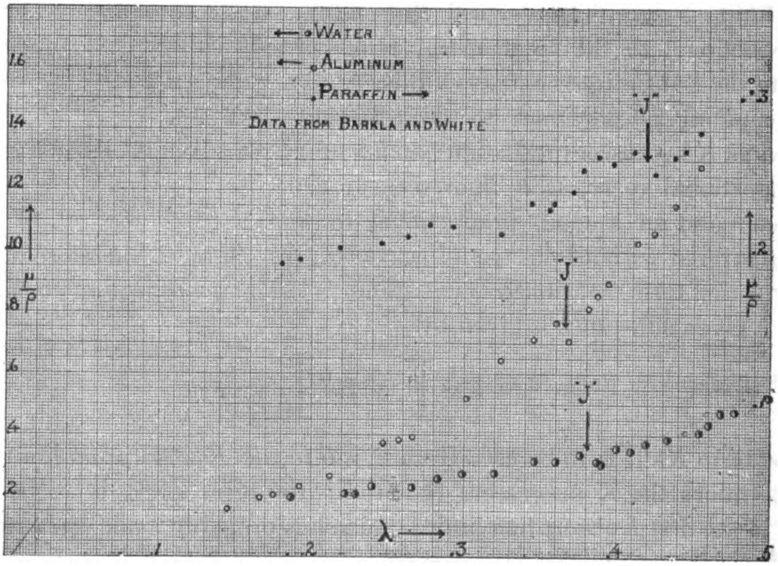

Fig. 11.

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[17] The plotting is my own. F.K.R.

Williams concluded that Barkla's discontinuity for aluminum was at too short a wave-length, and should be at .49 Å. Fig. 3 shows no discontinuity in aluminum at this wave-length, nor is there any apparent discontinuity in copper at .448 Å. as will be seen from Fig. 4. It is suggested that these apparent discontinuities were due to a rather large experimental error, combined with an accidental grouping of points. Thus Barkla and White's data would show apparent discontinuities at several other wave-lengths than those mentioned.

Dauvillier's apparent discontinuity in aluminum at .358 seems to be due to the *K* absorption limit of iodine in his ionization chamber, for no discontinuity is to be observed at this point in Fig. 2. His "*J*" for bromine is identical with Duane's α_3 line for tungsten. I have been unable to trace the origin of his suggestion of a "*J*" for silver at .178 Å. But Dauvillier states that if the square root of the frequencies for these three "*J*'s" be plotted, after Mosley, against atomic number, a straight line results. Such a graph predicts a discontinuity for copper at .250 Å. and for molybdenum at .196 Å. An inspection of Figs. 4 and 5 will show that there are no discontinuities for these elements at these points. The conclusion is that there is no evidence in favor of discontinuities in absorption curves indicating a radiation in the X-ray region shorter than the *K* series.

THE MASS ABSORPTION COEFFICIENT OF WATER AT VERY SHORT WAVE-LENGTHS.

It is known that the value of μ/ρ for the hard rays from radium *C* is of the order of .05 to .08 for various substances, much lower than would be expected by extrapolation to $\lambda = 0$ of the curves, say, in Fig. 2 for water and aluminum. The wave-length of these rays from radium *C* is not known but has been estimated by Rutherford and Andrade to be less than .01 Å. An attempt was made to see whether, in the case of water, there was any evidence of a downward bend in the curve, at very short wave-lengths, toward the value given for the rays from radium *C*. Although measurements were pushed down to .093 Å. there was no evidence of a downward bend. It is as if the mechanism operative at 0.1 Å. must differ materially from that which is responsible for the scattering of the much shorter rays from radium *C*. If we assume that the innermost group of electrons around the nucleus is responsible for the emission and absorption of *K* radiation, we must look to the nucleus itself for the emission and absorption (scattering) of still shorter groups of rays as the hard rays from radium *C* are thought to be. Rutherford has recently found evidence pointing to a structure of the nucleus, of elements up to oxygen, as made up of both positive and negative charges. Such a system should obviously both radiate and absorb. Perhaps further knowledge of the nucleus may throw light on the question of absorption of very short waves.

SUMMARY.

1. It has been shown that the mass absorption coefficient of Ag, Al, Cu, Mo, Pb and H_2O for X-rays is given in each case by an equation of the form

$$\frac{\mu}{\rho} = F\lambda^3 + \frac{\sigma}{\rho}, \tag{1}$$

so long as the values of λ are considerably smaller than the K absorption limit λ_K, and that as this limit is approached a term of the form $f(\lambda/\lambda_K)$ must be subtracted. This function is unknown.

2. Above the K absorption limit an equation of the same type holds, namely

$$\frac{\mu}{\rho} = F'\lambda^3 + \frac{\sigma}{\rho}. \tag{8}$$

3. If the atomic absorption coefficient μ_a be computed, equation (1) becomes

$$\mu_a = F_a\lambda^3 + \sigma_a \tag{4}$$

and it has been shown that F_a is proportional to the fourth power of the atomic number, *i.e.*,

$$F_a = kN^4,$$

where the numerical value of k is 2.29×10^{-27}. This combines with (1) to give the atomic absorption coefficient of substances (from Al to Ag at least) by the equation

$$\mu_a = 2.29 \times 10^{-27} N^4\lambda^3 + \sigma_a.$$

4. The value of the mass scattering coefficient seems to be independent of wave-length, since to a first approximation at least the value of σ/ρ is the same, for each substance, in equations (8) and (1).

5. There is no evidence, on the basis of absorption measurements, for the existence of a "J" radiation.

6. The value of μ/ρ for water, down as far as .093 Å. shows no evidence of an approach to the corresponding value for the hard rays from radium C.

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THE EFFECT OF A UNIFORM MAGNETIC FIELD ON THE MOTION OF ELECTRONS BETWEEN COAXIAL CYLINDERS.

By Albert W. Hull.

Synopsis.

Equations of motion for electrons are developed starting from a cylindrical cathode and moving toward a co-axial cylindrical anode, in a uniform magnetic field parallel to the common axis. The electrons will reach the anode if the ratio of potential difference to magnetic field is greater than a critical value, and will fail to reach it if the ratio is less than this value.

In the case of a *small cathode* in the axis of an anode of radius R at potential V, the critical magnetic field is

$$H = \sqrt{\frac{8m}{e}}\frac{V^{1/2}}{R}.$$

For a *small anode* of radius r_0 at potential V, in the axis of a cathode of radius R_0, the critical field is

$$H = \frac{r_0}{R_0}\sqrt{\frac{8m}{e}}\frac{V^{1/2}}{R_0} + \frac{2mv_0}{eR_0}.$$

In this case the initial velocity v_0 of the electrons cannot be neglected. This equation also applies, with appropriate R_0 and e/m, to *positive ions* produced by electrons from an *internal cathode.*

If the radii of *both cylinders are large* the solution reduces to the familiar one of plane parallel plates.

The *equation of the path* of the electrons is deduced, on the assumption that the space charge distribution is the same as without magnetic field. The path is given by $r = R(\sin \frac{3}{2}\theta)^{2/3}$. This is a close approximation to the true path, as calculated from the space charge distribution recently worked out by Langmuir. Experimental curves showing current at constant potential as a function of magnetic field, for different anode diameters, voltages, filament temperatures, degrees of symmetry, etc., are in agreement with theory within the limit of experimental error.

The *internal cathode* tube offers a means of measuring e/m, and the *internal anode* a means of measuring the distribution of initial velocities.

Introduction.

THE motion of electrons in uniform electric and magnetic fields has been studied by many investigators.[1] A few special cases of non-uniform fields have also been investigated; for example, the magnetic field due to current in a long straight wire,[2] and the electric field between a straight filament and concentric cylinder. In the latter case the effect of the space charge of the moving electrons has been taken account of.[3]

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[1] See, for example, J. J. Thomson, Conduction of Electricity through Gases, second Ed., pp. 104–116.

[2] J. J. Thomson, l.c., p. 108; O. W. Richardson, Roy. Soc. Proc., 90, 174, 1914.

A case of special interest, which appears not to have been investigated, is that in which the electric field is radial and symmetrical about a straight line, and the magnetic field is uniform and parallel to this line. This applies to an evacuated tube containing a straight filament and a concentric circular cylinder, in the axis of a long straight solenoid. The theoretical conditions can be accurately realized, and the apparatus is easy to construct and operate.

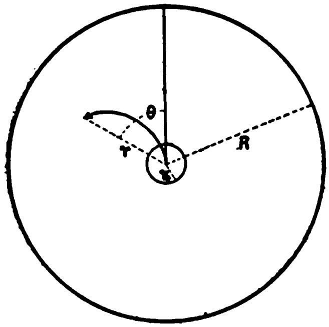

Fig. 1.
Small cathode in axis of cylindrical anode.

Relation between Magnetic Field, Voltage, and "Range" of the Electrons.—The equations of motion of an electron starting from a point at distance r_0 from an axis (Fig. 1), with initial velocity components

$$u_0 = \left(\frac{dr}{dt}\right)_0; \quad v_0 = r_0\left(\frac{d\theta}{dt}\right)_0; \quad w_0 = \left(\frac{dz}{dt}\right)_0;$$

and moving under the influence of a constant magnetic field of intensity H parallel to the axis, and a symmetrical radial electric field of intensity F (function of r only) are:

$$\frac{d^2r}{dt^2} - r\left(\frac{d\theta}{dt}\right)^2 = F\frac{e}{m} - H\frac{e}{m}r\frac{d\theta}{dt}, \tag{1}$$

$$\frac{1\,d}{r\,dt}\left(r^2\frac{d\theta}{dt}\right) = H\frac{e}{m}\frac{dr}{dt}, \tag{2}$$

$$\frac{d^2z}{dt^2} = 0. \tag{3}$$

Integrating (2) with respect to time gives:

$$\frac{d\theta}{dt} = \frac{r_0^2}{r^2}\left(\frac{v_0}{r_0} - \frac{He}{2m}\right) + \frac{He}{2m}. \tag{4}$$

The maximum radial distance from the axis that an electron can travel may be found either by equating the total work eV done by the electric force to the kinetic energy of tangential motion

$$\frac{1}{2}m\left(r\frac{d\theta}{dt}\right)^2,$$

given by Eq. 4; or by equating the radial velocity dr/dt to zero. The latter is found from integrating Eq. 1 with respect to r, which gives:

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[3] Langmuir, Phys. Rev., 2, 450, 1913; Schottky, Ann. d. Phys., 44, 1011, 1914.

$$\left(\frac{dr}{dt}\right)^2 = 2\frac{e}{m}V_r - H^2\left(\frac{e}{2m}\right)^2 r^2\left(1 - \frac{r_0^2}{r^2}\right)^2 - H\frac{e}{m}r_0v_0\left(1 - \frac{r_0^2}{r^2}\right) + v_0^2\left(1 - \frac{r_0^2}{r^2}\right) + u_0^2, \quad (5)$$

where

$$V_r = \int_{r_0}^{r} F dr$$

is the potential difference between the starting point r_0 and the point r.

Putting $dr/dt = 0$ gives the maximum distance r_m from the axis that the electron can reach, in terms of the potential V_r at this distance and the magnetic field H:

$$V_r = H^2\frac{e}{8m}r_m^2\left(1 - \frac{r_0^2}{r_m^2}\right)^2 + \left(\frac{Hr_0v_0}{2} - \frac{v_0^2}{2e/m}\right)\left(1 - \frac{r_0^2}{r_m^2}\right) - \frac{u_0^2}{2e/m}. \quad (6)$$

If a conducting cylinder of radius r_m is inserted in the position of the surface r_m, all the electrons will reach it if its potential is greater than V_r, and none will reach it if its potential is less than V_r. *Hence if one observes the current to the cylinder as a function of V with constant H, or as a function of H with constant V, this current should fall abruptly to zero at the critical values of V and H.* This is found to be true experimentally (Figs. 7–10).

The above equations refer to the motion of both electrons and ions, in either direction, starting at any point with any initial velocity. They become much simpler under practical conditions. Four special cases will be considered.

Case 1. Internal Cathode.—*Electrons start from a straight filament of radius r_0 and travel to a concentric cylindrical anode of radius R, large compared with the filament (Fig. 1).*

Equation 6 then becomes

$$V = H^2\frac{e}{8m}R^2 + H\frac{r_0v_0}{2} - \frac{u_0^2 + v_0^2}{2e/m}. \quad (7)$$

The last term on the right-hand side represents the initial energy of emission of the electrons, expressed in the same units as V.

If the initial energies are those of thermal emission, between zero and 2 volts, then the last two terms are negligible for the great majority of electrons, whose initial energies are less than 1/10 volt; and they are negligible even for electrons with the highest initial energies when V and H are large.

For example, in the case of a 4 mil filament in the axis of a 1 inch cylinder,

$$r_0 = .005 \text{ cm.}; \qquad R = 1.25 \text{ cm.};$$

$$\sqrt{u_0^2 + v_0^2} = 2 \times 10^7 \text{ (0.1 volt initial energy).}$$

If $H = 10$,

$$\left.\begin{aligned} V &= 3.5 \times 10^8 \pm 5 \times 10^5 - 10^7 \text{ E.M.U.}\\ &= 3.5 \pm .005 - 0.1 \text{ volts}\\ \text{If } H &= 100\\ V &= 350 + .05 - 0.1 \text{ volts,}\end{aligned}\right\} \text{initial energy } \frac{1}{10} \text{ volt,}$$

while for the very fastest electrons, with initial energies of 2 volts ($\sqrt{u_0^2 + v_0^2} = 8 \times 10^7$).

$$\left.\begin{array}{llll} \text{If } H = 10, & V = & 3.5 \pm .02 - 2 & \text{volts}\\ \text{If } H = 100, & V = & 350 \pm 0.2 - 2 & \text{''}\\ \text{If } H = 1000, & V = & 35000 \pm 2 - 2 & \text{''}\end{array}\right\} \text{initial energy 2 volts.}$$

It is seen that the term involving the first power of H is negligible under all conditions, so that Eq. 7 becomes

$$V = H^2 \frac{e}{8m} R^2 - \frac{1}{2}\frac{m}{e}(u_0^2 + v_0^2). \tag{7'}$$

The only correction due to initial velocities is the addition of the initial energy of emission, $\frac{1}{2}m(u_0^2 + v_0^2)$ to the work eV done by the electric field, as in all thermionic applications. This correction is negligible at reasonable voltages. Hence in all practical cases, for electrons moving from a straight filament to a concentric cylinder,

$$V = H^2 \frac{e}{8m} R^2 \tag{8}$$

or

$$H = \sqrt{\frac{8m}{e}}\, \frac{V^{1/2}}{R}. \tag{9}$$

V = potential difference between filament and cylinder.
R = radius of cylindrical anode.
H = magnetic field just sufficient to prevent electrons reaching anode.

The value of H corresponding to the middle of the steep portion of the $I - H$ curve (Figs. 7–10) can be accurately measured, so that Eq. 9 may be used for the experimental determination of e/m.

If e/m is taken as 1.77×10^7 E.M.U. and V is expressed in volts, H and R in Gauss and cm. resp., Eqs. (8) and (9) become

$$V = .0221\, H^2 R^2, \tag{8)'}$$

$$H = 6.72 \frac{V^{1/2}}{R}. \tag{9)'}$$

Case 2. *Internal Anode.—Electrons start from an outer cylinder of radius R_0 and move toward a concentric inner cylinder or wire of radius r_0, small compared with R_0 (Fig. 2).*

Eq. 6 becomes in this case:

$$V = \beta^2 \frac{e}{8m}\left(HR_0 - 2\frac{m}{e}v_0\right)^2 \quad (10)$$

or

$$H = \frac{1}{\beta R_0}\sqrt{\frac{8m}{e}}\,V^{1/2} + \frac{2mv_0}{eR_0}. \quad (11)$$

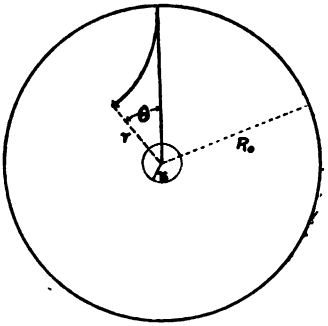

Fig. 2.

Small anode in axis of cylindrical cathode.

V = potential difference between outer and inner cylinders, reckoned positive when the inner cylinder is positive.
R_0 = radius of outer cylinder.
H = magnetic field just sufficient to prevent electrons reaching inner cylinder.
β = ratio of radii of outer and inner cylinders.
v_0 = tangential initial velocity of electrons.

For electrons with very small tangential initial velocities this becomes:

$$V = \beta^2 \frac{e}{8m} H^2 R_0^2. \quad (12)$$

This is the same as Eq. 8 for electrons starting from the filament, except for the factor β^2. The voltage against which the electrons can travel in the same magnetic field is β^2 times greater than in case 1; or, conversely, with the same potential difference between cathode and anode, the magnetic field required to prevent electrons reaching the anode is only $1/\beta$ as great with internal anode as with internal cathode. This is found to be true experimentally (see Figs. 15–20).

The effect of initial velocities is much greater in this case, however, than in the case of internal cathode, so that in practical cases only a very small fraction of the electrons have a sufficiently small initial velocity to obey Eq. 12. For the rest the critical magnetic field will depend on the initial velocity, and the curves representing current as a function of magnetic field will not be steep, as in the case of electrons starting from the filament, but will fall off slowly (see Figs. 15–20).

The magnitude of the correction for initial velocities can best be illustrated by an example. If the diameters of inner and outer cylinders are in the ratio 1 : 25, viz., 12 mils (r_0 = .0152 cm.) and 3/10 inches

(R = 0.38 cm.) respectively, and the potential difference 50 volts, Eqs. 7 and 11 give, for the cases of external anode and internal anode respectively,

$$H_1 = \frac{\sqrt{8m/e}}{R}\sqrt{V + \frac{1}{2}\frac{m}{e}(u_0^2 + v_0^2)}$$

$$= 17.7\sqrt{50 + 2.82 \times 10^{-16}(u_0^2 + v_0^2)} \quad \text{(external anode)},$$

$$H_2 = \frac{\sqrt{8m/e}}{\beta R}V^{1/2} + \frac{2m}{eR}v_0 = 5 + 3 \times 10^{-7}\,v_0 \quad \text{(internal anode.)}$$

The following table gives the values of H_1 and H_2 for different values of v_0 (taking u_0 = 0).

v_0		H_1 (External Anode).	H_2 (Internal Anode).
Cm./sec.	Volts.	Gauss.	Gauss.
0×10^7..........	0.	125	5.0
1 "	.028	125 + .035	5.0 ± 3.0
2 "	.112	125 + .141	5.0 ± 6
3 "	.25	125 + .317	5.0 ± 9
4 "	.45	125 + .563	5.0 ± 12
5 "	.70	125 + .880	5.0 ± 15
6 "	1.01	125 + 1.27	5.0 + 18
7 "	1.37	125 + 1.72	5.0 ± 21
8 "	1.80	125 + 2.25	5.0 ± 24

r_0 = .0152 cm.; R = 0.38 cm.; β = 25, V = 50 volts.

Case 3. *Parallel Planes.—The radii of both cylinders are large compared with their distance apart.*

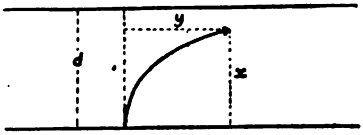

Fig. 3.

Plane parallel plates.

This applies to electrons moving between plane parallel plates, and includes the case of uniform electric and magnetic fields at right angles to each other. Putting

$$r - r_0 = x; \qquad r d\theta = dy; \qquad \frac{r}{r_0} = 1;$$

Eq. 6 becomes

$$V_x = H^2\frac{e}{2m}x^2 + Hv_0x - \frac{u_0^2}{2e/m}, \tag{13}$$

where x is the maximum distance that an electron starting from the plane x = 0, with initial velocity u_0 and v_0 in the x and y directions respectively, can travel from this plane in a magnetic field H parallel to Z; and V is the potential at the point x.

With large values of H and x the effect of initial velocities is small, and (13) becomes

$$V = H^2 \frac{e}{2m} d^2, \tag{14}$$

where d is the distance between plates, V the potential difference between them, and H the critical magnetic field that is just sufficient to prevent the electrons reaching the anode plate.

This value of V is 4 times greater than that against which electrons starting from a filament are able to travel the same distance in the same magnetic field (Eq. 8); or, for electrodes the same distance apart and at the same potential difference, only half as strong a magnetic field is required to turn electrons back in the case of parallel plates as is the case of filament and concentric cylindrical anode.

The effect of initial velocities is much greater in the case of the parallel plates, however, than for the filament. For example, for plates 1 cm. apart and electrons with 2 volts initial energy ($v_0 = 8 \times 10^7$), Eq. (13) gives

$$\left.\begin{array}{l} \text{for } H = 10,\ V = 8.8 \pm 8 - 2 \\ \text{“ } H = 100,\ V = 880 \pm 80 - 2 \end{array}\right\} \text{2 volt electrons}$$

while even for electrons of only 1/10 volt initial energy ($v_0 = 2 \times 10^7$)

$$\left.\begin{array}{l} \text{for } H = 10,\ V = 8.8 \pm 2 - 0.1 \text{ volts} \\ \text{“ } H = 100,\ V = 880 \pm 20 - 0.1 \text{ “} \end{array}\right\} \frac{1}{10} \text{ volt electrons.}$$

Hence, the curve representing current as a function of magnetic field will not be as steep in the case of plane parallel plates as in the case of filament and cylindrical anode.

CASE 4. *Positive ions are produced at a point r cm. from a straight filament where the potential is V with respect to the filament, with initial velocities u_0 and v_0, and move toward the filament in a magnetic field H.*

Equations (10) and (11) apply, with

$$\frac{e}{m} = \frac{10^4}{M},$$

where M is the molecular weight of the ion. The small value of e/m, as compared with that for electrons, more than offsets the factor β^2, so that a magnetic field which is capable of turning back electrons moving from a filament to concentric cylindrical anode will not be able to prevent the positive ions produced by these electrons from reaching the filament, except in the case of small diameter filament (large β) and ions of low atomic weight. The relative effect, upon the value of critical magnetic field, of initial velocities and voltage respectively is the same as in the case of electrons, for the same initial energy per unit charge. Hence the cutoff by a magnetic field of positive ions moving toward a filament will not be sharp except in the case of very strong magnetic and electric fields, and will be practically ineffective in the case of small diameter filaments and heavy ions.

PATHS OF THE ELECTRONS.

The phenomena discussed thus far are independent of the potential distribution in the tube. The complete determination of the paths of the electrons requires a knowledge of this potential distribution, which depends on the space charge of the electrons. Certain properties of the motion can be obtained, however, without this knowledge, and an approximate solution can be obtained on the assumption that the space charge distribution is the same as that which exists without the magnetic field. There is experimental evidence that this assumption is not far from the truth.

The angular velocity of the electron is given by Eq. 4 above;

$$\frac{d\theta}{dt} = \frac{r_0^2}{r^2}\left(\frac{v_0}{r_0} - \frac{He}{2m}\right) + \frac{He}{2m}. \tag{4}$$

In the case of an electron starting from a filament of radius r_0 with initial velocity

$$v_0 \leq r_0 \frac{He}{2m},$$

the first term on the right hand side becomes negligible compared with the second as soon as the electron has travelled a small distance r equal to about 10 times the radius of the filament. From this point on its angular velocity is constant and equal to $He/2m$. In the usual case this distance is of the order of 1/2 mm. For example, in the case of a 1 mil fil. (r_0 = .0012 cm.) and electrons with initial velocities of 2 × 10^6 cm./sec., if H = 200 gauss,

$$\frac{v_0}{r_0} = 1.7 \times 10^9; \qquad \frac{He}{2m} = 1.8 \times 10^9; \qquad 10r_0 = .012;$$

the electron acquires 99 per cent. of its maximum angular velocity within 1/10 mm. of the filament. Hence

$$\frac{d\theta}{dt} = \frac{He}{2m}\left\{\begin{array}{l} r > .01 \text{ cm.}, \\ H > \dfrac{2mv_0}{er_0}, \end{array}\right. \tag{15}$$

that is, for electrons starting with small initial velocity from a 1 mil filament, *except in the very small region within 1/10 mm. of the filament, that angular velocity of the electrons is constant and equal to He/2m.*

If an electron starts with initial angular velocity v_0/r_0 large compared with $He/2m$, this angular velocity will fall off rapidly at first, at a rate proportional to $1/r^3$, then less rapidly as it approaches the limiting value $He/2m$, and will not reach its limiting value in as short a distance as that given by Eq. (15). For example, in the case of the 1 mil filament and magnetic field of 200 gauss considered above, electrons with 1/10 volt initial energy ($v_0 = 2 \times 10^7$ cm./sec.) must travel 1 mm. before their angular velocity falls to within 1 per cent. of its constant value, and electrons with 2 volts initial energy ($v_0 = 8 \times 10^7$) must travel 4 mm. before their angular velocity is constant within 1 per cent.

From Eqs. 4 and 5 one obtains for the angle ϕ between the radius vector and the tangent to the path, neglecting initial velocities:

$$\tan \varphi = \frac{r d\theta}{dr} = \pm \frac{H \frac{e}{2m} (r^2 - r_0^2)}{\sqrt{2 \frac{e}{m} V r^2 - \left(H \frac{e}{2m} \right)^2 (r^2 - r_0^2)}}. \tag{16}$$

Since V is a function of r only, this has the same numerical value for an electron traveling in toward the axis as for one traveling out, hence *the electrons return in paths of exactly the same shape as those in which they go out.* This shows that the electrons, if they fail to reach the anode, do not continue circling around the cathode in approximately circular orbits forever (or until they strike gas molecules), but return periodically to the cathode. The total angle they describe before returning depends on the distribution of electric intensity as a function of r. If the space charge distribution were the same as without magnetic field this angle would be 270 degrees. The actual space charge, which has recently been calculated by Langmuir,[1] is slightly greater than without magnetic field and gives a total angle of 288.5 degrees.

There are two special cases of intensity distribution that lead to very simple solutions of the path. The first represents the initial condition, before space charge has had time to build up. The second represents very nearly the stationary condition.

CASE 1. *Space charge is negligible.*

The electric intensity is that due to the potential difference V between the cylinders, viz.,

$$F_r = \frac{V}{r \log \frac{R}{r_0}},$$

where R and r_0 are the radii of cylinder and filament respectively. If r_0

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[1] These calculations will be published soon by Dr. Langmuir.

is very small, the intensity is negligible except very close to the filament. The electron will therefore move with constant velocity, and its path will be a circle of radius

$$\frac{\sqrt{2\frac{m}{e}V}}{H}.$$

This solution represents the initial condition, before the electrons which fail to reach the cylinder have had time to pile up in the space in appreciable quantity. The steady state, however small the emission, must be that of full space charge.

CASE 2. *Space Charge the Same as Without Magnetic Field.*—Consider the case of electrons starting with zero initial velocities from a filament of very small diameter. The potential at any distance r from the axis is given by the equation[1]

$$i_0 = \frac{2\sqrt{2}}{9}\sqrt{\frac{e}{m}}\frac{V^{3/2}}{r}, \qquad (17)$$

where i_0 is the maximum electron current per cm. length from the filament to a concentric cylinder of radius r at potential V. Equations (4) and (5) give

$$\left.\begin{aligned} \left(\frac{d\theta}{dt}\right)^2 &= \frac{\alpha^2}{r^4}(r^2 - r_0^2), \\ \left(\frac{dr}{dt}\right)^2 &= 2\frac{e}{m}V - \frac{\alpha^2}{r^2}(r^2 - r_0^2) \end{aligned}\right\} ; \alpha = H\frac{e}{2m},$$

from which, if $r_0 = 0$,[2]

$$\frac{d\theta}{dr} = \frac{\alpha}{\sqrt{2e/m\,V - \alpha^2 r^2}}$$

and substituting the value of V from (17)

$$\frac{d\theta}{dr} = \frac{\alpha}{\sqrt{(9i_0e/m)^{2/3}r^{2/3} - \alpha^2 r^2}}. \qquad (18)$$

Eq. (18) may be written,

$$d\theta = \frac{3/2dx}{\sqrt{\beta^2/\alpha^2 - x^2}}, \text{ where } \left\{\begin{aligned} x &= r^{2/3}, \\ \beta^2 &= \left(9i_0\frac{e}{m}\right)^{2/3}, \\ \alpha &= \frac{He}{2m}. \end{aligned}\right.$$

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[1] Langmuir, PHYS. REV., 2, 455, 1913.

[2] The assumption that $r_0 = 0$ leads to no mathematical discontinuities. The solution thus obtained (Eq. 19) will therefore represent the actual solution, viz., the solution for finite r_0, to any desired degree of accuracy, if r_0 is taken sufficiently small.

The integral of this is

$$\theta = \frac{3}{2}\sin^{-1} x\frac{\alpha}{\beta} + \theta_0 .$$

Substituting the values of α, β and x, and determining the constant θ_0 from the condition that θ shall be zero when r is zero,[1]

$$\theta = \frac{3}{2}\sin^{-1}\left(\sqrt[3]{\frac{e^2}{72 i_0 m^2}}\, H r^{2/3}\right)$$

or

$$r = 6\sqrt{2 i_0}\,\frac{m}{e}\,H^{-3/2}\left(\sin\frac{2}{3}\theta\right)^{3/2}, \tag{19}$$

substituting for i_0 its value from Eq. 17 in terms of the diameter R and potential V_0 respectively of the outer cylinder,

$$r = 4\sqrt[4]{2}\left(\frac{m}{e}\right)^{3/4}\frac{V^{3/4}}{R^{1/2}H^{3/2}}\left(\sin\frac{2}{3}\theta\right)^{3/2}. \tag{20}$$

If $r_{\max}$ represents the maximum value of r in Eq. 20, that is the greatest radial distance the electron can travel in magnetic field H when the potential on the anode, of radius R, is V_0, Eq. (20) may be written

$$r = r_{\max}\left(\sin\frac{2}{3}\theta\right)^{3/2}, \tag{21}$$

where

$$r_{\max} = 4\sqrt[4]{2}\left(\frac{m}{e}\right)^{3/4}\frac{V^{3/2}}{R^{1/2}H^{3/2}}.$$

When $r_{\max} = R$, *i.e.*, when the electrons can just reach the outer cylinder, this reduces to

$$R = \sqrt{\frac{8m}{e}\,\frac{V^{1/2}}{H}},$$

which is identical with Eq. (9).

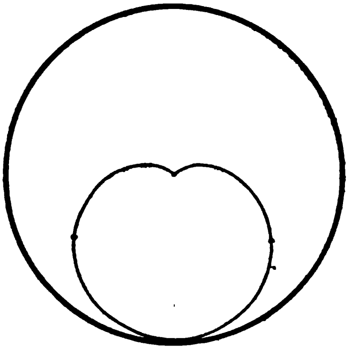

Fig. 4.

Plot of approximate equation of path, $r = R[\sin^{3/2}\theta^{3/2}]$.

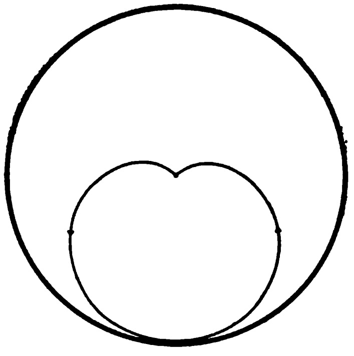

Fig. 5.

Exact path of electron (Langmuir's solution).

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[1] See footnote 2, previous page.

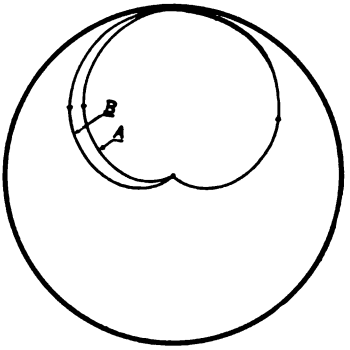

Fig. 6.

Path of electron. Degree of approximation represented by Eq. 21.

A, approximate path, Eq. 21; *B*, exact path (Langmuir).

The path given by Eq. 20 is shown in Fig. 4. The exact path, as calculated by Langmuir, is shown in Fig. 5. For the purpose of comparison the two are plotted together in Fig. 6. The difference is inappreciable up to $r = 9/10\ r_{\max}'$, *i.e.*, Eq. 21 represents the true path very closely except in the region near the maximum range. In this region the radial velocity is less than that given by Eq. 21, making the total angle traversed per cycle greater, viz., 288.5 degrees instead of 270 degrees.

EXPERIMENTS.

The experimental work was done in coöperation with Mr. E. F. Hennelly. More than 1,000 tubes have been studied, of various shapes and sizes, all conforming as closely as possible to the symmetry conditions postulated above. The following examples are typical:

A. Internal Cathode. Straight Filament in Axis of Cylinder. General Characteristics.—The diameter of the filament is always small compared with that of the anode, so that r_0^2/R^2 is negligible compared with unity. Hence, the relation between critical magnetic field, voltage, and anode radius is that given by equation (9):

$$H = \sqrt{\frac{8m}{e}}\frac{V^{1/2}}{R}. \tag{9}$$

H is the value of the magnetic field that is just sufficient to prevent electrons from reaching an anode of radius R at potential V with respect to filament. If V is kept constant and H gradually increased, the current should remain constant until H reaches the critical value given by equation (9), and should then fall abruptly to zero.

Figure 7, for which I am indebted to Dr. K. H. Kingdon, shows a typical example. The tube contained a straight tungsten filament .012 cm. in diameter, about 1 mm. from the axis of an anode 3.86 cm. in diameter, 12 cm. long. A rotating commutator applied voltage and heating current alternately, so that measurements were made while the filament was an equipotential surface. The potential difference between cathode and anode was 50 volts, and the filament temperature high enough so that the current was limited only by space charge.

It is seen that the current maintains nearly its full space charge value until the critical magnetic field is reached, and then falls very abruptly. The steepness is as great as to be expected from the degree of symmetry, *i.e.*, with filament 1 mm. out of center. The effect of high initial velocities is observable near the bottom of the curve.

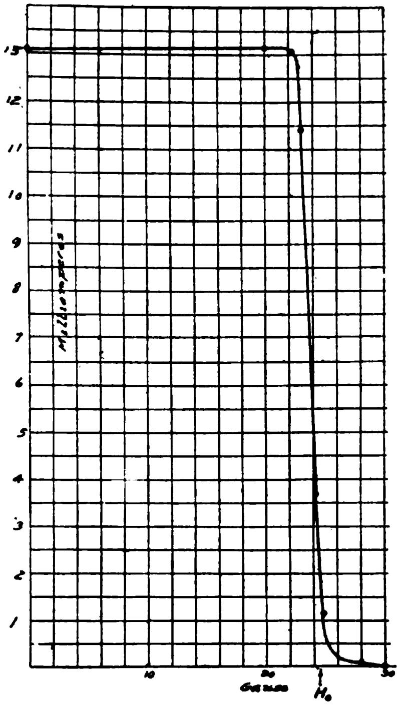

Fig. 7.

Typical Gauss-Ampere characteristic at 50 volts.

The theoretical "cut off" field is shown by the dotted curve, as calculated from Eq. 11, viz.,

$$H = 6.72\sqrt{\frac{50}{1.93}} = 24.56 \text{ Gauss.}$$

Effect of Heating Current through Filament.—The characteristic of Fig. 7 was taken with a rotating commutator, adjusted to cut off the heating current during the brief intervals when measurements were being made. If measurements are made while the heating current is flowing, the curves are less steep, due to the voltage drop along the filament. This effect is more marked at low than at high anode voltages. Most of the data which follows was taken with filament current flowing.

No effect due to the magnetic field of the heating current has been observed.

Effect of Anode Voltage.—Fig. 8 shows the characteristic of a tube with anode 3/4 in. diameter × 1½ in. long and cathode 4 mils diameter, at three different anode potentials, viz., 10.2, 21.7, and 57 volts. These measurements were taken by Dr. K. H. Kingdon with rotating commutators.

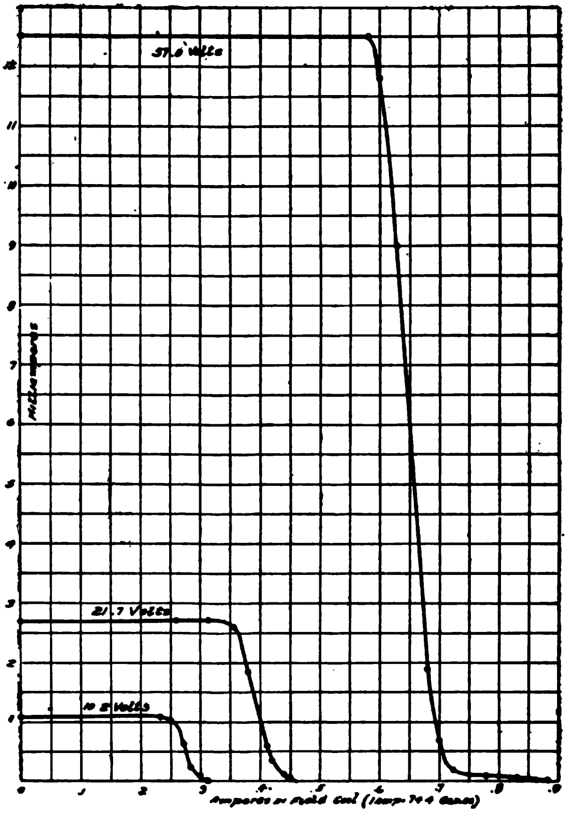

Fig. 8.

Characteristic of ¾ inch dia. anode, at different potentials.

Fig. 9 shows the $I - H$ characteristic of a tube with 1 in. × 4 in. anode and 10 mil cathode at four different anode voltages, 60, 125, 200, and 250 volts. The potential drop in the filament was 10 volts, and anode potentials were measured from the negative end of the filament. Hence in the 60-volt test, one end of the filament was at 50 volts with respect to the anode, and the other at 60.

It will be observed that the maximum currents in Figs. 8 and 9 are in the ratio of the 3/2 power of the voltage, and the critical magnetic fields are in the ratio of the 1/2 power of the voltage.

Figs. 10 and 11 show the lower portions of curves taken at higher potentials. The anode of Fig. 10 was 2 in. diam. × 2 in. long, that of Fig. 11 was 1 in. diam. × 4 in. long. The upper portions of these curves could not be obtained because of the excessive heating of the anode. The full current at 9,000 volts would have been 100 amps. Its reduction to 35 milliamperes was due solely to the magnetic field.

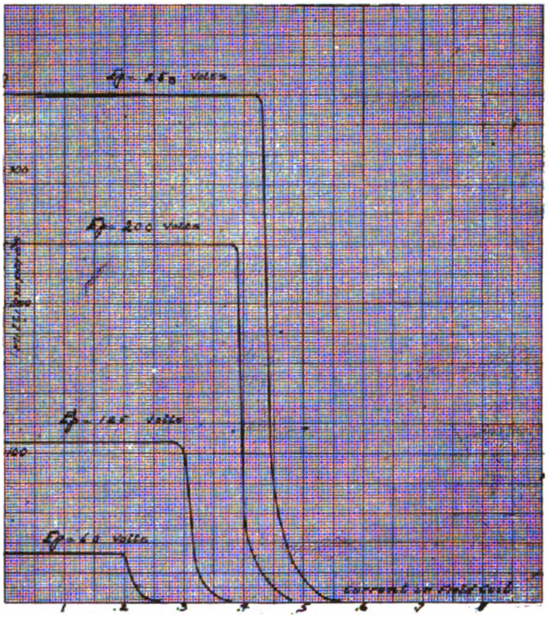

Fig. 9.

Characteristic of 1 inch dia. anode, at different potentials.

Effect of Filament Temperature.—Fig. 12 shows the characteristics of a tube of the same construction as that of Fig. 9, at three different filament temperatures. The potential difference was 250 volts. The steep portions of the curves are identical. There is a slight transition region between the horizontal and vertical portion of each curve. In the upper curve the current is limited by space charge, in the other two by filament temperature.

Effect of Diameter of Anode.—Fig. 13 shows the characteristics, at 250 volts, of 4 tubes of different diameters, from 2 inches to 3/8 inch. In order to bring them within the limits of the same plot, the currents in the smaller tubes have been limited by filament temperature to the same maximum value as for the largest diameter. The critical magnetic fields are proportional to the diameters within the limits of accuracy with which these diameters were known.

Effect of Symmetry.—Symmetry, both of electric and magnetic field, is essential for sharp cut-off. A numerical estimate of the effect of asymmetry is difficult, but an example will illustrate it. Fig. 14 shows two characteristics taken with the same tube. In curve *A* the filament was parallel to the magnetic field; in curve *B* the magnetic field was inclined 20 degrees to the filament.

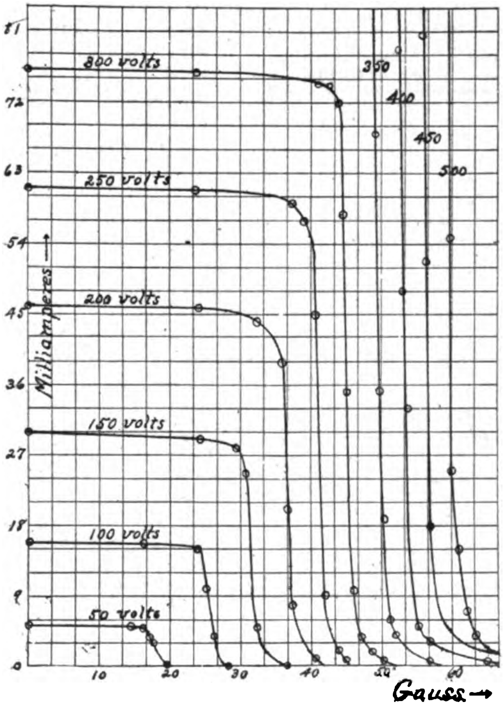

Fig. 10.

Characteristic of 2 inch dia. anode, at different potentials.

The effect of small pieces of iron in the magnetic field is very striking. With the magnetic field adjusted to the critical value, the point of a pen brought near the tube is sufficient to reduce the current to half value.

Effect of Gas.—The equations for cut-off are independent of potential distribution in the tube. The cut-off should not be affected, therefore, by the presence of positive ions, even in sufficient numbers to completely neutralize the space charge of the electrons, provided the mean free path is large compared with the dimensions of the tube. Hence with low gas pressure one should observe currents which are not limited by space charge, but are completely controllable by a magnetic field. At higher gas pressures the magnetic field becomes less and less effective, due to the fact that every electron which collides with a molecule will suffer a change, in direction at least, of its velocity, and a large fraction of these velocity changes will be in such direction as to require a stronger field to keep the electrons from the anode. It is probable, also, that the impact of positive ions on the cathode gives rise to the emission of electrons of relatively high initial velocity, some of which, according to Eq. 7, require a stronger field to turn them back.

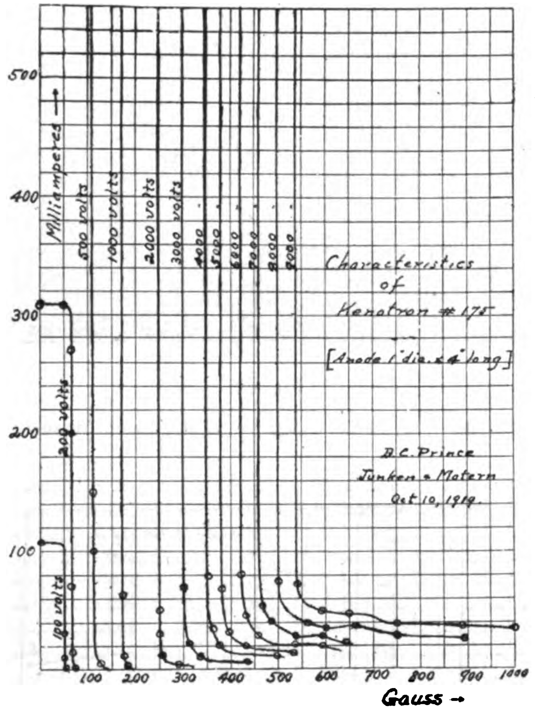

Fig. 11.

Characteristic of 1 inch dia. anode, at high potentials.

Figure 15 shows the typical behavior of tubes containing small amounts of gas. Curve *A* is the Gauss-ampere characteristic in good vacuum. The maximum current is limited by space charge, which increases slightly, with corresponding decrease in current, as the critical magnetic field is approached. The steepness of cut-off is limited mainly by symmetry and potential drop along the filament. Curve *B* shows the effect of a few hundredths of a bar of gas. The space charge of electrons is partially neutralized by positive ions so that the maximum current is slightly larger, especially in the neighborhood of critical magnetic field, where the paths of the electrons are longer and the ionization greater. The cut-off

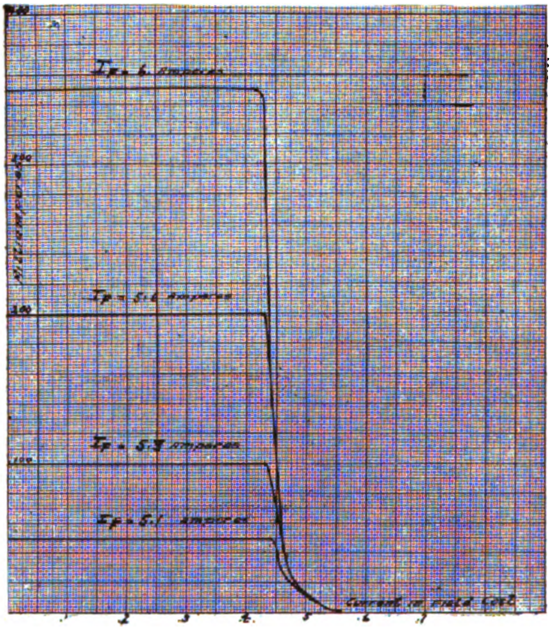

Fig. 12.

Characteristic of 1 inch dia. anode, at 250 volts, at different filament temperatures.

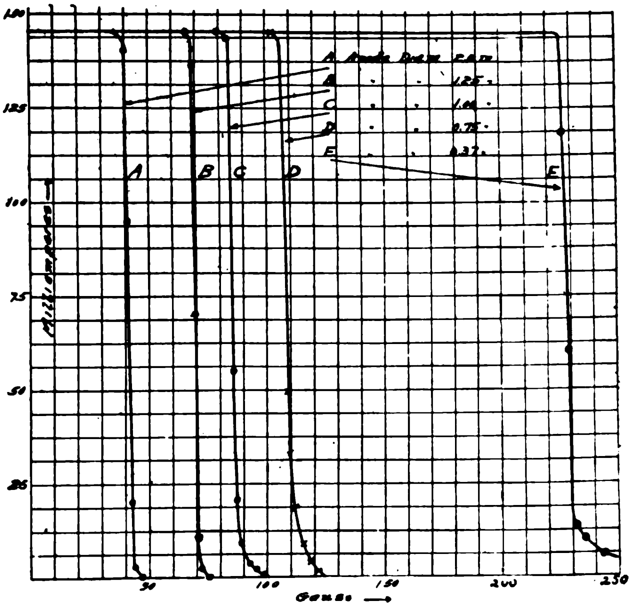

Fig. 13.

Characteristics of different anodes, at 250 volts.

is not appreciably affected. Curve C shows the effect of a larger amount of gas, of the order of 1 bar. The maximum current is very large, but the critical magnetic field is only slightly greater than for good vacuum. The curve has a long tail, however, representing the electrons which have collided or have abnormal initial velocities, and hence require a field much stronger than the critical value to keep them from the anode. At higher pressures, of the order of 10–20 bars, the magnetic cut-off is much less steep (Curve D) and a considerable fraction of the electrons are entirely uncontrollable by the magnetic field. At still higher pressures magnetic fields of this order of magnitude are practically ineffective.

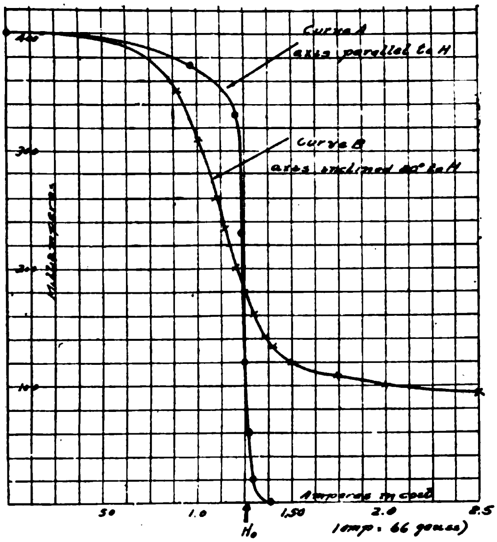

Fig. 14.

Effect of lack of symmetry. Magnetic field not parallel to axis.

B. Internal Anode. Straight Rod in Axis of Helical Filament. General Characteristics.—Equations (10) and (11) apply. The effect of initial velocities of the electrons is large, except in strong magnetic fields. Hence the curves representing current to the central anode as a function of magnetic field, at constant anode potential, will not be as steep as in the first case. On the other hand, the magnetic field required to produce a marked change in current is very much smaller than in the first case, so that the earth's magnetic field produces an easily measurable effect, and the field of the current which heats the filament practically prevents any electrons reaching the anode at potentials less than about 100 volts.

A typical example is shown in Fig. 16.[1] The cathode was a helical coil 0.63 cm. in diameter, 1.7 cm. long, consisting of 10 turns of 10 mil tungsten wire. The anode was a 10 mil wire in the axis of the helix. A rotating commutator applied heating current and voltage alternately, so that the measurements were taken when no current was flowing through the filament. The measurements were taken with the tube vertical, and the earth's magnetic field was not compensated. The abscissas are the values of the impressed magnetic field. It will be noted that the curve is symmetrical, not about the axis of ordinates, but about the ordinate $H = 0.6$ approximately, the vertical component of the earth's magnetic field.

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[1] For this and the following curves I am indebted to Mr. C. G. Found.

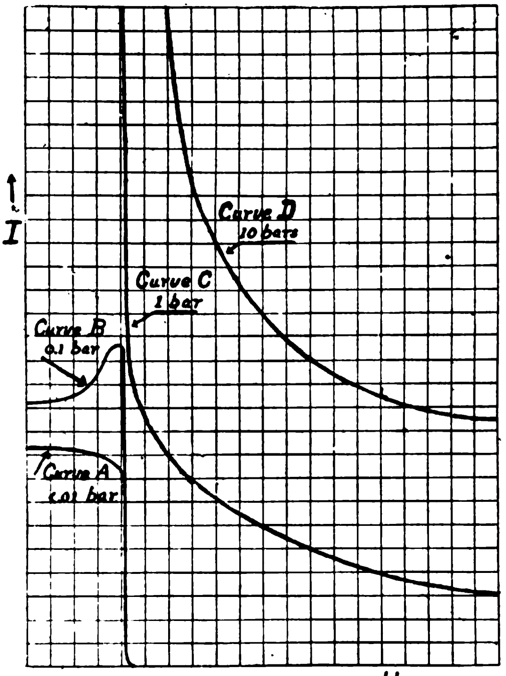

Fig. 15.

Effect of gas pressure upon magnetic cutoff.

The value of magnetic field just sufficient to prevent electrons of zero tangential initial velocity from reaching the anode, according to Eq. 12, is shown at H_0. It is seen that about half of the electrons, viz., those whose initial velocities are in the same direction as the magnetic deflection, are kept from the anode by a field less than H_0. The remainder, whose initial velocities are in the opposite direction to the deflection produced by the magnetic field, require a field greater than H_0.

It is possible to calculate the exact current to be expected at each value of magnetic field, assuming the tangential initial velocities to be distributed according to Maxwell's law.[1] The magnetic field just sufficient to keep back electrons with initial velocities less than v_0' or greater than v_0'' is given by Equation (11), viz.,

$$H = \pm \frac{1}{\beta R_0}\sqrt{\frac{8m}{e}V} + 2\frac{mv_0}{eR_0} \tag{11}$$

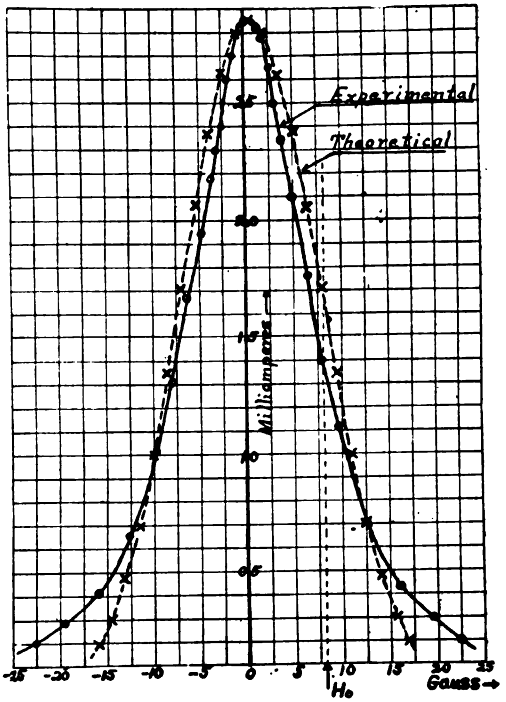

Fig. 16.

Typical internal anode characteristic at 110 volts. O, experimental points (solid curve). X, theoretical points (broken curve).

v_0' and v_0'' correspond to the plus and minus sign of the radical respectively. In the present case, with $\beta = 25$, $R_0 = .38$ cm. and $V = 110$ volts, this becomes

$$H = \pm 7.5 + 3v_0.$$

The values of H required for different values of v_0 are given in Table II. All electrons with initial velocities *between* any pair of values v_0' and v_0''

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[1] Richardson and Cooke, Phil. Mag., 16, 353, 1908.

will *not* be deflected from the anode by the corresponding H, *i.e.*, will reach the anode. This number, which represents the relative current to the anode for each value of H, is given by Maxwell's distribution law as

$$i_H = \int_{v_0'}^{v_0''} f(v_0)dv_0 = \int_{v_0'}^{v_0''} \left(\frac{m}{2\pi R_1 T}\right)^{1/2} e^{-\frac{mv_0^2}{2R_1T}} dv_0, \tag{22}$$

where m = mass of electron = 9.01×10^{-28},
R_1 = gas constant for 1 molecule = 1.372×10^{-16} ergs/degree,
T = absolute temperature = 2,200 degrees,
v_0 = initial (tangential) velocity of electron in cm./sec.

The values of i_H given by equation (22) are tabulated in column 4 of Table II. These values are percentages of the total number of electrons emitted by the filament. It is seen that only 83 per cent. of the electrons emitted reach the anode *even without magnetic field*. For comparison with the observed currents the values in column 4, multiplied by 3.44 to reduce them to the same scale, are tabulated in column 5, and plotted as crosses in Fig. 16. The agreement is remarkably good considering the degree of symmetry of the apparatus. With more perfectly constructed apparatus it should be possible to test accurately the law of distribution of initial velocities of the emitted electrons.

TABLE II.

nitial Velocity of Electrons.		Critical Magnetic Field (from Eq. 11).	Fraction of Emitted Electrons with Velocities between v_0' and v_0''.	Number of Electrons Reaching Anode.	
v_0' cm./sec. $\times 10^7$.	v_0'' cm./sec. $\times 10^7$.	H Gauss.	$i_H = \int_{v_0'}^{v_0''} f(v_0)dv_0$.	Milliamperes Calculated 3.44 i_H)	Observed Milliamperes.
− 2.5......	+ 2.5	0	0.828	2.85	2.85
− 2.0......	+ 3.0	1.5	.812	2.80	2.78
− 1.5......	+ 3.5	3.0	.765	2.63	2.45
− 1.0......	+ 4.0	4.5	.692	2.38	2.10
− 0.5......	+ 4.5	6.0	.600	2.07	1.80
0	+ 5.0	7.5	.497	1.71	1.45
+ 0.5......	+ 5.5	9.0	.391	1.35	1.15
+ 1.0......	+ 6.0	10.5	.292	1.01	0.97
+ 1.5......	+ 6.5	12.0	.207	.71	.72
+ 2.0......	+ 7.0	13.5	.138	.47	.56
+ 2.5......	+ 7.5	15.0	.086	.30	.47
+ 3.0......	+ 8.0	16.5	.050	.17	.39
+ 4.0......	+ 9.0	19.5	.015	.05	.28

Magnetic Field of Heating Current.—Fig. 17 shows the effect of the magnetic field of the heating current in the filament. The measurements were taken with a tube having the same cathode as that of Fig. 16, viz. a helix of .63 cm. diam. but with a 40 mil (1 mm.) anode. Curve *A* is taken with rotating commutator, that is the measurements were made while no heating current was flowing. Curve *B* was taken with D.C. heating current of 6.1 amperes.

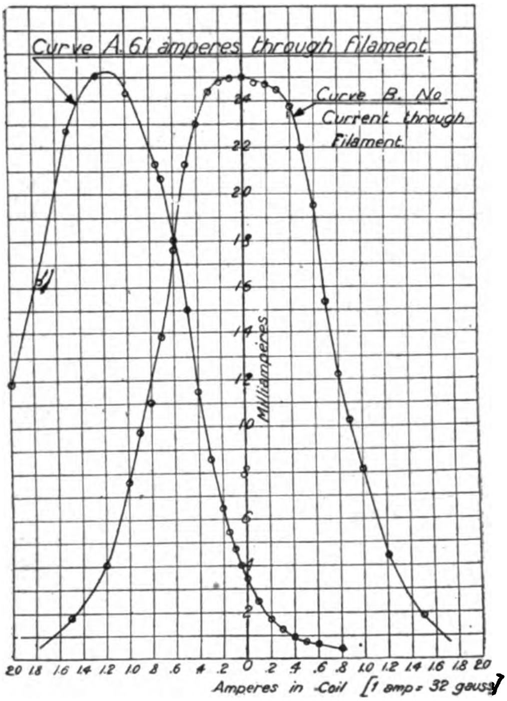

Fig. 17.

Internal anode. Effect of magnetic field of heating current in filament.

The magnetic field of the heating current reduced the electron current to 13 per cent. of its value with no magnetic field. The two curves are similar except that one is displaced to the left by 48 gauss, which is the approximate value of the field produced by the heating current of 6.1 amperes through a coil of 6 turns per cm. (10 turns in 1.7 cm.).

Effect of Voltage.—Figs. 18 and 19 show the variation of current with magnetic field for the same tube at different anode potentials. The tube is the same as of Fig. 17, viz., cathode .63 cm. diameter, 1.7 cm. long, anode 1 mm. diameter.

In Fig. 18 the currents are all plotted to the same scale, showing the relative magnitudes, and the relative variations in milliamperes with

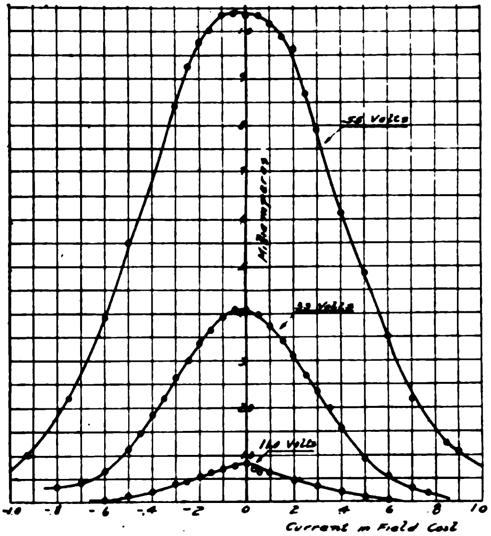

Fig. 18.
Characteristic of 1 mm. internal anode at different potentials.

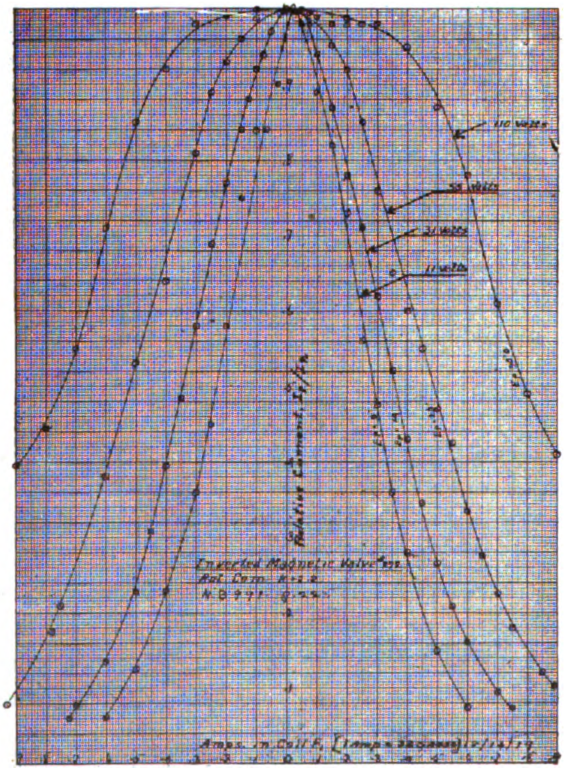

Fig. 19.
Characteristic of 1 mm. internal anode at different potentials. Currents expressed as fractions of maximum at each voltage.

magnetic field. The maximum currents at the different voltages are proportional to the 3/2 power of the voltages.

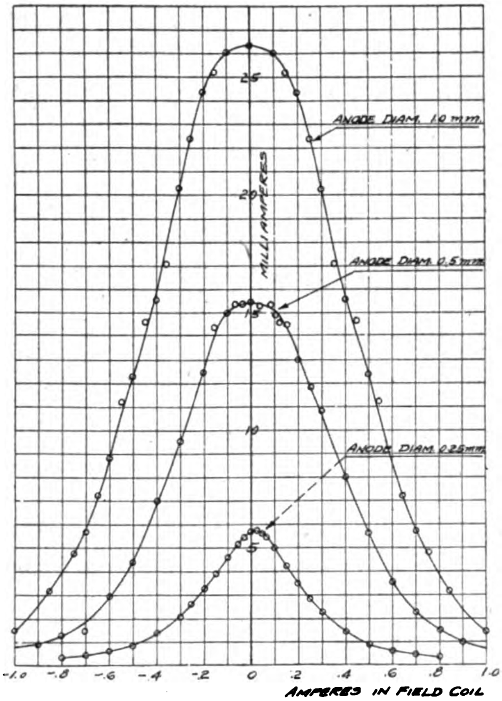

Fig. 20.

Different internal anodes at same potential (110 volts).

In Fig. 19 the maximum at each voltage is taken as unity, in order to show the relative *percentage* variation with magnetic field. It is seen that the field required to produce the same percentage decrease is larger the higher the voltage. At lower voltages, the variation is less rapid than the square root of the voltage, since the initial velocity term in equation 11 predominates. At the highest voltage the curve has a fairly flat top, showing smaller relative effect of initial velocities, and approaching the steep cutoff of internal cathode tubes.

Effect of Anode Diameter.—Figures 20, 21, and 22 show the characteristics of three similar tubes with different anode diameters. The cathodes of the three tubes were the same, viz., 10 turns of 10 mil tungsten wire wound in a helix 0.63 cm. in diam. 1.7 cm. long. The anodes were 0.25 mm., 0.50 mm., and 1.0 mm. diam. respectively.

Figure 20 shows the actual currents in the three tubes at the same anode voltage, plotted to the same scale. The difference in the maxima is due partly to space charge, but mostly to initial velocities, since by Eq. (10) only part of the electrons emitted can reach the anode even in the absence of magnetic field, viz., those whose initial energy of tangential motion, expressed in equivalent volts, is less than V/β. (V is the potential difference between anode and cathode, and β the ratio of cathode diameter to anode diameter.) Thus, the fraction that can reach the anode falls off very rapidly with increasing β, that is, with smaller anode diameter.

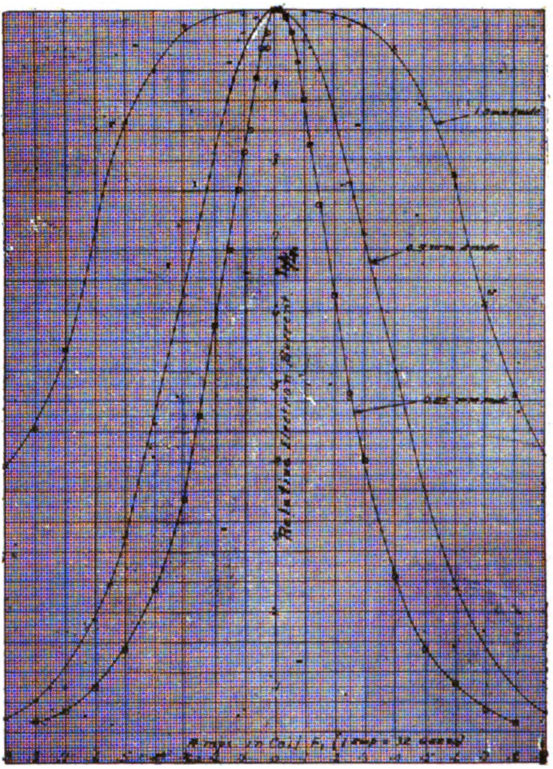

Fig. 21.

Different internal anodes at same potential (110 volts). Currents expressed as fractions of maximum for each anode.

In Fig. 21, the same curves are reproduced on different scales, the maximum of each being taken as unity. It will be noted that the smaller anode diameter gives the greater percentage decrease of current with magnetic field, and that weaker fields are required for smaller anode diameter, as predicted by Eq. 11.

Fig. 22 shows the characteristic of the same three tubes at different voltages, so chosen that the maximum space-charge limited current was

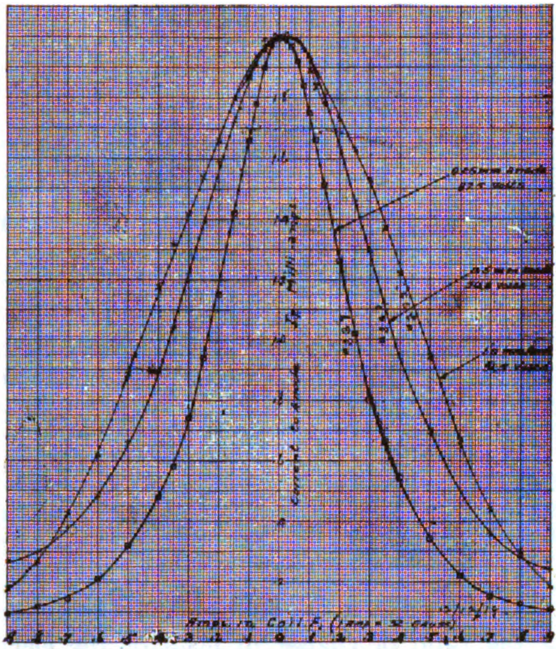

Fig. 22.

Comparison of different internal anodes at different potentials, so chosen as to give same maximum current for each.

the same for each. It is seen that even at the higher voltage required for equal current, the smaller anode is the more susceptible to magnetic field.

Conclusion.

The agreement between theory and experiment is well within the limits prescribed by the degree of symmetry of the apparatus. Experiments with more perfect apparatus are in progress by Dr. K. H. Kingdon, which, it is hoped, will show the limitations of the theory, and bring to light any new facts not hitherto taken account of.

Several useful applications have been investigated, which will be discussed elsewhere.

The above solution assumes constant e/m, and therefore applies only to potential differences of the order of 50,000 volts or less. Dr. Leigh Page has very kindly worked out the solution for the case of variable e/m, which is presented in the following paper.

Research Laboratory, General Electric Co.,
Schenectady, N. Y.,
March 3, 1921.

THEORY OF THE MOTION OF ELECTRONS BETWEEN CO-AXIAL CYLINDERS TAKING INTO ACCOUNT THE VARIATION OF MASS WITH VELOCITY.

BY LEIGH PAGE.

SYNOPSIS.

Theory of the Motion of Electrons between Coaxial Cylinders, the Electric Field being Radial, the Magnetic Longitudinal.—If in Hull's coaxial cylinder method of measuring e/m, large potential differences are used, it is necessary *to take account of the variation of mass with velocity*. Putting the expressions for transverse and longitudinal mass into the equations of motion and integrating we get: $e_0/m_0 = 8\pi^2c^2R^2V/(N^2 - 4\pi^2R^2V^2)$, which depends only on the larger radius R, the total magnetic flux N, and the total potential drop V, and is not affected by space charge. Because the longitudinal mass is involved, this equation provides a *method of experimentally verifying the theoretical expression for longitudinal mass.*

IN the preceding paper Dr. A. W. Hull has described a new method of measuring the ratio of charge to mass of the electron. If the difference of potential between the coaxial cylinders is sufficiently great, the electrons will acquire a velocity in the course of their motion comparable to that of light. Hence the variation of mass with velocity must be taken into account. The following is a rigorous solution of the problem insofar as concerns the term in the kinetic reaction of the electron which involves its mass, although the radiation reaction, and in fact all terms involving derivatives of the acceleration,[1] are neglected. The problem is interesting in that the solution involves both transverse and longitudinal masses. The theoretical expression for the transverse mass has been abundantly verified by Bucherer,[2] Neumann[3] and others, but so far as the author is aware, no experimental confirmation of the formula for the longitudinal mass has yet been obtained other than the rather indirect confirmation involved in Sommerfeld's[4] theory of the fine structure of spectral lines. Therefore experimental verification of the results of the theory to be presented would constitute a valuable confirmation of the theoretical formula for longitudinal mass. Furthermore the experimental data would provide a test between the symmetric electron here assumed (such as the Lorentz electron) and the asymmetric electron (such as that proposed by Parsons).

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[1] See L. Page, PHYS. REV., 11, p. 376, 1918.

[2] Phys. Zeit., 9, p. 755, 1908.

[3] Ann. d. Phys., 45, p. 529, 1914.

[4] Atombau und Spektrallinien, Chap. 5, 1921.

Let

$F_r \equiv$ component of force along radius vector,
$F_\theta \equiv$ “ “ “ perpendicular to radius vector,
$f_r \equiv$ “ “ acceleration along radius vector,
$f_\theta \equiv$ “ “ “ perpendicular to radius vector,
$\beta \equiv \frac{v}{c} =$ ratio of velocity of electron to that of light,
$m_0 \equiv$ rest mass of electron (a constant),
$e_0 \equiv$ charge on electron,
$a \equiv$ radius of inner cylinder or wire,
$R \equiv$ “ “ outer “ ,
$E \equiv$ electric field (radial),
$H \equiv$ magnetic field (axial),
$V \equiv$ difference between the potentials of the two cylinders,
$N \equiv$ magnetic flux through the part of the cross-section intercepted between the wire and outer cylinder.

Then, parallel to the path of the electron under consideration

$$F_r\dot{r} + F_\theta r\dot{\theta} = \frac{m_0}{(1-\beta^2)^{3/2}}[f_r\dot{r} + f_\theta r\dot{\theta}],$$

and perpendicular to the path

$$F_r r\dot{\theta} - F_\theta\dot{r} = \frac{m_0}{(1-\beta^2)^{1/2}}[f_r r\dot{\theta} - f_\theta\dot{r}],$$

where, expressed in Heaviside-Lorentz rational units,

$$F_r = e_0\left[E + \frac{r\dot{\theta}}{c}H\right],$$

and

$$F_\theta = -e_0\frac{\dot{r}}{c}H.$$

Put

$$e \equiv \frac{e_0}{m_0}E,$$

$$h \equiv \frac{e_0}{m_0}H.$$

Then the equations of motion are

$$\ddot{r} - r\dot{\theta}^2 = \sqrt{1-\beta^2}\left[\left(1 - \frac{\dot{r}^2}{c^2}\right)e + \frac{r\dot{\theta}}{c}h\right], \tag{1}$$

$$\frac{d}{dt}(r^2\dot{\theta}) = -r\sqrt{1-\beta^2}\left[\frac{\dot{r}}{c}h + \frac{\dot{r}r\dot{\theta}}{c^2}e\right], \tag{2}$$

from which is easily obtained the energy equation

$$\frac{1}{2}d(\beta^2) = (1-\beta^2)^{3/2}\frac{e}{c^2}\,dr. \tag{3}$$

The integral of (3) is

$$\frac{1}{\sqrt{1-\beta^2}} = 1 + \frac{1}{c^2}\int_a^r e\,dr. \tag{4}$$

Substituting the value of $\sqrt{1-\beta^2}$ from (4) into (2) this equation of motion becomes

$$d(r^2\dot{\theta}) = -\frac{\dfrac{hr}{c}+\dfrac{er^2\dot{\theta}}{c^2}}{1+\dfrac{1}{c^2}\displaystyle\int_a^r e\,dr}\,dr, \tag{2'}$$

of which the solution is

$$r^2\dot{\theta} = -\frac{\dfrac{1}{c}\displaystyle\int_a^r h\,r\,dr}{1+\dfrac{1}{c^2}\displaystyle\int_a^r e\,dr}.$$

Now

$$\begin{aligned}\frac{\dot{r}^2}{c^2} &= \beta^2 - \frac{r^2\dot{\theta}^2}{c^2}\\ &= 1 - \frac{1+\dfrac{1}{c^4r^2}\left[\displaystyle\int_a^r h\,r\,dr\right]^2}{\left[1+\dfrac{1}{c^2}\displaystyle\int_a^r e\,dr\right]^2}.\end{aligned}$$

The critical condition is that for which $\dot{r} = 0$ when $r = R$. But

$$\int_a^R e\,dr = \frac{e_0}{m_0}V,$$

$$\int_a^R h\,r\,dr = \frac{e_0}{m_0}\frac{N}{2\pi}.$$

Therefore

$$\frac{e_0}{m_0} = \frac{8\pi^2c^2V}{\dfrac{N^2}{R^2}-4\pi^2V^2}, \tag{5}$$

whereas if no account is taken of the variation of mass with velocity, (*i.e.*, if β is very small compared to unity),

$$\frac{e_0}{m_0} = \frac{8\pi^2c^2V}{\dfrac{N^2}{R^2}}. \tag{6}$$

It may be noted that the expression for the ratio of charge to mass involves only the total potential drop and total magnetic flux. So long as the electric field is radial, and the magnetic field has radial symmetry about the axis, it is immaterial what functions of r these fields may be. Hence the effect of space charge is completely eliminated. In electromagnetic units the expression (5) for the ratio of charge to mass becomes

$$\frac{e_0}{m_0} = \frac{8\pi^2 V}{\frac{N^2}{R^2} - 4\pi^2 \frac{V^2}{c^2}}.$$

Sloane Physics Laboratory,
Yale University.

THE TRANSMISSION OF WAVES THROUGH A SYMMETRIC OPTICAL INSTRUMENT.[1]

BY IRWIN ROMAN.

SYNOPSIS.

Gaussian Parameter Method of Studying the Transmission of a Wave Surface through a Centered System of Symmetric Lenses.—Since for each point of an incident wave surface there is a corresponding point on the refracting surface, the parameter β of the refracting surface may be expressed in terms of the parameter α of the wave surface, thus allowing the elimination of the parameter β. After expressing all quantities in terms of α, certain invariant combinations are found among the coefficients of the wave and lens surfaces, these furnishing the determination of the refracted surface. This gives a point by point correspondence between all surfaces of the system. As an application, the focal distance and longitudinal aberration are calculated and a numerical case given. The method is applicable to aspherical surfaces as simply as to spherical ones, and it is hoped that many problems in practical optics may be simplified by the present method, combined with proper choices of the parameter.

PART I. INTRODUCTION.

THE literature on the aberrations in an optical system contains a number of different methods of attack on the problems involved. Nearly all of these may be grouped into two classes, according as the chief emphasis is placed on the wave surface or the ray congruence. As a combination of the two points of view may be mentioned the methods using the characteristic function of Hamilton, which, in the hands of Bruns[2] and of Schwarzschild,[3] have yielded some valuable results. The characteristic function is essentially a relation between the coördinates of the points at the opposite ends of a particular ray segment, the results being studied without expressly finding the coördinates of one end in terms of those of the other end. The method is an implicit function method, and leads to fairly complicated analyses.

It was pointed out by Gullstrand[4] that the aberrations of an optical system could be studied more simply from the point of view of the wave surface than from that of the ray. He selected the equation of a particular wave in the form $z = f(x, y)$. So far as the present writer has been able to learn, Gullstrand has made no attempt to study the transformation of a wave surface, but restricts himself to the analysis of a particular surface, the aberrations being referred to that surface.

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[1] Presented to The University of Chicago, in partial fulfillment of the requirements for the degree of Doctor of Philosophy.

[2] Abh. d. math.-phys. Kl. d. Kgl. Sächs. Ges., Bd. 21, s325 (1895).

[3] Abh. d. Kgl. Ges. d. Wiss. zu Göttingen, math.-phys. Kl. Neue Folge, IV (1905–1906).

[4] Ann. d. Phys. (4), Bd. 18, S. 941–973; also K. Svenska Vetenskapsakad. Handl., 41. No. 3 (1906–7).

The purpose of the present investigation has been to replace the rectangular representation of the surface by a representation in terms of two Gaussian parameters such as are used in differential geometry. Let us consider a particular wave of a family incident on the surface of separation between two homogeneous and isotropic media. Suppose that the initial wave and the lens surface are determined in terms of separate parameters. At each regular point of the wave surface there is a unique normal, commonly called the ray. This ray, if effective in image formation, must pass through the lens surface, and the point of intersection determines a correspondence between the two sets of parameters. By means of this correspondence, either set of parameters may be expressed in terms of the other set, thus allowing the elimination of all but a single set. The use of this single set of parameters throughout the system, makes it possible to determine corresponding points on all surfaces of the system, since all points on a particular ray are given by the same values of the parameters throughout the entire system. It should be noted, however, that if the parameters have a simple geometric significance for one surface, they will, usually, not have a simple significance for another surface. Having determined the various parameters in terms of a common parameter, we may proceed to the study of corresponding surfaces and to the study of the transformation of the surface as the wave is transmitted through the system. Knowing the initial and the final wave of the system, we may proceed to the study of the aberrations in accordance with any desired classification or analysis.

Besides the advantage of directness in the method, there are several other advantages. Since we have a single set of parameters, we have a definite interpretation for the "order" of a quantity. While a few attempts have been made to retain powers of small quantities beyond the second, such terms are usually neglected, probably because of the complexity involved in their retention, and possibly also because there is usually a certain vagueness in the analysis. While a quantity may be of one order in terms of one parameter, it may be of a different order in terms of another parameter. A second advantage lies in the facility with which questions involving aspherical surfaces may be treated. In the usual treatments, the surfaces are assumed to be spheres or to depart only slightly from spheres.

In the preceding paragraphs, the method has been sketched for a general system. Because of the complexity which arises in carrying out the method, the investigation in this paper has been restricted to the case in which all the surfaces are symmetrical with respect to a common axis of revolution. This allows the selection of one of the two Gaussian parameters as the azimuth around the common axis, reducing the problem to a plane problem. Besides assuming that each surface is a surface of revolution around a common axis, we shall assume that each surface and its representation is non-singular in the portions to be studied. This means that for the axis and for a certain region around the axis, there shall be no singular points of the surface or in its representation. The parameters will be selected so as to reduce to zero for the axial point. This choice of parameters is a restriction of small importance, since practically all discussions of the symmetrical system are made in terms of the optical height or of the paraxial angle. The restrictions here imposed still allow a considerable freedom in the choice of parameters to suit particular needs.

In following out the methods here discussed, we shall use power series expansion in the parameter selected, retaining all terms by means of recursion formulas. Except for the application to the first four orders, no attempt will be made to reduce these to explicit forms. The complexity increases rapidly with the order of the term, and in numerical calculations, the recursion formulas may even be more convenient than the explicit forms.

The method also makes use of invariants of two classes, parametric and optical. A parametric invariant is a quantity whose value is independent of the choice of parameter. An optical invariant is a quantity whose value is independent of which particular wave of the family is selected. The parametric invariants are geometric and are incidental rather than fundamental in the optical problem. The optical invariants, however, are fundamental, as they furnish the new surface in terms of the old surface and the interface. Unless specified to the contrary, we shall understand the single word "invariant" to refer to optical invariant.

In order to avoid possible confusion, we shall define several terms ordinarily used with varying meanings. By a refraction, we shall understand the transformation from a wave surface in one medium to that in the other, the two surfaces coinciding with the interface at the axis. By a propagation, we shall understand a transformation from one wave surface to another in the same medium. By a transmission, we shall understand a refraction between two propagations. Thus a transmission carries an arbitrary wave surface in one medium to a wave surface in the other medium at an arbitrary distance. When convenient, and where no confusion is likely to arise, the term transmission may be used for any combination of propagations and refractions, or either alone.

Part II. The Correspondence of Parameters.

As was pointed out in the introduction, there corresponds to each point on the initial wave a point on the lens surface uniquely determined as the point where a normal to the wave at the point in question cuts the lens surface. Since the system is one of revolution about a single axis, we may select that axis as the x-axis, and study the system by means of a typical meridian plane, which we may take as $z = 0$. Then the y-axis will be in the plane under consideration and normal to the axis of revolution. The symmetry assumed may be expressed in the equations of the wave surface by:

$$x = \sum_{i=0}^{\infty} A_{2i}\alpha^{2i} \qquad y = \sum_{i=0}^{\infty} B_{2i+1}\alpha^{2i+1}. \tag{1}$$

The assumption that the representation is regular at the axis is equivalent to the condition $B_1 \neq 0$. The other coefficients may be assumed to be all finite. Thus, $y = 0$ for $\alpha = 0$. Likewise, we may assume the lens surface to be given by

$$\xi = \sum_{i=0}^{\infty} C_{2i}\beta^{2i}, \qquad \eta = \sum_{i=0}^{\infty} D_{2i+1}\beta^{2i+1}, \tag{2}$$

where $D_1 \neq 0$ and all the coefficients are finite.

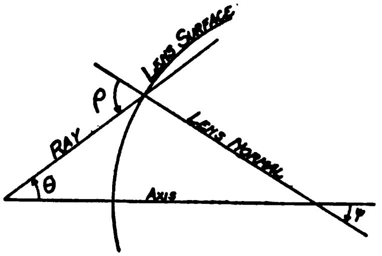

Fig. 1.

Let the normal to the lens surface make an angle ψ with the x-axis, measured as shown in the figure, positive angles being counterclockwise, as usual. Let the ray make an angle θ with the x-axis and an angle ρ with the lens surface normal. Then, since the ray is normal to the wave, it follows that

$$\tan\theta = -\frac{dx}{d\alpha}\Big/\frac{dy}{d\alpha} \qquad \tan\psi = -\frac{d\xi}{d\beta}\Big/\frac{d\eta}{d\beta}. \tag{3}$$

If the point (ξ, η) of the lens surface corresponds to the point (x, y) of the wave surface, then it follows that

$$(\eta - y)/(\xi - x) = \tan\theta = -\frac{dx}{d\alpha}\Big/\frac{dy}{d\alpha}$$

or

$$(\eta - y)\frac{dy}{d\alpha} + (\xi - x)\frac{dx}{d\alpha} = 0. \tag{4}$$

Equation (4) represents the correspondence between α and β and enables us to determine E_i where

$$\beta = \sum_{i=0}^{\infty} E_i\alpha^i. \tag{5}$$

From equation (1), we have

$$\left.\begin{aligned} \frac{dx}{d\alpha} &= \sum_{i=0}^{\infty}(2i+2)A_{2i+2}\alpha^{2i+1}, \\ \frac{dy}{d\alpha} &= \sum_{i=0}^{\infty}(2i+1)B_{2i+1}\alpha^{2i}. \end{aligned}\right\} \tag{6}$$

For $\alpha = 0$, $y = dx/d\alpha = 0$ and $dy/d\alpha = B_1 \neq 0$, so that equation (4) requires that $\eta = 0$ for $\alpha = 0$ or that the solution we want is the one for which $E_0 = 0$, giving

$$\beta = \sum_{i=1}^{\infty} E_i\alpha^i. \tag{7}$$

To express ξ and η in terms of α, we need the various powers of β in terms of α. For this purpose, let

$$\beta^k = \sum_{j=0}^{\infty} I_j^k\alpha^{j+k}. \tag{8}$$

Inspection of equations (7) and (8) shows that I_j^k has the following defining properties:

(*a*) I_j^k is the sum of terms each involving k of the E_i, one from each of the factors β.

(*b*) Since the order, or subscript, of each E_i denotes the power of α that it accompanies from β, the sum of the subscripts in each term of I_j^k is $(j + k)$.

(*c*) The numerical coefficient of each term in I_j^k is the number of distinct permutations possible among the subscripts of that term. If E_i occurs n_i times in a particular term, this number is

$$\frac{k!}{\Pi_i(n_i!)} \quad \text{where} \quad \sum_i n_i = k \quad \text{and} \quad \sum_i in_i = j + k.$$

Hence, we have

$$I_j^k = \sum_u \frac{k!}{\Pi_i(n_i!)}\prod_i(E_i^{n_i}), \tag{9}$$

where $\sum_i n_i = k$ and $\sum_i in_i = j + k$ while $\sum_u$ extends over all possible partitions of $(j + k)$ things into k parts.

In particular, I_j^k has the following useful properties:

(A) The highest order E_i among all the terms of $I_j{}^k$ is in $E_1{}^{k-1}E_{j+1}$.

(B) If j is odd, each term of $I_j{}^k$ must have at least one E_i of even order. This follows from the fact that if each E_i were of odd order, the sum would be even or odd with k, while $(j + k)$ is the opposite.

(C) $I_0{}^0 = 1$, $I_j{}^0 = 0$ for $j > 0$, $I_j{}^1 = E_{j+1}$.

Inserting equations (8) in equations (2), we get

$$\xi = \sum_{i,j=0}^{\infty} C_{2i} I_j{}^{2i} \alpha^{2i+j}, \qquad \eta = \sum_{i,j=0}^{\infty} D_{2i+1} I_j{}^{2i+1} \alpha^{2i+j+1}. \tag{10}$$

By means of equations (1), (6) and (10), equation (4) becomes

$$\sum_{i,j,k=0}^{\infty} \left\{ \begin{aligned} & C_{2i} I_j{}^{2i} (2k + 2) A_{2k+2} \alpha^{2i+j+2k+1} \\ & \qquad - (2j + 2) A_{2j+2} A_{2i} \alpha^{2i+2j+1} \\ & \qquad + D_{2i+1} I_j{}^{2i+1} (2k + 1) B_{2k+1} \alpha^{2i+j+2k+1} \\ & \qquad - (2j + 1) B_{2j+1} B_{2i+1} \alpha^{2i+2j+1} \end{aligned} \right\} = 0. \tag{11}$$

The coefficient of α^{2m} is

$$\begin{aligned} \sum_{k=0}^{m-1} \sum_{i=0}^{m-k-1} [(2k + 2) A_{2k+2} C_{2i} I^{2i}_{2m-2k-2i-1} & \\ + (2k + 1) B_{2k+1} D_{2i+1} I^{2i+1}_{2m-2k-2i-1}] &= 0. \end{aligned} \tag{12}$$

Since $j = 2m - 2k - 2i - 1$ is odd, each term in equation (12) contains an E_i of even order. Since $I^0_{2m-1} = 0$, the highest order E_i occurs for $k = i = 0$ and is E_{2m}, the term being $B_1 D_1 I^1_{2m-1} = B_1 D_1 E_{2m}$. Hence by (12), E_{2m} is the sum of terms each of which involves a factor E_i of even order below $2m$. In particular, for $m = 1$, we have, by equation (12), $k = i = 0$ so

$$2A_2 C_0 I_1{}^0 + B_1 D_1 L^1_1 = B_1 D_1 E_2 = 0. \tag{13}$$

Since B_1 and D_1 were selected different from zero, it follows that $E_2 = 0$. Further, we have

$$E_{2m} = 0 \qquad \text{and} \qquad I^k_{2j+1} = 0. \tag{14}$$

As a consequence of equations (14), we may write equations (8) and (9) as follows:

$$\left. \begin{aligned} & \beta^k = \sum_{j=0}^{\infty} I^k_{2j} \alpha^{2j+k}, \text{ where} \\ & I_{2j}{}^k = \sum_u \frac{k!}{\Pi_i(n_i!)} \prod_i E^{n_i}_{2i+1}, \\ & \sum_i n_i = k, \\ & \sum_i (2i + 1) n_i = k + 2j \\ & \text{and where } \sum_u \text{ extends over all partitions of } (k + 2j) \text{ into } k \text{ parts.} \end{aligned} \right\} \tag{15}$$

By means of equations (14), equation (11) becomes

$$\sum_{i,j,k=0}^{\infty}\left\{\begin{array}{l} [(2k+2)A_{2k+2}C_{2i}I_{2j}{}^{2i} \\ \qquad\qquad + (2k+1)B_{2k+1}D_{2i+1}I_{2j}{}^{2i+1}]\alpha^{2i+2j+2k+1} \\ \qquad\qquad - [(2j+2)A_{2i}A_{2j+2} \\ \qquad\qquad + (2j+1)B_{2i+1}B_{2j+1}]\alpha^{2i+2j+1} \end{array}\right\} = 0. \qquad (16)$$

The coefficient of α^{2m+1} is

$$\left.\begin{array}{l} \displaystyle\sum_{k=0}^{m}\sum_{i=0}^{m-k}[(2k+2)A_{2k+2}C_{2i}I^{2i}_{2m-2k-2i} \\ \qquad\qquad\qquad + (2k+1)B_{2k+1}D_{2i+1}I^{2i+1}_{2m-2k-2i}] \\ \quad \displaystyle -\sum_{i=0}^{m}[(2i+2)A_{2i+2}A_{2m-2i} + (2i+1)B_{2i+1}B_{2m-2i+1}] \end{array}\right\} = 0. \qquad (17)$$

Since

$$\sum_{k=0}^{m}\sum_{i=0}^{m-k}\varphi(i,k) = \varphi(0,0) + \sum_{i=1}^{m}\varphi(i,0) + \sum_{k=1}^{m}\sum_{i=0}^{m-k}\varphi(i,k),$$

equation (17) becomes

$$\left\{\begin{array}{l} [2A_2C_0I_{2m}{}^0 + B_1D_1I_{2m}{}^1] \\ \quad \displaystyle + \sum_{i=1}^{m}[2A_2C_{2i}I^{2i}_{2m-2i} + B_1D_{2i+1}I^{2i+1}_{2m-2i}] \\ \quad \displaystyle + \sum_{k=1}^{m}\sum_{i=0}^{m-k}[(2k+2)A_{2k+2}C_{2i}I^{2i}_{2m-2k-2i} \\ \quad + (2k+1)B_{2k+1}D_{2i+1}I^{2i+1}_{2m-2k-2i}] \\ \quad \displaystyle - \sum_{i=0}^{m}[(2i+2)A_{2i+2}A_{2m-2i} + (2i+1)B_{2i+1}B_{2m-2i+1}] \end{array}\right\} = 0. \qquad (18)$$

Since $I_{2m}{}^1 = E_{2m+1}$, this gives the recursion formula:

$$E_{2m+1} = \frac{1}{B_1D_1}\left\{\begin{array}{l} \displaystyle\sum_{i=0}^{m}[(2i+2)A_{2i+2}A_{2m-2i} + (2i+1)B_{2i+1}B_{2m-2i+1}] \\ \displaystyle - 2A_2C_0I_{2m}{}^0 - \sum_{i=1}^{m}[2A_2C_{2i}I^{2i}_{2m-2i} \\ \qquad\qquad\qquad\qquad + B_1D_{2i+1}I^{2i+1}_{2m-2i}] \\ \displaystyle - \sum_{k=1}^{m}\sum_{i=0}^{m-k}[(2k+2)A_{2k+2}C_{2i}I^{2i}_{2m-2k-2i} \\ \qquad\qquad + (2k+1)B_{2k+1}D_{2i+1}I^{2i+1}_{2m-2k-2i}] \end{array}\right\} \qquad (19)$$

Equation (19) furnishes E_{2m+1}, after which equations (15) furnish $I_{2j}{}^k$ so that substitution in equation (10) gives:

$$\left.\begin{aligned}\xi &= \sum_{m=0}^{\infty} F_{2m}\alpha^{2m}\\ \eta &= \sum_{m=0}^{\infty} G_{2m+1}\alpha^{2m+1}\end{aligned}\right\}\text{ where }\left\{\begin{aligned}F_{2m} &= \sum_{i=0}^{m} C_{2i}\Gamma^{2i}_{2m-2i}\\ G_{2m+1} &= \sum_{i=0}^{m} D_{2i+1}\Gamma^{2i+1}_{2m-2i}\end{aligned}\right\}. \qquad (20)$$

While equations (20) give the value of G_{2m+1}, it may be expressed in a more convenient form as follows. By equations (1), (4), (6), and (20), we have

$$\sum_{i,j=0}^{\infty} [(2i + 1)(B_{2j+1} - G_{2j+1})B_{2i+1} + (2i + 2)(A_{2j} - F_{2j})A_{2i+2}]\alpha^{2i+2j+1} = 0. \qquad (21)$$

The coefficient of α^{2m+1} is

$$\sum_{j=0}^{m} [(2m - 2j + 1)(B_{2j+1} - G_{2j+1})B_{2m-2j+1} + (2m - 2j + 2)(A_{2j} - F_{2j})A_{2m-2j+2}] = 0. \qquad (22)$$

Since $\sum_{j=0}^{m} \varphi(j) = \varphi(m) + \sum_{j=0}^{m-1} \varphi(j)$, we have from equation (22)

$$\left.\begin{aligned}G_{2m+1} = B_{2m+1} &+ \frac{1}{B_1}\sum_{j=0}^{m-1} (2m - 2j + 1)(B_{2j+1} - G_{2j+1})B_{2m-2j+1}\\ &+ \frac{1}{B_1}\sum_{j=0}^{m} (2m - 2j + 2)(A_{2j} - F_{2j})A_{2m-2j+2}\end{aligned}\right\}. \qquad (23)$$

We have thus expressed in β terms of α, and are able to eliminate the parameter β from the problem, expressing everything in terms of the parameter α. We have also found explicit values for the coördinates of each point of the lens surface in terms of the value of the parameter specifying the corresponding point on the incident wave surface. Since the coefficients F_{2m} and G_{2m+1} are for the lens surface in terms of α, it follows that they are independent of the choice of β. Thus, when F_{2m} and G_{2m+1} are expressed in terms of A_{2m}, B_{2m+1}, C_{2m} and D_{2m+1}, the coefficients of A_{2m} and B_{2m+1} will be combinations of the C_{2m} and D_{2m+1} which will be the same for all parameters β. Hence these give parametric invariants of the curve which describes the lens surface and hence of all curves with the assumed type of symmetry. The details of this will appear more clearly in the special cases treated in Part IV.

PART III. TRANSFORMATION OF THE WAVE SURFACE.

Having expressed the coördinates of the lens surface in terms of the parameter of the incident wave, we may now proceed at once to the study of how the wave is transformed in its passage through the system. We shall assume that the incident wave

$$x' = \sum_{i=0}^{\infty} A'_{2i}\alpha^{2i} \qquad y' = \sum_{i=0}^{\infty} B'_{2i+1}\alpha^{2i+1} \tag{1^1}$$

is propagated to the lens surface

$$\xi = \sum_{i=0}^{\infty} F_{2i}\alpha^{2i} \qquad \eta = \sum_{i=0}^{\infty} G_{2i+1}\alpha^{2i+1}, \tag{20}$$

is there refracted in the usual way and is finally propagated into the wave

$$x'' = \sum_{i=0}^{\infty} A''_{2i}\alpha^{2i} \qquad y'' = \sum_{i=0}^{\infty} B''_{2i+1}\alpha^{2i+1}. \tag{1''}$$

We shall thus want to determine A''_{2i} and B''_{2i+1} in terms of A'_{2i}, B'_{2i+1}, F_{2i}, G_{2i+1} and A_0'', the latter quantity specifying which wave of the refracted family we are considering. The same value of α specifies corresponding points (x', y'), (ξ, η) and (x'', y'').

The law of refraction is equivalent to the optical invariance of $\mu \sin \rho$ where μ is the index of refraction of the medium and ρ is the angle between the normals to the lens surface and the wave surface, respectively. If we use primes to refer to the first medium and double primes to refer to the second medium, this may be stated in the form $\mu' \sin \rho' = \mu'' \sin \rho''$. From the figure, we have

$$\sin \rho = \sin (\theta - \psi) = \sin \theta \cos \psi - \cos \theta \sin \psi. \tag{24}$$

By means of equations (3) and (24) and setting

$$\mu \sin \rho = \sum_{i=0}^{\infty} L_{2i+1}\alpha^{2i+1} \tag{25}$$

we find that

$$\left.\begin{aligned}
\sum_{i=0}^{\infty} L_{2i+1}\alpha^{2i+1} &= \frac{\mu \sum\limits_{i=0}^{\infty} M_{2i+1}\alpha^{2i+1}}{\sum\limits_{j=0}^{\infty} H_{2j}\alpha^{2j} \sum\limits_{k=0}^{\infty} J_{2k}\alpha^{2k}} \quad \text{where} \\
\sum_{i=0}^{\infty} M_{2i+1}\alpha^{2i+1} &= \frac{d\xi}{d\alpha}\frac{dy}{d\alpha} - \frac{dx}{d\alpha}\frac{d\eta}{d\alpha} \\
\sum_{j=0}^{\infty} H_{2j}\alpha^{2j} &= \sqrt{\left(\frac{dx}{d\alpha}\right)^2 + \left(\frac{dy}{d\alpha}\right)^2} \\
\sum_{k=0}^{\infty} J_{2k}\alpha^{2k} &= \sqrt{\left(\frac{d\xi}{d\alpha}\right)^2 + \left(\frac{d\eta}{d\alpha}\right)^2}
\end{aligned}\right\} \tag{26}$$

By means of equations (1), (20) and (26), after a few simple reductions, we find

$$\left.\begin{aligned}
&H_0 = B_1, \qquad J_0 = G_1, \qquad M_1 = 2(F_2B_1 - A_2G_1),\\
&\qquad\qquad\qquad\qquad L_1 = \frac{2\mu}{B_1G_1}(F_2B_1 - A_2G_1)\\
&\text{while for } m > 0, \text{ we get}\\
&H_{2m} = \frac{1}{2B_1}\left\{\sum_{i=0}^{m-1}(2i+2)(2m-2i)A_{2m-2i}A_{2i+2}\right.\\
&\qquad + \sum_{i=0}^{m}(2i+1)(2m-2i+1)B_{2i+1}B_{2m-2i+1}\\
&\qquad \left. - \sum_{i=1}^{m-1}H_{2i}H_{2m-2i}\right\}\\
&J_{2m} = \frac{1}{2G_1}\left\{\sum_{i=0}^{m-1}(2i+2)(2m-2i)F_{2m-2i}F_{2i+2}\right.\\
&\qquad + \sum_{i=0}^{m}(2i+1)(2m-2i+1)G_{2i+1}G_{2m-2i+1}\\
&\qquad \left. - \sum_{i=1}^{m-1}J_{2i}J_{2m-2i}\right\}\\
&M_{2m+1} = \sum_{i=0}^{m}(2i+2)(2m-2i+1)(F_{2i+2}B_{2m-2i+1}\\
&\qquad\qquad - A_{2i+2}G_{2m-2i+1})\\
&L_{2m+1} = \frac{1}{B_1G_1}\left\{\mu M_{2m+1} - \sum_{i=0}^{m-1}L_{2i+1}\sum_{j=0}^{m-i}H_{2j}J_{2m-2i-2j}\right\}
\end{aligned}\right\} \quad (27)$$

If we define

$$S_{2m+1} = \frac{1}{G_1}\sum_{i=0}^{m}L_{2i+1}J_{2m-2i} = L_{2m+1} + \frac{1}{G_1}\sum_{i=0}^{m-1}L_{2i+1}J_{2m-2i}, \quad (28)$$

we get

$$S_{2m+1} = \frac{1}{B_1G_1}\left\{\mu M_{2m+1} - \sum_{i=0}^{m-1}L_{2i+1}\sum_{j=1}^{m-i}H_{2j}J_{2m-2i-2j}\right\}. \quad (29)$$

The quantities F_{2m}, G_{2m+1}, J_{2m}, L_{2m+1} and S_{2m+1} are optical invariants and have the same value in the two sets of primed symbols, *i.e.*, in the two media. The invariants G_{2m+1} and S_{2m+1} involve B_{2m+1} and A_{2m+2} along with terms of lower order and F_{2m} and G_{2m+1}, which are invariant. Hence the equations

$$G'_{2m+1} = G''_{2m+1} \quad \text{and} \quad S'_{2m+1} = S''_{2m+1}, \quad (30)$$

which are linear in B_{2m+1} and A_{2m+2} if $m > 0$, may be solved for B''_{2m+1} and A''_{2m+2}. In the case $m = 0$, the equations are not linear, but, as shown in Part IV., the solution may be effected without difficulty. The invariant L_{2m+1} might be used instead of S_{2m+1}, but the latter is simpler. While S_{2m+1} involves G_{2m+1}, this may be eliminated by means of equation (23).

The solution for A''_{2m+2} and B''_{2m+1} completes the problem of finding the new wave in terms of the old wave and the refracting surface. The solution of equations (30) becomes increasingly complex as m increases.

PART IV. TERMS OF THE FIRST FOUR ORDERS.

To illustrate the use of the preceding formulas, we shall calculate the results for the cases $m = 0$ and $m = 1$. The case $m = 0$ is the elementary Gaussian theory, while the case $m = 1$ is essentially the theory of first order aberrations, so far as these aberrations have a direct interpretation in the case of a system of revolution. For the case of $m = 0$, we shall calculate the case of a general transmission, but for the case of $m = 1$, we shall calculate the cases of propagation and refraction separately, because of the complexity of the formulas. As a matter of notation, let

$$l' = F_0 - A_0', \quad l'' = F_0 - A_0'', \quad \text{and} \quad l = l' - l'' = A_0'' - A_0'. \qquad (31)$$

Case I. Gaussian Theory. $m = 0$.—By substitution in the various formulas (23), (27), (29) and (30) ,we get

$$\left.\begin{aligned} G_1' &= B_1' - 2l'(A_2'/B_1') = G_1'' = B_1'' - 2l''(A_2''/B_1'') \\ S_1' &= L_1' = 2\mu'\left(\frac{F_2}{G_1} - \frac{A_2'}{B_1'}\right) = S_1'' = 2\mu''\left(\frac{F_2}{G_1} - \frac{A_2''}{B_1''}\right) \end{aligned}\right\}. \qquad (32)$$

$$\left.\begin{aligned} B_1'' - 2l''(A_2''/B_1'') &= B_1' - 2l'(A_2'/B_1') \\ (A_2''/B_1'') &= \frac{\mu'}{\mu''}\left(\frac{A_2'}{B_1^1}\right) + \frac{F_2(\mu'' - \mu')}{G_1\mu''} \end{aligned}\right\}. \qquad (33)$$

Solving for B_1'' and (A_2''/B_1''), we get

$$\left.\begin{aligned} B_1'' &= B_1' - \frac{2A_2'(l'\mu'' - l''\mu')}{B_1'\mu''} + \frac{2l''F_2(\mu'' - \mu')}{G_1\mu''} \\ A_2'' &= \left[\frac{\mu'A_2'}{\mu''B_1'} + \frac{F_2(\mu'' - \mu')}{G_1\mu''}\right]B_1'' \end{aligned}\right\}. \qquad (34)$$

For refraction, $l' = l'' = 0$ so that

$$B_1'' = B_1' = G_1 \qquad \text{and} \qquad A_2'' = [\mu'A_2' + F_2(\mu'' - \mu')]/\mu''. \qquad (34r)$$

For propagation, $\mu' = \mu'' = 1$ so that

$$B_1'' = B_1' - 2l(A_2'/B_1') \qquad A_2'' = A_2' - 2l(A_2'/B_1')^2 \qquad (34p)$$

Case II. First Order Aberrations. $m = 1$.—In this case, we have, by equations (23), (27) and (29),

$$\left.\begin{aligned}
G_3 &= B_3 + [3(B_1 - G_1)B_3 + 4(A_0 - F_0)A_4 + 2(A_2 - F_2)A_2]/B_1\\
H_2 &= 3B_3 + (2A_2^2/B_1)\\
M_3 &= 6(F_2B_3 - A_2G_3) + 4(F_4B_1 - A_4G_1)\\
S_3 &= (\mu M_3/B_1G_1) - (L_1H_2/B_1)
\end{aligned}\right\} \quad (35)$$

For refraction, $A_0'' = A_0' = F_0$, $B_1'' = B_1' = G_1$, so that

$$\left.\begin{aligned}
G_3 &= B_3' + 2A_2'(A_2' - F_2)/B_1' = B_3'' + 2A_2''(A_2'' - F_2)/B_1''\\
L_1 &= 2\mu(F_2 - A_2)/B_1\\
M_3 &= (F_2 - A_2)(6B_1B_3 + 12A_2^2)/B_1 + 4B_1(F_4 - A_4)\\
S_3 &= \frac{4\mu'}{B_1'}\left[(F_4 - A_4') + \frac{2A_2'^2}{B_1'^2}(F_2 - A_2')\right]\\
&= \frac{4\mu''}{B_1''}\left[(F_4 - A_4'') + \frac{2A_2''^2}{B_1''^2}(F_2 - A_2'')\right]
\end{aligned}\right\} \quad (36r)$$

$$\left.\begin{aligned}
B_3'' &= B_3' + \frac{2(A_2' - F_2)}{B_1'}\left(A_2' - \frac{\mu'}{\mu''}A_2''\right)\\
A_4'' &= F_4 + \frac{\mu'}{\mu''}(A_4' - F_4) + \frac{2\mu'}{B_1'^2\mu''}(A_2' - F_2)(A_2'^2 - A_2''^2)
\end{aligned}\right\} \quad (37r)$$

A_2'' is known by equations (34r). For propagation, we may select the initial wave as coinciding with the refracting surface, so that $A_{2k}' = F_{2k}$ and $B'_{2k+1} = G_{2k+1}$. Then, by equations (34p),

$$B_1' - B_1'' = G_1 - B_1'' = 2lA_2'/B_1', \qquad A_2' - A_2'' = 2lA_2'^2/B_1'^2,$$
$$A_2''/B_1'' = A_2'/B_1'.$$

Hence

$$\left.\begin{aligned}
&\left(1 - \frac{6A_2'l}{B_1'B_1''}\right)B_3'' + \frac{4l}{B_1''}A_4'' = B_3' + 4l\left(\frac{A_2'}{B_1'}\right)^3,\\
&L_1'' = 2\mu''\left(\frac{A_2'}{B_1'} - \frac{A_2''}{B_1''}\right) = 0,\\
&M_3'' = 6(A_2'B_3'' - A_2''B_3') + 4(A_4'B_1'' - A_4''B_1'),\\
&S_3'' = \frac{\mu''}{B_1'B_1''}\{6(A_2'B_3'' - A_2''B_3') + 4(A_4'B_1'' - A_4''B_1')\} = 0,\\
&3A_2'B_3'' - 2B_1'A_4'' = 3A_2''B_3' - 2A_4'B_1''.
\end{aligned}\right\} \quad (36p)$$

Solving the first and last of equations (36*p*), we get

$$\left.\begin{aligned} B_3'' &= B_3' + l\left[\frac{6A_2'B_3'}{\mathrm{B}_1'^2} - \frac{4A_4'}{B_1'} + 4\left(\frac{A_2'}{B_1'}\right)^3\right], \\ A_4'' &= A_4' + l\left[6\left(\frac{A_2'}{B_1'}\right)^4 + 12\left(\frac{B_3'}{B_1'}\right)\left(\frac{A_2'}{B_1'}\right)^2 - 8\left(\frac{A_2'}{B_1'}\right)\left(\frac{A_4'}{B_1'}\right)\right]. \end{aligned}\right\} \quad (37p)$$

The calculations for F_0, F_2 and F_4 are made without special difficulty and the results are as follows:

$$\left.\begin{aligned} F_0 &= C_0I_0^0 = C_0 \qquad F_2 = C_0I_2^0 + C_2I_0^2 = C_2E_1^2, \\ F_4 &= C_0I_4^0 + C_2I_2^2 + C_4I_0^4 = 2C_2E_1E_3 + C_4E_1^4, \\ E_1 &= \frac{B_1}{D_1} + \frac{2A_2(A_0 - C_0)}{B_1D_1}, \\ E_3 &= \frac{2A_2^2}{B_1D_1} + \frac{4B_3}{D_1} - \frac{2A_2C_2E_1^2}{B_1D_1} - \frac{D_3E_1^3}{D_1} + \frac{4A_4(A_0 - C_0)}{B_1D_1} \\ &\qquad - \frac{3B_3E_1}{B_1}. \end{aligned}\right\} \quad (38)$$

For pure refraction, $C_0 = A_0$ so that equations (38) become

$$\left.\begin{aligned} E_1 &= \frac{B_1}{D_1}, \qquad E_3 = \frac{2A_2^2}{B_1D_1} + \frac{B_3}{D_1} - \frac{2A_2C_2B_1}{D_1^3} - \frac{D_3B_1^3}{D_1^4}, \\ F_0 &= A_0, \qquad F_2 = \frac{C_2B_1^2}{D_1^2}, \\ F_4 &= \left(\frac{C_2}{D_1^2}\right)(4A_2^2 + 2B_1B_3) - 4A_2B_1^2\left(\frac{C_2}{D_1^2}\right)^2 \\ &\qquad - B_1^4\left(\frac{2C_2D_3 - C_4D_1}{D_1^5}\right). \end{aligned}\right\} \quad (38r)$$

As was noted above (see end of part III.), the coefficients of the wave surface symbols are combinations of the lens surface coefficients which are invariant under a transformation of the parameter β of the type assumed. Hence we have two parametric invariants $P_2 = C_2/D_1^2$ and $P_4 = (C_4D_1 - 2C_2D_3)/D_1^5$. That these are actually parametric invariants may be verified by direct calculation, assuming that $\beta = a_1\gamma + a_3\gamma^3$. While it may appear that the assumption of pure refraction is a loss of generality, such is not the case. The values for F_2 and F_4 using the general values in equations (38) are quite complicated and lead to the same values of P_2 and P_4 as given above.

PART V. CONCLUDING REMARKS.

While no exhaustive analysis of the preceding results has been made, several conclusions may be mentioned.

The invariant S_{2k+1} may be considered as a generalization of the so-called "zero invariant." Since the axial curvature of the wave is $K = 2A_2/B_1^2$ and of the lens is $k = 2F_2/G_1^2$, the invariant S_1 is

$$S_1 = \mu\left(\frac{2F_2}{G_1} - \frac{2A_2}{B_1}\right) = \mu(kG_1 - KB_1).$$

For pure refraction, $G_1 = B_1$, so that we get the usual zero invariant

$$Q = S_1/G_1 = \mu(k - K). \tag{39}$$

While the assumption that the entire system is one of revolution makes a full analysis of the image formation unsatisfactory, we may nevertheless calculate the longitudinal aberration. Let the normal to the wave at the point (x, y) cut the axis at the point $(X, 0)$. Then X will vary with α, unless the surface is spherical. If we set

$$X = \sum_{m=0}^{\infty} \lambda_{2m}\alpha^{2m}, \tag{40}$$

λ_{2m} will be the longitudinal aberration of order $2m$. It may be shown without difficulty, by the preceding methods, that

$$\lambda_{2m} = \frac{1}{A_2}\left[\frac{1}{2}\sum_{i=0}^{m}(2i + 1)B_{2i+1}B_{2m-2i+1} + \sum_{i=0}^{m}(i + 1)A_{2i+2}A_{2m-2i} - \sum_{i=1}^{m}(i + 1)A_{2i+2}\lambda_{2m-2i}\right]. \tag{41}$$

For $m = 0$, we have $\lambda_0 = A_0 + B_1^2/2A_2$. Since the value of X reduces to λ_0 for $\alpha = 0$, and since the wave surface cuts the axis at A_0, the quantity $B_1^2/2A_2$ represents the axial distance from the wave to the cusp of the caustic. If the initial wave is plane, this distance reduces to the ordinary focal length. The ordinary value of the longitudinal aberration (the so-called spherical aberration) is

$$\lambda_2 = A_2 + 2B_1B_3/A_2 - A_4(B_1/A_2)^2.$$

The results calculated by this method agree very satisfactorily with the results obtained by the usual methods. It may be mentioned that in this method the emphasis is placed on the wave and not on the aberrations. In order to study the aberrations and the effects of the transmission on them, we have merely to calculate the aberrations for the initial and for the final surfaces of the wave family and to subtract the one from the other.

As an example of the agreement between the present method and the usual one, let us consider a particular telescopic system as follows. Let the front lens have a front curvature of 2.0538 and a rear curvature of

− 3.0538. Let the rear lens have a front curvature of − 3.0538 and a rear curvature of − 0.4394. Let the indices of refraction of the front and rear lenses be 1.51115 and 1.61624, respectively, with respect to air, the system being used in air. Let the lenses have axial thicknesses of 0.0065 and 0.0050, respectively, the axial separation being 0.0005. Let the aperture be 1/30. Then the present method gives the following results, for a plane incident wave:

	B_1	A_2	B_3	A_4
Approaching first surface.......	1.	0.	0.	0.
Leaving " "	1.	0.34735	0.47208	0.53027
Approaching second "	0.99548	0.34578	0.46578	0.52570
Leaving " "	0.99548	1.29596	−5.55684	−3.53208
Approaching third "	0.99418	1.29427	−5.56713	−3.56175
Leaving " "	0.99418	0.22537	0.94578	0.16635
Approaching fourth "	0.99191	0.22486	0.94914	0.16785
Leaving " "	0.99191	0.49663	0.43533	0.30213

The focal point is 0.99057 beyond the axial point of the last surface. The usual theory gives this value as 0.99059. The longitudinal aberration here is 1.03034 per unit aperture or 0.0011448, the usual theory giving 0.0011446.

While some of the formulas have been given for a refraction between two propagations, it is usually more convenient to treat the cases of refraction and of propagation separately, especially in numerical calculations. The formulas so divided are readily adapted to the use of a calculating machine, although not to the use of logarithms.

As a particular case,[1] let us consider the case of a plane wave refracted through a single spherical surface of curvature k, and let us write μ for μ'/μ''. Let us take the optical height as parametric. Then

$$\left.\begin{aligned} x &= a, \\ y &= \alpha, \\ \xi &= a + \tfrac{1}{2}k\alpha^2 + \tfrac{1}{8}k^3\alpha^4, \\ \eta &= \alpha, \end{aligned}\right\} \qquad \left\{\begin{aligned} A_0 &= C_0 = a, \\ A_2 &= A_4 = B_3 = D_3 = 0, \\ B_1 &= D_1 = 1, \\ C_2 &= \tfrac{1}{2}k, \\ C_4 &= \tfrac{1}{8}k^3, \end{aligned}\right.$$

$$\left.\begin{aligned} \frac{C_2}{D_1^2} &= \frac{1}{2}k, \\ \frac{C^4}{D_1^4} - \frac{2C_2D_3}{D_1^5} &= \frac{1}{8}k^3, \end{aligned}\right\} \qquad \left\{\begin{aligned} F_0 &= C_0, \\ F_2 &= \tfrac{1}{2}k, \\ F_4 &= \tfrac{1}{8}k^3. \end{aligned}\right.$$

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[1] See, *e.g.*, Preston, Theory of Light, § 73.

The refracted wave has

$$
\begin{aligned}
B_1 &= 1, \\
A_2 &= \tfrac{1}{2}k(1 - \mu), \\
B_3 &= \tfrac{1}{2}k^2\mu(1 - \mu), \\
A_4 &= \tfrac{1}{8}k^3(1 - \mu)(1 + 2\mu - 2\mu^2),
\end{aligned}
$$

The second order longitudinal aberration is

$$\lambda_2 = \frac{k\mu^2}{2(\mu - 1)},$$

This shows that if the optically denser medium is on the concave side of the refracting surface, the surface is over-corrected for aberration, for λ_2 is positive which means that the cusp of the caustic is towards the lens surface. The focal length of the surface is $B_1^2/2A_2 = 1/k(1 - \mu)$.

While no attempt has been made to apply the foregoing results to the problems of practical optics, it is to be hoped that these problems may be materially simplified by the proper choice of parameters to meet the demands of each particular case. Since the geometrical significance of the parameter is of no essential value, there is more freedom in the choice of parameters than is customary in most treatments of geometrical optics. To review what has been said, the use of a single parameter, besides the azimuth, clarifies the interpretation of orders of small quantities. The recursion formulas, furnish terms of all orders, to be used when occasion demands. Since the sphericity of the lens surfaces has no essential part in the method, it follows that the method is directly applicable to the cases of aspherical lenses. Besides furnishing the final surface as a whole, the present method gives a point by point correspondence between all the surfaces of the system. While many of the quantities involved become indeterminate at a focal point, this furnishes no special difficulty in tracing a surface through a focal point.

In closing, the writer wishes to thank Professor Arthur C. Lunn, who suggested the present investigation and who made numerous helpful suggestions during its development.

AN EXTENSION OF THE RANGE OF THE McLEOD GAUGE.

BY A. H. PFUND.

SYNOPSIS.[1]

Modification of the McLeod Gauge.—In order to extend the range of a McLeod gauge, the pressure of the gas which is forced into the capillary is measured by means of *a hot-wire gauge.* Since the ratio of the compression is known, it is possible to determine the true pressure. The hot-wire gauge consists of a loop of tungsten wire, sealed into the closed end of the capillary and connected so as to form part of a Wheatstone bridge. The hot wire gauge is calibrated directly against the McLeod gauge. Because of the fact that the cooling effect varies with the character of the gas, a separate calibration for each gas is necessary.

Tests Made on the Gauge with Air.—Careful tests prove that the results obtained for air, the only gas used, are reliable. *The limit of the hot-wire McLeod gauge* was found to be 1.7×10^{-7} mm. hg. or 2.25×10^{-4} bar, while that of the unaided McLeod gauge was found to be 5.26×10^{-5} mm. Hg. The range of the McLeod gauge is therefore extended three hundred fold. By using a sensitive wall galvanometer in place of the portable galvanometer, a sensibility of 3.4×10^{-10} mm. or 4.5×10^{-1} bar was realized.

THE sensibility limit of a McLeod gauge has been reached when the length of the trapped air column and the difference in level of the two mercury column's are of the order of magnitude of 1 mm. Under these conditions, however, the trapped air is subjected to the pressure of 1 mm. of merucry—which is large from the standpoint of high vacua. If it were possible to measure the pressure of this trapped air, then, knowing the ratio of compression, it would be possible to extend greatly the range of the McLeod gauge.

This idea is realized by sealing a fine loop of tungsten wire into the top of the closed capillary. Using this arrangement (which is the well-known "hot-wire" gauge) in conjunction with a Wheatstone Bridge and a portable galvanometer, it is possible to extend the range of the McLeod gauge about 300-fold.

DESCRIPTION OF APPARATUS.

The apparatus used is shown in Fig. 1. This particular form of the gauge was adopted because of the convenience of baking out the glass. This is accomplished by inverting a small electric furnace over the top of the gauge. The filament (f) was made of 2 mil tungsten wire and had an over-all length of 6 mm. The resistances r_1, r_2, r_3, forming part of a simple Wheatstone bridge, were made of No. 22 chromel wire. While no definite information can be given as to the highest temperature reached by the tungsten filament in a high vacuum, it may be stated that when the current flowing through the filament was increased to 130 m.a. the tip of the filament appeared faintly red in a darkened room. The filament current actually used was about 100 m.a. For several reasons the bore of the capillary was chosen large: 2.0 mm. Not only is the range of the gauge on the high-pressure side increased but such defects as mercury sticking to the walls, negative pressures, etc,. are reduced to a minimum. Careful tests showed that no measurable amount of gas was caught between the rising column of mercury and the glass walls of the closed capillary. No difficulty was experienced in sealing in the filament without distorting the adjacent portions of the capillary. In fact, filaments were sealed successfully into capillaries having a bore less than 1 mm. The large bulb (*B*) had a volume of about 60 c.c. and the ratio of the volume of a length of 1 mm. of capillary to that of the bulb was 5.26×10^{-5}. A Gaede mercury pump was used to produce a moderately high vacuum while a liquid-air trap containing cocoanut charcoal, introduced between pump and gauge, was used to attain the highest vacua.

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[1] Presented at Washington meeting of Am. Phys. Soc. (1920).

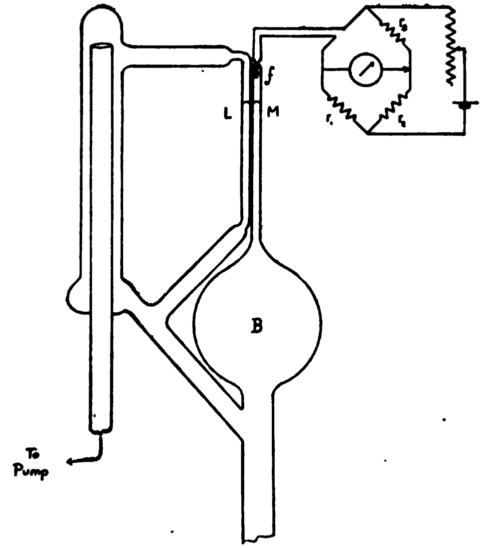

Fig. 1.

Calibration of Hot-wire Gauge.

Unfortunately, the indications of the hot-wire gauge are dependent upon the character of the gas used. This shortcoming is, however, not a vital defect, since this gauge may be calibrated against the McLeod gauge itself. The mode of procedure consisted in surrounding the charcoal bulb with liquid air so as to remove all vapors, hydrocarbon, water, etc., from the air which had been admitted into the pump. After measuring the pressure by means of the McLeod gauge, the resultant steady galvanometer deflection was noted (bulb and capillary empty). By carrying out a number of such pairs of observations at different pressures, a calibration curve of the type shown in Fig. 2 was obtained.

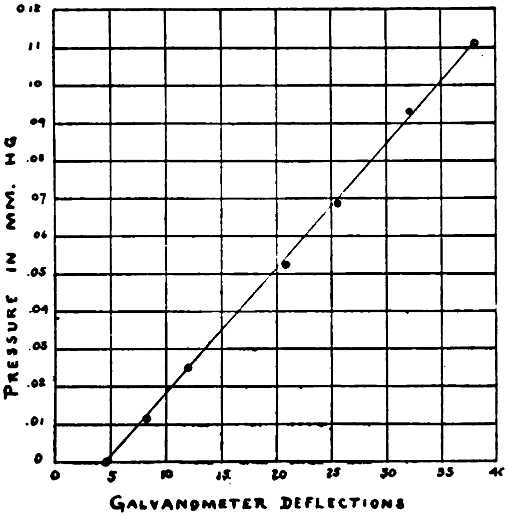

Fig. 2.

It is apparent that the points plotted lie, very approximately, on a straight line. Since mercury vapor is always present, it is evident that only partial pressures, to the exclusion of that of mercury vapor, are being measured.

Reliability and Accuracy.

Upon reducing the pressure below the point at which any difference of level in the two mercury columns L and M (Fig. 1) is discernible, the hot-wire gauge still shows large deflection. As previously stated, this is due to the fact that the original, large volume of gas, after having been forced into the capillary, now exists at a relatively high pressure and hence cools the hot tungsten filament. Since the hot-wire gauge is *used* when the gas is compressed into a limited volume, while it is *calibrated* when capillary and bulb are empty, it is entirely possible that the original calibration does not apply under the conditions attendant upon the actual use of the gauge. To test this point, a certain volume of air was compressed so that it filled the entire capillary; then, by successive steps, the volume was decreased until the mercury stood 2 mm. below the tip of the tungsten filament. Galvanometer readings, corresponding to these successive volumes were recorded. A characteristic series of results is presented in Table I.

TABLE I.

h	Galvanometer Reading.	P' mm.	P_0 mm.
∞	4.6		
60 mm.	7.0	0.0084	2.66×10^{-5}
30	10.0	0.0175	2.76
15	15.0	0.035	2.76
10	19.9	0.051	2.68
8	23.7	0.064	2.70

h = length of air-column in closed capillary.
P' = pressure of gas in closed capillary.
P_0 = pressure in system outside of gauge.

Referring back to the original calibration curve (Fig. 2) the pressures P' in the capillary were evaluated. Since the ratio of a compression is known in each case, it is possible to calculate P_0, the true pressure of the gas in the vacuum system outside the gauge. If the complications, previously referred to, exist, then P_0 might be expected to vary pronouncedly. As a matter of fact P_0 remains nearly constant even though the top of the mercury in the closed capillary is at widely different distances from the filament.

While the constancy of P_0 *points* to the conclusion that the gauge-readings are correct, it does not *establish* the proof. Most fortunately, the McLeod and hot-wire gauges overlapped in a small range of pressures, so that a decisive test was possible. The results are presented in Table II

TABLE II.

	McLeod Gauge.			Hot-wire Gauge.
	h	Δ	P_0	
I.	8.0	1. mm.	2.3×10^{-4} mm.	2.0×10^{-4} mm.
II.	6.0	0.8	3.35×10^{-4}	4.0×10^{-4} mm.

h = length of trapped air-column in mms.
Δ = difference in level of the two mercury columns in mm.
P_0 = true pressure in mm.

The agreement is well within experimental error which is rather large in the case of the McLeod gauge because of the fact that differences in level (Δ) amounting to a millimeter or less must be read.

An attractive feature of the mode of procedure here presented is the ease with which the hot-wire gauge may be calibrated. It is only necessary to obtain galvanometer deflections (bulb and capillary empty) at two known pressures, preferably as far apart as possible. As a final test, the two pressures were respectively near the upper and lower limits of the McLeod gauge. Upon laying off these two points on coordinate paper, as in Fig. 2, and connecting them by a straight line, a calibration curve was obtained which, when tested for constancy of P_0, showed an average deviation from the mean of only 2.2 per cent.

The range of pressures which may be measured with this arrangement and the magintude of the lowest pressures detectable are rather remarkable. Since the scale divisions on the portable galvanometer were about 3 mm. long, it was easy to read deflections to one-tenth division. Readings were reliable only to .2 divisions so, calculating the galvanometer sensibility on this basis it turned out to be 2.4×10^{-6} amperes. With this combination of gauges as used, it is possible to cover the entire range of pressures extending from 0.275 mm. to 1.7×10^{-7} mm Hg (or 2.25×10^{-4} bars). It is, of course, obvious that the limit of sensibility is determined only by that of the galvanometer. Since the limit of the unaided McLeod gauge is 5.26×10^{-5} mm. it appears that, through the addition of the hot-wire gauge, pressures as low as 1/300th of the above may be measured.

At a time when the true pressure P_0 was 8.4×10^{-7} mm. and the portable galvanometer yielded a deflection of 1 division, a wall galvanometer of sensibility 5×10^{-9} amp. was substituted. As might have been foreseen, a deflection too large to read was obtained. While the calculated sensibility was 3.4×10^{-10} mm. or 4.5×10^{-7} bars it is not feasible to use so sensitive a galvanometer unless the Wheatstone bridge, hot-wire gauge, etc., are surrounded by constant temperature oil-baths, as is customary in bolometer practice.

Johns Hopkins University,
February, 1921.

NEW BOOKS.

Conrse de Cinematique Theorique. By H. LACAZE. Paris: Gauthier-Villars et Cie, 1929. Pp. 1 + 138.

The five chapters of this brief treatise deal respectively with:

Vectors; Kintmatics of a point; Movement of a Solid; The Composition of Accelerations; The Displacement of a Vector in a Plane.

More than half of the volume is given to appendices in which the subject matter is developed in somewhat greater detail.

Physical Laboratory Manual. By E. L. CHAFFEE AND F. A. SAUNDERS. Cambridge: Harvard University Press, 1920. Pp. vi + 138.

A set of about fifty well-chosen elementary experiments forming the laboratory portion of an undergraduate course in general physics. The treatment is chiefly quantitative and the distribution of material among the various parts of physics a reasonable one.

Thèses Prèsentees A La Faculté des Sciences de L'Université de Paris. By M. PAUTHENIER. Paris: Masson et Cie, 1920. Pp. 1 + 68.

M. Pauthenier's thesis deals with a new method of producing instantaneous charges, with applications to the Kerr Effect.

The New Physics. By ALBERT C. CREHORE. San Francisco: Journal of Electricity, 1920. Pp. 1 + 111.

This little volume contains a non-mathematical statement of the author's electromagnetic theory of gravitation; a theory which connects the gravitational force directly with the motion of the electrons of matter and which is to be regarded as supplementary to the Einstein theory.

Laboratory Projects in Physics. By FREDERICK F. GOOD. New York: The Macmillan Company, 1920. Pp. xiii + 267.

A school book in which "in accordance with recent tendencies in the teaching of science" numerous familiar household appliances and mechanisms are utilized in the illustration of the principles of physics. The applications are more than commonly sound and successful.

Report of the Chief Signal Officer United States Army to the Secretary of War. Washington: Government Printing Office, 1920. Pp. 1 + 64.

This report deals chiefly with the activities of the U. S. Signal Corps at home and abroad since the signing of the armistice.

Lessons in Mechanics. By WM. S. FRANKLIN AND BARRY MACNUTT. Bethlehem, Pa.: Franklin and Charles, 1919. Pp. xi + 221. Price, $2.00.

Eminent Chemists of our Time. By BENJAMIN HARROW. New York: D. Van Nostrand Company, 1920. Pp. xvl + 248. Price, $2.50 net.

A volume of very readable sketches of Perkin, Mendeléeff, Ramsay, Richards, Van' T Hoff, Arrhenius, Moisson, M. Curie, V. Meyer, Remsen and E. Fischer.

Lessons in Heat. By WM. S. FRANKLIN AND BARRY MACNUTT. Bethlehem, Pa.: Franklin and Charles, 1920. Pp. xi + 147. Price, $2.00.

Lessons in Electricity and Magnetism. By WM. S. FRANKLIN AND BARRY MACNUTT. Bethlehem, Pa.: Franklin and Charles, 1919. Pp. xvi + 254. Price, $2.25.

These three companion volumes are planned for use in elementary undergraduate courses in Physics. They are concise and concrete and conform throughout to the method of treatment characteristic of the authors.

Second Series *August, 1921* *Vol. XVIII, No. 2*

THE

PHYSICAL REVIEW.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE 109TH MEETING, WASHINGTON, APRIL 22 AND 23, 1921

THE 109th regular meeting of the American Physical Society was held at the Bureau of Standards in Washington, D. C., on Friday and Saturday, April 22 and 23, 1921. There were four sessions for the reading of papers. President Lyman presided and there were about 250 members and friends in attendance.

At the meeting of the Council held on April 22, 1921, fourteen persons were *elected to membership*, as follows: E. C. Bain, Ben C. Bellamy, James M. Douglas, Arthur R. Bauder, George Dobson, H. E. Fox, Charles F. Hagenow, L. E. Hildebrand, E. H. Kurth, J. W. Miles, Charles L. Swisher, J. R. Jenness, Victor C. Stechschulte, John R. Weeks, Jr.

It was voted that the Society coöperate with the International organization which has charge of the publication of Annual Tables of Constants.

The program contained 64 papers, twelve of which were read by title only. The program is given below:

Crystal Structure of Titanium, Zirconium, Cerium, Thorium and Osmium. ALBERT W. HALL.

The Width of X-Ray Spectral Lines. L. GILCHRIST.

Characteristic Low Voltage X-Radiation from Arcs in Metallic Vapors. F. L. MOHLER and PAUL D. FOOTE.

Secondary High Frequency Radiation. A. H. COMPTON.

Polarization of Secondary X-Rays. A. H. COMPTON and C. F. HAGENOW.

A Remeasurement of the Radiation Constant, by Means of X-Rays. WILLIAM DUANE, H. H. PALMER and CHI-SUN YEH.

Soft X-Rays of Characteristic Type. E. H. KURTH.

The Decrease of the Mobility of the Positive Ions in Air with Age. HENRY A. ERIKSON.

The Absolute Sizes of Certain Monovalent Ions. WHEELER P. DAVEY.

Forces Within a Static Atom. IRVING LANGMUIR.

Effect of Roughening of Surface on the Dispersion of Cannon Ball in Falling Through Air. F. C. BROWN.

On The Determination of Instantaneous Pressures in Guns by Means of a Piezo-Electric Gage and a Ballistic Galvanometer. J. C. KARCHER.

A New General Law of Deformation. P. G. NUTTING. (Read by title.)

Contractions and Expansions of Amalgams with Time. ARTHUR W. GRAY.

Measurement of Mercury Vapor Pressure from 0° to 35° by the Knudsen Pressure Gauge. C. F. HILL. (Read by title.)

Thermal Conductivity of White Marble and "Neat" Cement. ROY A. NELSON. (Read by title.)

The Effect of Pressure on the Thermal Conductivity of Metals. P. W. BRIDGMAN.

The New Fixed Point on the Thermometric Scale. FREDERICK BATES and F. P. PHELPS.

A Disappearing Filament Optical Pyrometer Free from Diffraction Effects at the Filament. C. O. FAIRCHILD.

The Organ Pipe as a Coupled System. ARTHUR TABER JONES.

On the Measurement of the Acoustic Output and Efficiency of Fog-alarm Apparatus. LOUIS V. KING.

A Simple Apparatus for Testing Pitch Control. PRESTON EDWARDS.

Some New Consequences of the Theory of Ultimate Rational Units. GILBERT N. LEWIS.

Present Status of the Electric and Magnetic Units. J. H. DELLINGER.

Discussion of a Kinetic Theory of Gravitation II., and Some New Experiments in Gravitation. CHARLES F. BRUSH.

Defects in Symmetric Optical Systems. IRWIN ROMAN. (Read by title.)

The High-Frequency Resistance of Inductance Coils. G. BREIT. (Read by title.)

A Method of Measuring Coil Capacities and Standardizing Wavemeters. G. BREIT. (Read by title.)

Tables for the Calculation of the Inductance of Circular Coils of Rectangular Cross Section. FREDERICK W. GROVER. (Read by title.)

On Some New Formulæ for the Direct Numerical Calculation of the Coefficient of Mutual Induction of Coaxial Circles. LOUIS V. KING.

Note on the Earth's Magnetic Field. W. F. G. SWANN.

The Electrical Resistance of the Human Body. N. FORMAN, J. S. MARTIN and F. WENNER.

New Methods for Maintaining Constant Frequency in High-frequency Circuits. W. G. Cady.

Magnetic Rotatory Dispersion in Transparent Liquids. R. A. Castleman, Jr., and E. O. Hulburt.

A Direct Reading Spectrophotometer for Measuring the Transmissivity of Liquids. Irwin G. Priest.

The Excitation of Enhanced Spectra in Low Voltage Arcs. Paul D. Foote, W. F. Meggers and F. L. Mohler.

On the Photoelectric Effect of Alkali Vapors. Jakob Kunz.

Photo-Electric Potentials from the Retina. E. L. Chaffee and W. T. Bovie.

The Variation of Metallic Conductivity with Electrostatic Charge. Henry A. Perkins.

The Distributed Capacity of Inductance Coils. G. Breit.

Some Effects of the Distributed Capacity of Coils to Ground. G. Breit. (Read by title.)

Total Emissive Powers and Resistivities of Tungsten at Incandescence. A. G. Worthing and W. E. Forsythe.

Color Temperature of High Efficiency Lamps. W. E. Forsythe.

The Detecting Efficiency of the Resistance-Captivity Coupled Electron Tube Amplifier. E. O. Hulburt.

Radio Signal Fading Phenomena. J. H. Dellinger and L. E. Whittemore.

Some Physical Problems of Aircraft Radio. L. E. Whittemore.

The Design of a Standard Wavemeter. R. T. Cox.

The High-Frequency Impedance of Radio Telephone Receivers. C. T. Zahn.

A Record of Radio Time Signals. E. A. Eckhardt and J. C. Karcher.

An International Series of Radio Audibility and Directional Measurements. Carl Kinsley.

Detection Coefficient of Thermionic Valves. E. L. Chaffee.

The Radio Research Field. J. H. Dellinger and L. E. Whittemore.

On Emergent β Rays from Materials of Different Atomic Weight and Different Thickness Produced by γ Rays; Studies by the Statistical Method. Alois F. Kovarik. (Read by title.)

High Frequency Limits of X-Ray Spectra at Different Angles with the Cathode Stream. D. L. Webster.

A Generalization of the Electrodynamic Equations with Applications to the Structure of the Electron and Radiationless Orbits. Leigh Page.

Resistance and Inductance of a Three-Phase Cable. Harvey L. Curtis.

Thickness of Stratified Soap Films. P. V. Wells. (Read by title.)

Temperature of an Electrically Heated Filament. Chester Snow.

Standard Wave Lengths and the Constant Frequency Differences in the Spectra of Higher Gases. W. F. Meggers.

Note on the Diurnal Variation of the Atmospheric Electric Potential Gradient. S. J. Mauchly.

Peculiar Ice Formations. H. G. Dorsey.

On the Exploded Wire Spectrum of Calcium. R. A. Sawyer and A. E. Becker.

On the Radiation of Energy from Coils. A. G. Webster. (Read by title.)

On the Vibration of Gun Barrels. A. G. Webster. (Read by title.)

George B. Pegram,
Secretary pro tem.

CRYSTAL STRUCTURE OF TITANIUM, ZIRCONIUM, CERIUM, THORIUM AND OSMIUM.

By Albert W. Hull.

X-ray powder diffraction patterns of these metals were taken in the manner previously described,[1] using a molybdenum water-cooled X-ray tube, with zirconium filter. The positions of the lines on the patterns were measured on a slide rule, and the corresponding planar spacings calculated by the Bragg formula.

Since there is no crystallographic data regarding these substances, it was necessary to consider all possible crystal systems and axial ratios. For the hexagonal and tetragonal systems (including cubic) plots were available[2] giving theoretical planar spacings as a function of axial ratio. Fortunately these plots were found to be sufficient, four of the metals being hexagonal, and the fifth cubic. The entire process of discovering the crystal system, axial ratio, type of lattice, and dimensions of the lattice was thus accomplished in a few minutes.

Titanium was found to crystallize in the hexagonal system, holohedral class, with axial ratio 1.59 ± .01. The arrangement of atoms is a slightly distorted form of "hexagonal close packing" of solid spheres, being shortened in the direction of the principal axis in the ratio 1.633 : 1.59. It is a close packed arrangement of oblate spheroids whose axes of revolution are 2 per cent. shorter than the lateral axes.

The side of the unit triangular prism is 2.97 Å, and its height 4.72 Å. The lattice is made up of two sets of these triangular prisms, the atoms of one set being in the center of the prisms of the other set.

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[1] Phys. Rev., 10, 661, 1917.

[2] Phys. Rev., 17, 549, 1921.

Zirconium has a crystal structure identical with Titanium, with axial ratio also 1.59. The side of its unit triangle is 3.23 Å and its height 5.14 Å.

Cerium shows the same structure as Ti and Zr, with axial ratio 1.62. The accuracy of the data is not sufficient in this case to distinguish with certainty between the experimental value 1.62 and true hexagonal close packing of spheres, 1.633. The side of the elementary triangle is 3.65 Å, and height 5.96 Å (taking $c = 1.633$).

There is present also a face centered cubic form, with side of cube 5.12 Å giving the same density as the hexagonal form. Cerium thus appears to be composed of a mixture of the hexagonal and cubic forms of close-packed arrangements of atoms, like cobalt. The effect of annealing has not been tried. It is possible, however, that the apparent dimorphism is due to impurity, as the sample, though supposed to be very pure, has not been analyzed and its pattern was almost identical with that of "misch metal," which contains about 8 per cent. iron. Further investigations are in progress.

Thorium is a face-centered cubic lattice with side of cube 5.04 Å.

The atoms of these four metals are very similar in structure, each being composed of a stable symmetrical shell (inert gas) surrounded by four electrons. In the case of the lighter elements of similar structure, C and Si, the four external electrons give perfect tetrahedral symmetry to the atom, as evidenced both by chemical properties and crystal structure. With the heavier metals here investigated the effect of the four outer electrons is much less marked. In the case of Ti and Zr, the crystal structures indicate that the four outer electrons merely increase the peripheral dimensions of the spherical shell, making it a slightly oblate spheroid; while in the case of Ce and Th the spherical symmetry is not affected at all—the atoms pack like perfect spheres.

Osmium is likewise found to crystallize in a hexagonal close packed lattice, with axial ratio 1.59. The side of the unit triangle is 2.714 Å, and its height 4.32 Å.

The analysis of osmium completes the eighth group metals. The results may be briefly summarized:—The second and third elements of each row, viz., Co, Ni, Rh, Pd, Ir, Pt, are *face centered cubic arrangements.* The first members, Fe, Ru and Os, are different from the rest. Fe is *body centered cubic;* Ru and Os are *hexagonal close packed arrangements of oblate spheroids*, the axial ratio of each being 1.59.

Research Laboratory,
General Electric Co.,
April 5, 1921.

The Width of X-ray Spectral Lines.

By L. Gilchrist.

Since certain instrumental factors which affect the width of spectral lines of ordinary light are not effective in the case of spectral lines, it would appear to be reasonably justifiable to investigate the conditions that might exist in the source of the x-rays, *i.e.*, in the target of the x-ray tube which would give the line width.

A consideration of the Doppler effect which was presented some time ago led to a value for the velocity of the source which was much too high, or conversely if the velocity of the source in the target is lower than that of an atom of gas, as given by the kinetic theory the width produced thereby would be inappreciable and would probably be masked by the widening produced from other causes.

The assumptions are now made that

(*a*) The x-rays are produced only by the impinging electrons which penetrate close to the nucleus of the atom to approximately 10^{-12} cm. from the center.

(*b*) The wave-length produced is given by

$$\text{P.D.} \times e = h\gamma = \frac{hc}{\lambda}$$

or

$$\tfrac{1}{2}mv^2e = \frac{hc}{\lambda},$$

where m = mass of electrons,
v = velocity of electrons,
c = velocity of x-rays.

(*c*) The line is widened because of the motion of the atom so that

$$(\tfrac{1}{2}M\xi^2 + \tfrac{1}{2}mv^2)e = \frac{hc}{\lambda_1}$$

where ξ is the component of the velocity of the atom along the direction of the impinging electron.

I. To obtain maximum change in wave-length and the resolving power necessary to show it, use the formula—

$$\text{P.D.} \times e = h\gamma = \frac{hc}{\lambda} \text{ and P.D.} = \tfrac{1}{2}mv^2,$$

where m = mass of the impinging electron,
e = charge of the electron,
v = velocity of the impinging electron.

Then

$$\tfrac{1}{2}mv^2e = \frac{hc}{\lambda}.$$

Then the changed wave-length is given by—

$$\tfrac{1}{2}mv^2e + \tfrac{1}{2}MV^2e = \frac{hc}{\lambda_1},$$

where V = the velocity of the atom in the target,
M = the mass of the atom.

$$\therefore \quad \frac{\frac{1}{2}MV^2e}{\frac{1}{2}mv^2e} = h\frac{\left(\frac{1}{\lambda} - \frac{1}{\lambda_1}\right)}{h\frac{1}{\lambda}} = \frac{\frac{\lambda - \lambda_1}{\lambda\lambda_1}}{\frac{1}{\lambda}} = \frac{\lambda - \lambda_1}{\lambda_1} = \frac{d\lambda}{\lambda_1};$$

or resolving power necessary to show this when a tungsten target is used is

$$\frac{\lambda_1}{d\lambda} = \frac{mv^2}{MV^2} = \frac{(10^{10})^2}{4 \times 184 \times 1800 \times 10^{10}},$$

if $V = 10^5$ cm. per sec. $M = 184 \times 1800 \times$ mass of the electron.

$$\therefore \frac{\lambda_1}{d\lambda} = \frac{10^8}{4 \times 184 \times 18} = \frac{10^4}{1.3248} = 7.5 \times 10^3,$$

i.e., the resolving power.

(*a*) $R = 7.5 \times 10^3$ which is necessary to show this width. If the velocity of the atoms is 10^4 cm. per sec. the resolving power necessary to show the maximum width is

(*b*) $R = 7.5 \times 10^5$ cm. per sec., or for (*a*) a change in λ of

$$d\lambda = \frac{1}{7.5 \times 10^3}$$

of λ_1 would have to be measurable or for (*b*) a change in λ of

$$d\lambda = \frac{1}{7.5 \times 10^5}$$

of λ_1 would have to be measurable. Even the latter would appear to be detectable and even measurable, unless the slits through which the x-ray beam passes are too wide. Even then it would appear that sections of a wedge-shaped slit could be used to eliminate the effect of the slit in much the same way as Merton at Oxford uses a wedge for spectral lines of ordinary light.

II. To obtain the effect of a change of temperature in the target from 300° absolute to 2100° absolute

$$\frac{\lambda_1}{d\lambda} = \frac{mv^2}{M\frac{2100}{300}V^2} = \frac{mv^2}{7MV^2}$$

i.e.,

$$d\lambda = \frac{7}{7.5 \times 10^3} \text{ of } \lambda_1$$

or 7 × as large as in 1 (*a*) or 1 (*b*).

III. To obtain the distribution of intensity in the x-ray spectral line which will be a representation of the distribution of intensity in the source of the x-ray, the expression for the intensity is

$$I = 2\int_{\lambda_1}^{\lambda_2} \psi(\lambda)\left(1 + \cos 4\pi\frac{D}{\lambda}\right)d\lambda$$

where $2D$ is the path difference between the interfering beams.

Now let Λ = natural wave-length N = natural frequency.
λ = actual wave-length n = actual frequency.

Now

$$\frac{\lambda}{\Lambda} = \frac{\frac{1}{2}mv^2 - \frac{1}{2}M\xi^2}{\frac{1}{2}mv^2} = 1 - \frac{M}{m}\frac{\xi^2}{V^2},$$

where ξ is the component velocity of the atom in the direction of the moving electron, or

$$\frac{\Lambda}{\lambda} = \frac{\frac{1}{2}mv^2}{\frac{1}{2}mv^2 - \frac{1}{2}M\xi^2} = \left(1 + \frac{M}{m}\frac{\xi^2}{V^2}\right),$$

or

$$\frac{\frac{1}{\lambda}}{\frac{1}{\Lambda}} = \left(1 + \frac{M}{m}\frac{\xi^2}{V^2}\right),$$

or

$$\frac{1}{\lambda} = \frac{1}{\Lambda}\left(1 + \frac{M}{m}\frac{\xi^2}{V^2}\right);$$

$$\therefore \frac{I}{2} = \int_{-\infty}^{+\infty} e - \beta\xi^2\left\{1 + \cos 4\pi\frac{D}{\Lambda}\left(1 + \frac{M}{m}\frac{\xi^2}{V^2}\right)\right\}d\xi$$

$$= \int_{-\infty}^{+\infty} e - \beta\xi^2\left\{1 + \cos 4\pi\frac{D}{\Lambda}\cos\left(4\pi\frac{D}{\Lambda}\frac{M}{m}\frac{\xi^2}{V^2}\right)\right\}d\xi.$$

Now

$$\int_{-\infty}^{+\infty} - \beta\xi^2 d\xi = \sqrt{\frac{\pi}{\beta}}.$$

Now put in

$$V = \frac{2}{\sqrt{\pi\beta}} \text{ or } \sqrt{\beta} = \frac{2}{\sqrt{\pi}\,V} \text{ or } \beta = \frac{4}{\pi V^2},$$

where

V = the root mean square velocity of the atom.

Then

$$I = \pi V + \cos 4\pi\frac{D}{\Lambda}\sqrt{\frac{\frac{\pi}{2}\frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{D}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{D}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}$$

$$+ \sin 4\pi\frac{D}{\Lambda}\sqrt{\frac{\frac{\pi}{2} - \frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{D}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{D}{\Lambda}\frac{M}{m}\frac{1}{v^2}\right)^2}}. \qquad (5)$$

From this V cannot be evaluated directly since the amplitude of the x-ray pulse at its source has been arbitrarily put equal to unity. But when $2D = 0$ I is a max, then

$$I \max = \pi V + \sqrt{\frac{\frac{\pi}{2}\,\frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{\Delta}{\Lambda}\,\frac{M}{m}\,\frac{1}{v^2}\right)}}{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{\Delta}{\Lambda}\,\frac{M}{m}\,\frac{1}{v^2}\right)^2}}$$

$$= \frac{3}{2}\pi V.$$

And if $2D_1$ = path difference where

$$I = \frac{1}{10} I \max.$$

as shown by the spectral line. Then

$$\frac{3}{20}\pi V = \pi V + \cos 4\pi\frac{\Delta_1}{\Lambda}\left\{\sqrt{\frac{\frac{\pi}{2}\,\frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{\Delta_1}{\Lambda}\,\frac{M}{m}\,\frac{1}{v^2}\right)^2}}{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{\Delta_1}{\Lambda}\,\frac{M}{m}\,\frac{1}{v^2}\right)^2}}\right.$$

$$+ \sin 4\pi\frac{\Delta_1}{\Lambda}\left\{\sqrt{\frac{\frac{\pi}{2} - \frac{4}{\pi V^2} + \sqrt{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{\Delta_1}{\Lambda}\,\frac{M}{m}\,\frac{1}{v^2}\right)^2}}{\frac{16}{\pi^2 V^4} + \left(4\pi\frac{\Delta_1}{\Lambda}\,\frac{M}{m}\,\frac{1}{v^2}\right)^2}}\right. . \qquad (6)$$

From equation (6) V may be determined since the values of all the other terms are known.

This equation may be somewhat simplified by certain modifications and approximations but this may be left in abeyance until it is known whether the actual width of the source may be determined from the spectral line and whether the temperature effect on the source may be detected by a corresponding change in the width of the spectral line.

Observations on the L series of spectral lines seem to give a little better promise of measurable results than observations on the K series of lines, for if all the energy of the impinging electrons goes into the x-ray pulse produced, then, according to Whiddington[1] the velocity of the electrons is given by

$$v = (N - 15)10^8 \text{ cm. per sec.},$$

where N is the atomic number of the material in the target. For tungsten $N = 74$. Then the required resolving power in I (a) becomes $R = 2.5 \times 10^3$, and in I (b) becomes $R = 2.5 \times 10^5$.

D. L. Webster[2] shows that the relationship $P.D. \times e = h\nu$ does not hold as a relationship connecting the characteristic radiations or their components.

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[1] Phil. Mag., 39, June, 1920, pp. 694–696.

[2] Phys. Rev., VII., No. 6, July, 1916, pp. 599–613.

It is not assumed in these calculations that this relationship holds. But it is assumed that the small width which any characteristic line has is of the nature of a general radiation to which the equation will apply.

Further it would appear that the velocities of the atoms in the target as calculated from equation (6) are still too high even though they are much lower than those which were obtained on the assumption that the width of the line is due to the Doppler effect.

The following modifications have therefore been made

(*a*) The characteristic x-ray spectra lines are produced by a disturbance of the electrons in the inner orbits of the atom, *i.e.*, the only orbits which exist in the case of hydrogen, helium, lithium.

(*b*) The ordinary ultra-violet, visible, and infra-red spectral series of hydrogen are its characteristic x-ray spectral groups.

(*c*) It is assumed that the critical velocity of the electrons necessary to produce the characteristic lines of Tungsten is of the order of 10^{10} cm. per second and the corresponding velocity necessary to produce the characteristic lines of hydrogen is of the order of 10^7 cm. per second. This is supported by the speed of the electrons emitted by a metal when these radiations fall upon it.

(*d*) The further assumption is made that this critical velocity of the electrons, viz., 10^7 cm. per second, which is required to produce hydrogen lines, is the same for all substances, but in the case of those atoms with a large number of electronic orbits about the nucleus it is necessary that the impinging electrons should have a higher velocity than this in order that they may penetrate to the inner orbits, and in the process of penetration their velocity is decreased in a manner similar to that of β particles shot into a cloud forming atmosphere. The critical velocity of the electrons for the production of x-rays is $\therefore = 10^7$ cm. per sec. and is independent of the nature of the target.

Some of the spectral lines given by William Duane,[1] have been examined and their widths and the angular distance between the successive orders measured and by means of equation (6) the velocities of the atoms in the targets have been calculated.

The results show that the velocities are of the order—12 to 14 meters per second.

UNIVERSITY OF TORONTO,
April 2, 1921.

CHARACTERISTIC LOW VOLTAGE X-RADIATION FROM ARCS IN METALLIC VAPORS.

BY F. L. MOHLER AND PAUL D. FOOTE.

A SEARCH for critical potentials above the resonance potentials and first ionization potentials has been made by the following methods. The metals were boiled in tubes containing four electrodes, a hot wire cathode surrounded by two grids and a cylindrical plate. A variable accelerating potential V between the cathode and inner grid maintained the arc. With the outer grid negative and the plate positive all electrons and ions are shielded from the plate and the photo-electric effect of the arc radiation on the plate can be measured as a function of V. With the plate negative the ionization can be measured. The plate current divided by the cathode current was plotted against the accelerating potential. It happens that the radiation voltage curves are then straight lines between critical potentials, so that the critical radiating potentials are shown very sensitively as points where the slope changes. Ionization points above the first are more difficult to locate. Initial potentials can be computed from the observed first ionization point. The following results have been obtained.

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[1] PHYS. REV., XVI., No. 6, Dec., 1920.

Potassium, a sharp break in the radiation curve at 23 ± 1 volts. With increased vapor pressure another break at 20 ± 1 volts. Ionization curves under some conditions show a break at 23 volts.

Sodium, a break in the radiation curve at 35 ± 1 volts much less pronounced than in potassium and not evident at low vapor pressure. A break was also evident in the ionization curves at 35 volts under some conditions.

Magnesium, a break in the radiation curve at 46 ± 1 volts. Above this the curve was a straight line. Below thirty volts there was marked curvature but the critical points were not measurable under the conditions of the experiments. The break at 46 volts was similar in intensity to that obtained in sodium.

The x-ray wave-length L_{α_1}, for magnesium can be computed by the law of Duane in the form $L_{\alpha_1} = K_a - K_\alpha$. It gives L_{α_1} at $\lambda = 263$ Å or a potential of 46.9 volts. The observed value was 46. For sodium L_{α_1} can not be directly computed as K_a has not been observed. Extrapolation by the Moseley relation of values of L_{α_1} obtained as above gives about 30 volts. The observed value 35 ± 1 may point to a marked deviation from Moseley's law at the lower end of the L series. However the observed point may be the potential required to doubly ionize the atom which would be higher than the limit of the L series by approximately the ionization potential, 5 volts. The experimental value agrees exactly with this hypothesis.

Theories of atomic structure indicate that the critical potentials observed in potassium should be related to the M series. It is impossible to compute this frequency accurately from x-ray data. All we can say is that the order of magnitude is correct. The principal point at 23 volts corresponds to $\lambda = 535$ Å.

Bureau of Standards, Washington, D. C.,
April 5, 1921.

SECONDARY HIGH FREQUENCY RADIATION.

By ARTHUR H. COMPTON.

THE experiments performed by Florance and Ishino on the scattering of gamma rays showed a considerable difference in hardness between the primary and the secondary radiation, an effect difficult to explain if the secondary radiation was truly scattered. An experimental method of distinguishing between truly scattered and fluorescent radiation was therefore devised. This consisted of placing an absorbing screen alternately in the path of the primary and of the secondary beam. Any truly scattered radiation should remain of the same intensity for both positions of the absorbing screen, while the fluorescent rays, being in general softer than the primary rays exciting them, would be more strongly absorbed when the screen is in the path of the secondary radiation. The ratio of the truly scattered to the total secondary rays is the limit of the ratio of the intensity observed with the screen in the path of the secondary to that when the screen is in the path of the primary beam as the thickness of the screen is increased.

The experiments showed that with gamma rays the amount of truly scattered radiation at angles greater than 90° as determined by this criterion is a negligible part of the total secondary rays. At 45° about 3 per cent. of the radiation seems to be of the scattered type.

Similar experiments were performed with x-rays of wave-lengths between 0.12 and 0.50 Å.U. using a modified Bragg spectrometer with a recording device to register the intensity of the secondary radiation for different angles of the ionization chamber. These rays showed an effect identical in character with that observed with gamma rays, but not so prominent. With $\lambda = 0.12$ Å.U. the scattered rays form about 15 per cent. at 90° with the primary beam and 70 per cent. of the total radiation at 30°. For $\lambda = 0.50$ the scattering is relatively much more important, but the fluorescent radiation is still a large part of the total radiation. These results, both in the case of the gamma rays and of the x-rays, seem to be independent of the material used as radiator except in those cases where a perceptible amount of characteristic fluorescent radiation enters the ionization chamber.

The experiments therefore indicate the existence of a type of fluorescent high frequency radiation whose wave-length is independent of the particular substance used as radiator, depending only on the frequency of the exciting primary rays.

The measurements made by Barkla, Crowther, Owen and others represented the total secondary radiation, only a part of which, according to the present measurements, represents truly scattered rays. A direct measurement of the absolute value of the scattering shows that at small angles the scattering of x-rays approaches that predicted according to Thomson's theory if the number of effective electrons per atom is equal to the atomic number. At large angles however, the scattering is much less than predicted by this theory, especially for the shorter wave-lengths, being less than 1/1,000th of the theoretical value in the case of gamma rays. The results are in much better accord with the hypothesis that the electron is of sufficient size (about 3×10^{-10} cm.) to give rise to interference effects between the rays scattered from its different parts.

Washington University,
Saint Louis, April 21, 1921.

The Polarization of Secondary X-Rays.

By A. H. Compton and C. F. Hagenow.

In the previous paper by one of us, evidence is presented for the existence of a type of fluorescent radiation whose wave-length is independent of the material of the radiator, being a function only of the wave-length of the primary radiation. The explanation suggests itself that this general fluorescent radiation may be the x-rays excited by the impact of the secondary cathode rays liberated in the radiator by the action of the primary radiation. While such an explanation accounts for many of the characteristics of this radiation, quantitative measurements showed that at least in some cases as much as 50 per cent. of the energy of the primary rays appeared as general fluorescent radiation. Since the efficiency of production of x-rays by cathode rays is less than 1 per cent., it does not seem that this can be the real origin of the secondary rays. It occurred to us that some information on this point might be obtained from a careful measurement of the polarization of the fluorescent radiation,

Barkla, in his classic measurement of the polarization of secondary x-rays, found that at 90° the rays were approximately 80 per cent. polarized. The remaining 20 per cent. might be accounted for in part by experimental error, and the remainder Barkla ascribed as due possibly to a real lack of polarization due to forces acting on the scattering electrons. We employed much more sensitive methods of measuring the tertiary radiation than those used by Barkla and his followers. The first radiator was placed about 12.5 cm. directly above the second radiator, which was in turn placed on the crystal table of a Bragg spectrometer. The degree of polarization could then be measured by noting the scattering from the second radiator parallel and at right angles with the primary beam. The geometrical conditions were such that at the 90° position about 4 or 5 per cent. of the radiation should appear to be unpolarized (the apertures being so large that the scattering angles were not exact right angles).

For thick blocks of scattering material our results confirmed those of Barkla, It was found, however, that when thin sheets of the radiators were employed. the polarization became much more complete. As one might have anticipated, multiple scattering in the case of the thick radiators was responsbile for a considerable lack of polarization. Extrapolating our results to zero thickness we find the ratio of the intensity of the tertiary rays at 90° to that at 0° to be for a paper radiator, 0.05, for aluminium, 0.04 and for sulphur, 0.05. When these results are corrected for the imperfection of the geometrical conditions, it means that the polarization is complete within experimental error.

Since under the conditions of our experiment approximately 70 per cent. of the total secondary radiation at 90° is of the general fluorescent type, this result means that the fluorescent as well as the scattered rays are completely polarized. Barkla's experiments show that the general radiation from an x-ray tube is only partly polarized, which makes it difficult to support the view that the general fluorescent radiation is excited in exactly the same manner as the primary x-rays. The hypothesis that this radiation is emitted at the instant of liberation of the secondary cathode rays from the atoms, however, seems to offer a possible explanation of both the degree of polarization and the efficiency of production of these fluorescent rays.

WASHINGTON UNIVERSITY,
SAINT LOUIS, April 21, 1921.

A REMEASUREMENT OF THE RADIATION CONSTANT, *h*, BY MEANS OF X-RAYS.

BY WILLIAM DUANE, H. H. PALMER AND CHI-SUN YEH.

SINCE the discovery of the fact[1] that the quantum law, $Ve = h\nu$, applies to the short wave-length limit of the continuous x-ray spectrum, a number of scientists have used the law in measurements of the value of h.[2] The most accurate measurement of h made in our laboratory by this method[3] gave the value $h = 6.555 \times 10^{-27}$. In this research we measured the voltage, V, by means of an electrostatic voltmeter, which we calibrated by passing a known current through a metallic resistance of several hundred thousand ohms.

Several years ago our laboratory purchased a series of resistance coils aggregating 10,000,000 ohms for the purpose of comparing high voltages directly with the E.M.F. of a standard cell by means of the simple potentiometer principle. The object of the research reported in this paper has been to measure the value of h using this high resistance potentiometer to measure the voltage, V. We took extreme care in insulating the various electrical circuits and instruments, and in determining the *ratios* of the several resistances. Two Weston cadmium cells served as standards of E.M.F., for each of which the Bureau of Standards had furnished a certificate, the error mentioned in the certificates being .01 per cent. Considering the fact that the ratios of resistances can be determined with great precision, and that we had two standard cells at our disposal, it seems probable that our error in measuring the voltage, V, did not exceed .01 per cent. This means that the error in V is negligible as compared with those in the other quantities entering into the above quantum equation.

In measuring the wave-lengths (and frequencies, ν) we used a somewhat better x-ray spectrometer than that employed before, and a different calcite crystal. The x-ray tube had a thin mica window attached to it through which the x-rays emerged. This diminished the absorption of the x-rays, and enabled us to use a narrower spectrometer slit. We thereby reduced the correction for the slit's width to about 45″ of arc.

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[1] Duane and Hunt, PHYS. REV., Aug., 1915, p. 166.

[2] See "Data Relating to X-Ray Spectra," by William Duane, published by the National Research Council.

[3] Blake and Duane, PHYS. REV., Dec., 1917, p. 624.

The glancing angle of incidence, θ, could be determined in a good experiment to within about 7″. As θ averaged about 290′ this means a precision of one part in 2,500.

As the weighted mean value of $V \sin \theta$ in five complete experiments we obtained

$$V \sin \theta = 2040 \pm 1.$$

Our 1917 value of $V \sin \theta$ was 2040 ± 3 about.

Taking Millikan's value of $e = 4.774 \times 10^{-10}$ e.s.u. and the value $2d = (6.056 \pm .004) \times 10^{-8}$ cm.for the grating constant of calcite we get for the value of h

$$h = (6.556 \pm .009) \times 10^{-27}.$$

This does not differ from the value previously obtained by as much as the errors of experiment. It is a more accurate value, however, in that the error in the voltage measurement has been eliminated.

An interesting series of measurements of h by means of x-rays has been made recently by Wagner. His most recent value is 6.52×10^{-27}, which differs from ours by about half of one per cent.

Experiments with x-rays coming from the tube at an angle of 45° from the stream of cathode rays indicate that the value of h is the same for them as for x-rays at right angles to the stream.

Harvard University,
April 5, 1921.

Soft X-Rays of Characteristic Type.

By E. H. Kurth.

This paper covers some further results secured in the investigation which I reported upon at the last meeting of this society.

Radiations from several other elements including oxygen have been investigated. The latter element was worked with in the compound copper oxide.

Radiation corresponding to an x-ray series of the N type has been definitely observed for copper and iron. This series will be further investigated for elements of higher atomic number.

In the following table the small corrections for the filament drop and for the

Element.	K Series.	L Series.	M Series.	N Series.
Carbon	42.6 Å	375 Å		
Oxygen	23.8 Å	248 Å		
Iron		16.3 Å	54.3 Å	247 Å
Copper		12.3 Å	41.6 Å	119 Å

initial velocity of the electrons have been ignored in computing the wavelengths because of the uncertainty regarding the correction to be applied for the energy involved in the absorption of the electrons by the target.

Princeton, University.

THE CHANGE OF MOBILITY OF THE POSITIVE ION IN AIR WITH AGE.

By Henry A. Erikson.

At the December, 1920, meeting[1] of the American Physical Society, the writer reported results indicating that there is very little difference in the mobility of the positive and negative ions in air if the mobility is measured. immediately after the ions are formed but that after an interval of the order of a second the mobilities assume the normal difference giving a ratio of 1.4.

The following is a report of the results of a further investigation of this effect.

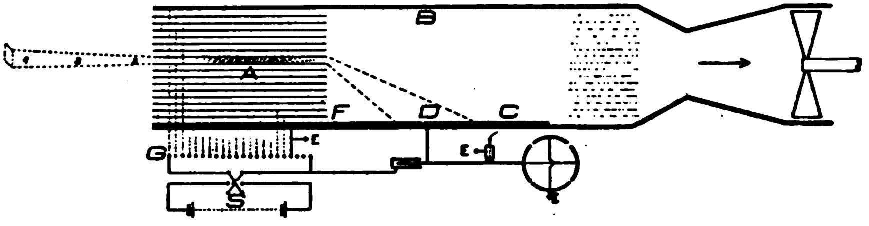

Fig. 1.

The apparatus used was essentially the same as in the earlier investigation Ions prduced at *A*, Fig. 1, by means of polonium are passed between two parallel plates *B* and *C* by means of a current of air. The field between *B* and *C* separates the ions. The number of ions arriving at the different down stream distances on the lower plate *C* is determined by means of a narrow strip *D* which is connected to a quadrant electrometer. The apparatus was arranged so that the distance *DF* could be changed without altering the field or the velocity of the air. In order to secure a steady flow of air a fan driven by a synchronous motor was used and a series of parallel plates were inserted above and below the plates between which the ions were produced. These

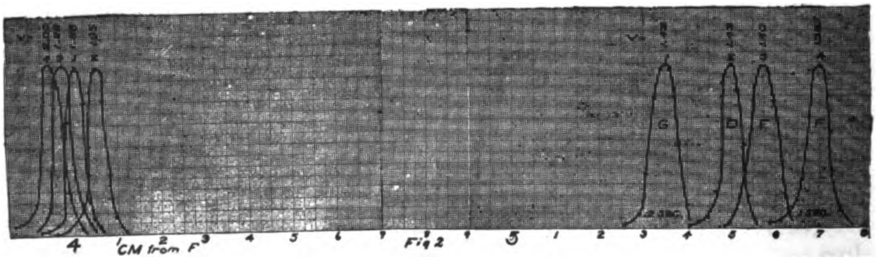

Fig. 2.

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[1] Phys. Rev., XVII., p. 400.

plates were connected to points of increasing potential as shown at *G*. The uniformity of the field was secured by means of wires connected to the plates and passing parallel on the two sides of the stream of air. An investigation was first made to determine if there is any change in the mobility of the negative ion with age or if the change takes place in the positive only. An extension was inserted so that the ions could be produced at points indicated by *A*, 2, 3, 4, Fig. 1. An air velocity such as to give an average age of 0.2 seconds for point *A* up to one second for point 4 was used. It is seen from Fig. 2, that the shift takes place in the positive ion only. The irregular shift of the negative curve is undoubtedly due to turbulence and experimental error. In these curves the scale for the width of each curve was arbitrarily diminished.

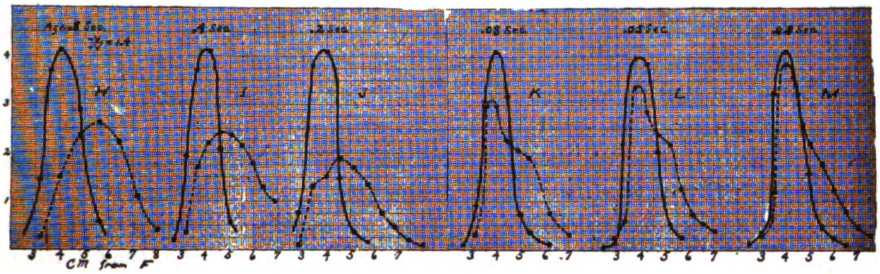

Fig. 3.

The next point investigated was to determine the nature of the shift in the case of the positive ion. The wind velocity was adjusted so as to give an average life of 0.8 seconds and the curves shown at *H*, Fig. 3, were obtained. The full curve represents the negative and the dotted curve the positive ion. Both curves were obtained simultaneously by reversing the field at *S*, Fig. 1. By using the maxima for these curves the normal mobility ratio of 1.4 is obtained.

By increasing the wind velocity so as to give an age of .4 seconds, the curves *I* were obtained. These show a shift of the positive towards the left, *i.e.*, the positive mobility has increased.

The curves *J* are for an age of 0.2 seconds and show a further shift of the positive to the left.

The curves *K* are for an age of 0.08 seconds. Here there was a further shift of the positive towards the left and an offset in the curve came into evidence. Curves *L* are for an age of .05 seconds. For an age of .03 seconds, curves *M*, the mobility of the positive ion has become essentially the same as that of the negative. A small offset is still present extending beyond the negative curve.

Owing to the loss of ions by recombination the currents measured diminished as the time increased. In the figure the ordinates have been made alike.

Physical Laboratory,
University of Minnesota,
April 16, 1921.

THE ABSOLUTE SIZES OF CERTAIN MONOVALENT IONS.

BY WHEELER P. DAVEY.

PROF. W. L. BRAGG has shown[1] from x-ray crystallographic data, that the difference in size between two ions is approximately constant and is independent of the state of combination of the ions as long as the valences are unaltered. The dimensions of the unit crystals alone, however, are not sufficient to determine the absolute size of the component ions, for there are always fewer equations than unknowns. Bragg attempted to use the dimensions of the crystals of elements as a means of elimination of one of the unknowns, and arrived at a remarkable series of numbers which he believed to represent approximately the absolute size of the ions. By assuming a structure for certain other crystals, he predicted from their densities what the dimensions of their unit crystals should be, and from this the size of the ions. The results seemed to be quite consistent among themselves, but did not agree with experimental data in this laboratory on the crystals themselves.[2]

Bragg's calculations are open to the objection that the atoms of metals are considerably larger than their corresponding ions, so that the size of the atoms cannot be substituted correctly in an equation dealing only with the size of ions. This may be well illustrated by the fact that the radius of the Ca atom, as shown by the crystal structure of the element, is 3.93 Å,[3] while the distance from $\overset{++}{\mathrm{Ca}}$ to $\overset{++}{\mathrm{Ca}}$ centers in CaO is only 3.42 Å.[4]

It seemed therefore desirable to recalculate the size of ions, using only assumptions dealing with ions, rather than atoms. The assumption chosen was, that $\overset{+}{\mathrm{K}}$, $\overset{+}{\mathrm{Rb}}$ and $\overset{+}{\mathrm{Cs}}$ were equal in size to $\overset{-}{\mathrm{Cl}}$, $\overset{-}{\mathrm{Br}}$ and $\overset{-}{\mathrm{I}}$, respectively. This assumption is in agreement with Langmuir's theoretical deductions on the volumes of ions of elements having nearly the same atomic numbers as the inert gases.[5]

From the standpoint of diffraction of x-rays, it may be justified as follows. Since KCl gives the diffraction pattern of a single cube of ions, the scattering powers of $\overset{+}{\mathrm{K}}$ and $\overset{-}{\mathrm{Cl}}$ must be approximately equal. Since, in addition to this, the number of electrons in the ions is the same, the ions must be of nearly the same size, *i.e.*, must have nearly the same *density* of electrons. If the nuclear charges had been identical in amount, then the two ions would have been of exactly the same size. This condition is approximated more nearly in RbBr and CsI. Bragg's values for $\overset{+}{\mathrm{K}}$ and $\overset{-}{\mathrm{Cl}}$ and for $\overset{+}{\mathrm{Rb}}$ and $\overset{-}{\mathrm{Br}}$ differ by almost 100 per cent. His value for $\overset{+}{\mathrm{Cs}}$ is about 70 per cent. larger than for $\overset{-}{\mathrm{I}}$.

With the exception of NaCl, which was taken as a starting point in calculating the wave-length of x-rays used, only data obtained in this laboratory were used in the calculations. The dimensions assigned to various crystals will be found to differ slightly from those published by Bragg and his co-workers. This is because our distances are all calculated from the x-ray data instead of being calculated from the measured density of the crystal and the mass and shape of the unit crystal. In this way our results are independent of the errors in the published data on density, caused by impurities, inclusions in the crystals, etc.—and are mutually consistent.

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[1] W. L. Bragg, Phil. Mag., Aug., 1920.

[2] Davey and Wick, PHYS. REV., March, 1920.

[3] A. W. Hull, PHYS. REV., Jan., 1921.

[4] Davey, PHYS. REV., May, 1920.

[5] I. Langmuir, Jour. Am. Chem. Soc., 1, pp. 911–913, 1919.

Most of the structures and dimensions of crystals given below have already been published.[1] Table I. gives the name, structure, and dimensions of crystals by which the size of ions may be directly calculated. Table II. shows that the values so obtained are consistent with the dimensions of other crystals formed from the same ions.

TABLE I.

KCl, — Simple cube of ions	$d = 3.13$ A	$\overset{+}{\mathrm{K}} = 1.56$ Å $\overset{-}{\mathrm{Cl}} = 1.56$
RbBr, — Simple cube of ions	$d = 3.465^*$	$\overset{+}{\mathrm{Rb}} = 1.73$ $\overset{-}{\mathrm{Br}} = 1.73$
CsI, — Centered cube of ions	$d = 4.58$	$\overset{+}{\mathrm{Cs}} = 1.98$ $\overset{-}{\mathrm{I}} = 1.98$
NaCl, — Simple cube of ions	$d = 2.81$	$\overset{+}{\mathrm{Na}} = 1.25$
$\overset{+}{\mathrm{Na}} = \mathrm{NaCl} - \overset{-}{\mathrm{Cl}}$	$2.81 - 1.56$	
KF, — Simple cube of ions	$d = 2.69$	$\overset{-}{\mathrm{F}} = 1.13$
$\overset{+}{\mathrm{K}} = \mathrm{KF} - \overset{-}{\mathrm{K}}$	$2.69 - 1.56$	

TABLE II.

NaF	$d = 2.34$	$\overset{+}{\mathrm{Na}} + \overset{-}{\mathrm{F}} = 1.25 + 1.13 = 2.38$	Error < 2 per cent.
NaBr	$d = 3.01$	$\overset{+}{\mathrm{Na}} + \overset{-}{\mathrm{Br}} = 1.25 + 1.73 = 2.98$	" 1 " "
NaI	$d = 3.25$	$\overset{+}{\mathrm{Na}} + \overset{-}{\mathrm{B}} = 1.25 + 1.98 = 3.23$	" < 1 " "
KBr	$d = 3.30$	$\overset{+}{\mathrm{K}} + \overset{-}{\mathrm{Br}} = 1.56 + 1.73 = 3.29$	" < 1 " "
KI	$d = 3.55$	$\overset{+}{\mathrm{K}} + \overset{-}{\mathrm{I}} = 1.56 + 1.98 = 3.54$	" < 1 " "
CsCl	$d = 3.54$	$\overset{+}{\mathrm{Cs}} + \overset{-}{\mathrm{Cl}} = 1.98 + 1.56 = 3.54$	" < 1 " "

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[1] A. W. Hull, Proc. Am. Inst. Elec. Eng., 37, 1919. W. P. Davey, Phys. Rev., March, 1921. W. P. Davey and Frances G. Wick, Phys. Rev., March, 1921.

The first five crystals in Table II. are simple cubes of ions. The last is a body centered cube of ions *d* 4.10. It will be noted that F comes out smaller than Na. This is hard to explain at present.

In order to extend this work to ions having a valence of two it is desirable to make measurements on SrSe and BaTe, treating these results as in the case of KCl, RbBr and CsI, but we have so far been unable to obtain any of these compounds. CaS can not be used in this way owing to the great percentage differences in the nuclear charges. The same end might be obtained using the halides of ions having a valence of two, provided that two different halides of the same ion could be employed, one as a check on the other. In this case the difficulties are considerable. Some of these compounds are so hydroscopic as to dissolve completely during the process of preparing the crystal powder. Others tend to decompose when heated to drive off water of crystallization. Still others give patterns which we have been so far unable to interpret. A report on bivalent ions will therefore be deferred until these difficulties are sufficiently overcome.

Research Laboratory,
General Electric Company.

Forces Within a Static Atom.

By Irving Langmuir.

If in addition to the Coulomb forces between charged particles we assume the existence of another force (quantum force) equal to

$$F_q = \frac{1}{mr^3}\left(\frac{nh}{2\pi}\right)^2$$

acting between an electron and a nucleus, we find that a stationary electron is in stable equilibrium when its distance *r* from a nucleus is the same as the radius of a circular orbit corresponding to a stationary state in Bohr's theory. The total energy of the electron is also the same as that given by Bohr's theory. The frequency of oscillation about the position of equilibrium is identical with the frequency of revolution of the electron in the Bohr atom. Thus the Rydberg constant and the Balmer series can be deduced from this law of force without assuming moving electrons. It seems possible to construct satisfactory models of the helium atom and the hydrogen molecule on this basis by assuming that each electron acts towards the nucleus like a dipole acting on a charge while the quantum action between electrons is like that between dipoles. Thus the forces depend on the orientation of the electrons as well as on their distance. This view suggests that quantum relationships may have their origin in the structure of the electrons and each change in the quantum number associated with an electron corresponds to a change in the structure of the electron.

Research Laboratory,
General Electric Co.,
Schenectady, N. Y.

The Effect of Roughening of Surface on the Dispersion of Cannon Ball in Falling Through Air.[1]

By F. C. Brown.

Dr. Duff and other eminent physicists in this country and abroad proposed the substitution of spherical bombs for the stream line bombs used during the war with the expectation of reducing irregularities of ballistic performance.

Experiments were carried out at Aberdeen Proving Ground, in coöperation with the Balloon Section of the Air Service, showing that the variability of flight of spherical bombs was several times as great as that of the best stream line form.

Further experiments were carried out to determine the effect of roughening of the surface of spheres on the dispersion constant. The dispersion constant was taken as the mean dispersion, divided by the altitude, and for convenience is expressed in angular measure.

The tests on the smooth spherical bombs were carried out with 8-inch iron spheres salvaged from the Civil War; and also some spherical German bombs were used in the experiments. Four of these were dropped simultaneously and the positions of impact were plotted. To determine the effect of roughening the 8-inch cannon balls were drilled so as to give the appearance, when reduced, of dimpled golf balls. The accompanying photograph shows the appearance of the spheres. The plots of the positions of the smooth and roughened cannon balls at impact are shown in the accompanying diagram. The angle of dispersion for each trial is indicated by θ.

The following tables give the dispersion of the smooth as well as the roughened spheres:

Smooth Spheres Dropped Below 2000 Feet.

Altitude	Tan θ
1400 feet	.0048
1400 "	.0189
1400 "	.0103
1100 "	.0052
1000 "	.0196
1000 "	.0129
1000 "	.0140
Mean	.0122 ± .002

Smooth Spheres Dropped at 2000 Feet and Above.

Altitude.	Tan θ
2700 feet	.0120 +
2000 "	.0074
2000 "	.0104
2000 "	.0215
2000 "	.0147
Mean	.013 ± .002

$\tan\theta = .0125$

$\theta = .72°$ average for all altitudes.

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[1] Abstract of paper to be presented at the Washington Meeting of the American Physical Society.

Roughened Spheres Dropped Below 2000 Feet.

Altitude.	**Tan θ**
1700 feet	.0025
1400 "	.0046
1400 "	.0099
Mean	.0057 ± .0030

$\theta = .33'$

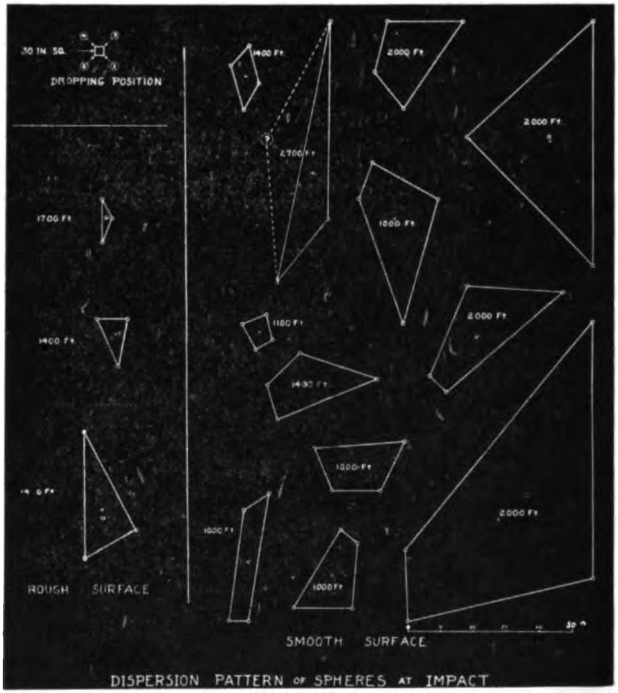

Fig. 1.

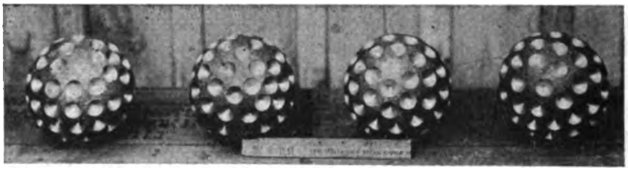

Fig. 2.

Even if we could roughen the surface of spherical bombs we would not then obtain as uniform ballistic properties as are obtained in the best stream line form.

The principal conclusion from this work is that the roughening of iron spheres reduced the dispersion constant to about one half.

Bureau of Standards.

ON THE DETERMINATION OF INSTANTANEOUS PRESSURES IN GUNS BY MEANS OF A PIEZO-ELECTRIC GAGE AND A BALLISTIC GALVANOMETER.

BY J. C. KARCHER.

ONE or more plates of quartz which have been cut from a quartz crystal in such a way as to display the piezo-electric property are stacked up with suitable electrodes in the manner of a plate condenser. The plates are subjected to the gun pressure by means of a piston. The whole with suitable leads for removing the charge is assembled in a kind of capsule which screws into the breech block of a gun.

From the well-known Piezo-electric property of quartz one has the relation

$$q = Kp,$$

where p = the gun pressure at any instant, K is a constant of the gage and q is the quantity of charge liberated. This relation was found to hold for pressures up to 50,000 pounds per square inch. If the charge q is permitted to flow through a galvanometer the current

$$i = K\frac{dp}{dt}$$

and if θ is the angular displacement of the moving system of the galvanometer the following relation exists

$$iM = KM\frac{dp}{dt} = I\frac{d^2\theta}{dt^2} + k\frac{d\theta}{dt} + c\theta.$$

Since the pressure-time curve of a gun usually extends over a period of several hundredths of a second, a ballistic galvanometer may be used in which case the last two terms in above equation may be neglected and

$$\frac{dp}{dt} = \frac{I}{MK}\frac{d^2\theta}{dt^2},$$

$$p = \frac{I}{MK}\frac{d\theta}{dt} + c'. \quad (p = 0 \text{ when } \theta = 0 \therefore c' = 0.)$$

Displacement-time curves are obtained by photographing the galvanometer deflections on a uniformly moving film. The differential of this curve obtained either graphically or analytically is the time-pressure curve.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
April 4, 1921.

A NEW GENERAL LAW OF DEFORMATION.

BY P. G. NUTTING.

AN investigation of pure shear in pitches possessing *both* elastic and viscous properties has led to the new general law of deformation s as a function of time t

$$s = at^n F^m$$

and stress F, in which n and m are independent of s, t and F, and of the units and method employed. For $n = 0$ and $m = 1$ this reduces to Hooke's law as a special case while for $n = 1$ and $m = 1$ it gives the law of viscous flow.

The corresponding forms for the electrical case

$$q = at^nX^m \quad \text{and} \quad i = ant^{n-1}X^m$$

give Ohm's law as a special case when $n = 1$ and $m = 1$. Data by Shrader on leakage through dielectrics is represented very well by the new law.

WESTINGHOUSE ELECTRIC AND MANUFACTURING CO.,
March 28, 1921.

CONTRACTIONS AND EXPANSIONS OF AMALGAMS WITH TIME.

BY ARTHUR W. GRAY.

THE dimensional changes that occur during the hardening of a dental amalgam are of such importance in connection with the tooth-restoring properties of this valuable filling material that determinations of these changes are nearly always included in tests of amalgam alloys. A contracting amalgam may shrink sufficiently to admit oral fluids and bacteria between the filling and the cavity walls; an expanding amalgam may swell sufficiently to extend above the margins of the cavity, or even to split the tooth. The ideal amalgam expands just enough to make sure that a properly inserted filling will remain firmly in contact with the tooth.

No theory yet advanced gives a satisfactory reason for the existence of these dimensional changes, which I call *reaction expansions* to differentiate them from *thermal expansions*. The prevailing belief is that silver causes expansion and tin contraction. Adjusting the composition of a dental alloy so that its amalgam will show a desirable reaction expansion is, therefore, regarded as "balancing" the opposing tendencies of the component metals.

A study of the modifications produced by systematically varying the conditions that affect reaction expansion has finally led me to formulate a simple theory that seems consistent with all facts at present known. I shall not enter into a consideration of this theory here, but shall merely present some of the experimental evidence upon which it is based.

Black had observed that some amalgams contracted a little before they expanded.[1] In a paper before the American Institute of Mining Engineers[2] I pointed out that the typical reaction expansion of a dental amalgam is characterized by four consecutive stages:

1. Rapid contraction to a minimum.
2. Somewhat slower expansion to a maximum.
3. Considerably slower contraction to a second minimum.
4. Very much slower expansion to a second maximum.

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[1] G. V. Black, The Physical Properties of the Silver-Tin Amalgams, Dental Cosmos, 38, 982, 1896.

[2] A. W. Gray, Metallographic Phenomena Observed in Amalgams, Am. Inst. Min. and Met. Eng. Trans., 60, 684 and 693, 1919; Journ. National Dental Assn., 6, 913 and 918, 1910.

One or more of these stages can be masked by suitable mechanical treatment of the amalgam.

The gradual changes in shape that the reaction expansion curve can be made to undergo by systematic variation of influencing conditions are particularly well shown by a series of forty-two curves obtained during an investigation of effects produced by varying both the size of particle in which a dental alloy is cut and the pressure under which its amalgam is molded into a test specimen. For this investigation a large quantity of alloy filings from the same melt was sorted by standard sieves into five samples. The particles of the first sample passed a sieve with 100 meshes and were retained by one with 115 meshes to the inch. The particles of the fifth sample passed a 200-mesh sieve. Unsorted alloy, comprising a mixture of all sizes passing 100 meshes, furnished a sixth sample.

The reaction expansion at 37.5° C. was determined for seven specimens from each of the six samples of alloy. The packing pressures under which amalgam from each sample was molded were increased in geometric progression from 25 to 1,600 kg. per cir. cm. In order to obtain the required plasticity a greater mercury-alloy ratio was used with the finer samples than with the coarser ones, but otherwise packing pressure and size of alloy particles were the only factors that were varied.

Fig. 1 shows how variations in packing pressure affect the reaction expansion of amalgam from the sample of alloy that passed a 200-mesh sieve. The distance from one horizontal line to the next represents a change of 2.5 microns (1/10,000 inch) in the diameter of a freely expanding amalgam cylinder one centimeter in diameter, *i.e.*, a linear expansion of 0.025 per cent. Since an observation was taken every minute during the first seven hours after a cylinder was molded, the plotted points merge into a continuous line for each specimen. The change in 24 hrs. is indicated by a single dot at the right of the chart. Subsequent changes have been followed for several months and are still under observation.

The chart shows that each increase in packing pressure is accompanied by an increase in the rate of reaction (solution of the alloy in the mercury followed by crystallization of the amalgam), each prominent feature of the expansion making its appearance earlier than in the preceding curve. The influence of pressure change is very much greater at the lower pressures. For example, the amalgam packed under 25 kg. per cir. cm. requires about a month and a half to reach the first maximum. Doubling this pressure results in the maximum being reached during the first 24 hrs. Another doubling shortens the time to about 2.5 hrs. Additional increments of pressure progressively hasten the appearance of this maximum until under 1,600 kg. it is reached in less than ten minutes. At the same time, the increase of pressure progressively obliterates the initial rapid descent preceding the first minimum, and it finally obliterates some of the following rise towards the first maximum. It also hastens the appearance of the second minimum, which is reached in about three weeks with 50 kg. and in about three hours with 1,600 kg. With this alloy the rise from the second minimum to the final maximum is negligible, being no more than one micron per centimeter, or 0.01 per cent.

Fig. 2 shows how variations in size of alloy particles affect the reaction expansion of amalgams packed under 25 kg. per cir. cm. Six more similar groups of curves have been charted, one for each of the other six pressures represented in Fig. 1. The figures to the right of each curve give the number of meshes per inch of the sieve that passed and the sieve that retained the alloy used for the amalgam represented by the curve. The curve at the top of the chart was obtained from the unsorted dental alloy, that is to say, from alloy in which particles of all the various sizes represented by the other curves were mixed in proportions that yielded a commercial product of the highest grade.

The control of the reaction expansion produced by this blending of various sized particles is evident on comparing the curve of specimen R464 with the other curves on the same chart, especially when the comparison is extended to include observations made at intervals during a period of several months. For example, specimen R467 expanded 0.18 per cent. beyond its first minimum, or 0.11 per cent. beyond its diameter when observations began a few minutes after molding, before it attained its first maximum a month and a half later. The other amalgams from the sorted particles of alloy behaved very much like this, although gradual changes in the expansion curve are noticeable as the alloy becomes progressively coarser.

In contrast with the specimens from sorted particles, R464 attains its first maximum in a few hours, with an expansion of less than 0.05 per cent. beyond its first minimum, or about 0.03 per cent. beyond its initial diameter. Contraction of about 0.02 per cent. to the second minimum, and final expansion too small for reliable measurement, leave the amalgam about 0.02 per cent. larger than when first measured. The same amalgam when packed under 50 kg. gives an expansion curve that is practically the same in all respects as the curve of R464.

The limitation of dimensional changes exhibited by the curve of R464 is not an accident of experimental procedure. It is a direct consequence of blending alloy particles that differ in size; this is shown by many almost identical curves obtained during routine tests of different lots of similarly made alloy.

The charts not reproduced in this paper show that, as the packing pressure is increased, the effect of variation in size of alloy particles becomes progressively less.

The contraction found when amalgam from a properly adjusted dental alloy is packed under a very high pressure does not in any way prevent the making of a tight tooth filling, because packing hard enough to cause contraction in such an amalgam will stretch the resilient dentin more than enough to make it follow the slight shrinkage of the filling. In fact, moderate contraction after very tight packing is an advantage, in that it relieves to some extent the straining of the tooth. The tighter the packing, the better the filling, because heavy packing pressure not only adds to the strength of the amalgam, but also shortens considerably the time required for it to complete all its dimensional changes and become stable. Moreover, it secures much better adaptation of the filling to the cavity walls and, consequently, reduces liability to leakage.

The packing pressures used in this investigation extend from well within the dental range to considerably beyond it. The mean effective packing pressures employed by different individuals, in condensing amalgam fillings with dental instruments, vary widely. I found the average for four individuals to be 68 kg. per cir. cm. The average deviation from this was 64 per cent.; the difference between the highest (234 kg.) and the lowest (17 kg.) was 321 per cent.[1] Such deviations taken in conjunction with the pressure effects shown in Fig. 1 indicate that, regardless of the accuracy of the particular device employed for measuring the dimensional changes, too much reliance must not be placed on comparisons of dental alloys that are based on expansion tests of amalgams packed by hand pressure or by mallet blows.

Control of temperature is also important. Since the linear thermal expansivity of a dental amalgam is about 0.000025 per degree centigrade, a temperature variation of one degree will change the length of a one-centimeter specimen 0.25 micron. Results obtained at uncontrolled room temperatures with an interferometer sensitive to 0.005 micron are, therefore, not nearly so reliable as results obtained with a less but sufficiently sensitive instrument and accurate

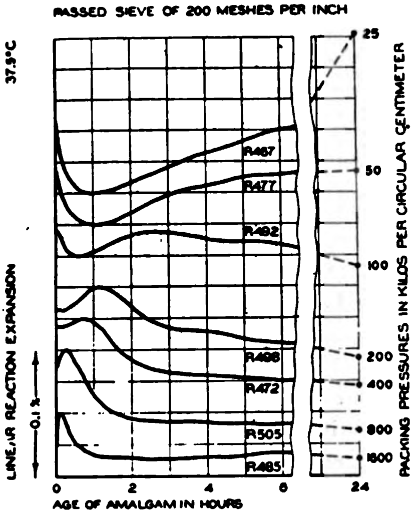

Fig. 1.

Influence of packing pressure on the reaction expansion of a dental amalgam made from an alloy containing silver, tin, copper and zinc. Approximately 69 per cent. silver.

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[1] See reply to discussion of my paper entitled "Transition Phenomena in Amalgams," presented at the Columbus meeting of the American Institute of Mining and Metallurgical Engineers, October, 1920.

control of temperature. The dilatometer used for obtaining the curves of Figs. 1 and 2 is accurate to 0.05 micron; the temperature of the specimen during measurements seldom varies 0.01° from 37.5° C., or whatever other temperature is selected for the measurements.

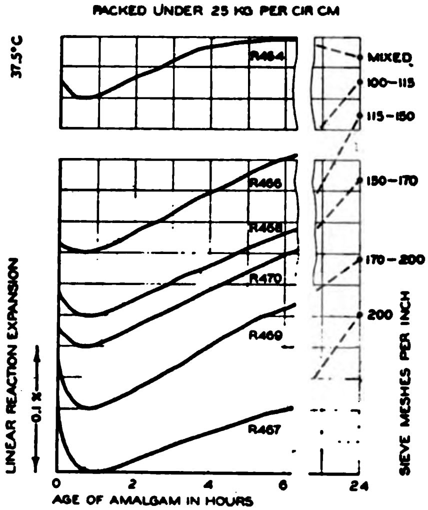

Fig. 2.

Influence of size of alloy particles on reaction expansion. Amalgam from alloy of same composition as in Fig. 1.

The reaction expansions of over five hundred amalgam specimens have been measured under accurately controlled conditions that have been systematically varied. Some specimens have been observed at regular intervals for about two years. In every case the expansion curve was found to conform to the behavior outlined above. Many examples were obtained of all four stages showing in the same curve.

From among the many interesting cases that might be cited as examples, only two will now be mentioned to illustrate long continued dimensional changes of considerable magnitude. Both specimens were prepared from commercial dental alloys by amalgamating according to the directions furnished by the manufacturers and packing under 50 kg. per cir. cm. One specimen, made from a coarse-grained non-zinc alloy, took more than a year to reach its first maximum. During this period it expanded as much as 0.48 per cent. The other, from an alloy containing zinc, reached its first minimum in half an hour, after contracting about 0.04 per cent. It then expanded 0.01 per cent. to its first maximum, which was reached within three hours after molding. The second minimum was reached in about a day, after a contraction of 0.02 per cent. below the first minimum, or 0.06 per cent. below the diameter when observations began. After this, the amalgam steadily expanded towards its second maximum, which was not reached in eight months, when observations had to be discontinued because the cylinder had grown so large that readjustment of the dilatometer would have been necessary for further measurement. The diameter of the specimen had increased 1.28 per cent. and was still expanding. This abnormal expansion is not attributable to the zinc contained in the alloy.

The phenomena described in this paper are just what ought to be expected from a consideration of conditions that influence diffusion, solution, and crystallization. Accurate measurements of the dimensional changes that I have termed *reaction expansions* ought to throw light on other problems of metallography and physical chemistry.

Physical Research Laboratory,
The L. D. Caulk Company,
Milford, Del.,
April 5, 1921.

Measurement of Mercury Vapor Pressure from 0° to 35° C. by the Knudsen Pressure Gauge.

By C. F. Hill.

Nineteen values for the vapor pressure of mercury at various temperatures from – .7° to 34.9° C. have been made by means of the Knudsen pressure gauge, calibrated against a McLeod gauge. These values all lie within about ± 6 per cent. of the mean curve, most of them lying within ± 2 per cent. The value at 0° is .000350, at 10° is .000775, at 20° is .00182, at 30° is .00407, and extrapolated to 40° gives .00787 mm. mercury. This last value agrees closely with the extrapolated values of Ramsey and Young. The slope of the curve also agrees with that of Ramsey and Young, and agrees fairly closely with that of Morley. The results of the present method indicate that the values given by Knudsen in 1909, are much too low, probably as much as 50 per cent.

The complete account of these measurements will be published soon.

University of Illinois,
Urbana, Illinois,
April 1, 1921.

Thermal Conductivity of White Marble and "Neat" Cement.

By Roy A. Nelson.

The thermal conductivity of white marble has been determined by a number of investigators. The principal method used has been the "plate" or "wall" method. It was thought that it might be of value to determine the thermal conductivity of this material by the "cylinder" method both for a check on this method of measuring the thermal conductivity of poor conductors and to give another value of the conductivity of white marble, which is of fairly definite composition.

The sample of marble tested was a very good grade of marble with a fine grained structure. It was called "Alabama White" marble and chemical analysis showed that it was principally calcium carbonate with a small amount of magnesium carbonate. In order to be sure that the marble was free from moisture it was heated in a large oven at 130° C. for four hours.

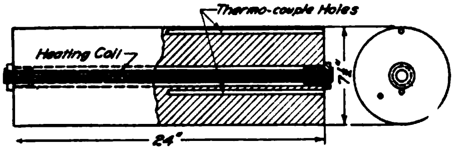

Fig. 1.

The figure shows sections of the cylinders. The central hole for inserting the heating coil was 3.5 cm. in diameter and the holes for inserting the thermocouples were 0.4 cm. in diameter. Cylinders made of Portland cement and water were also tested for the thermal conductivity.

The diffusivity or the thermometric conductivity was calculated for these materials. This was done by determining the densities and the specific heats as well as the conductivities.

Perhaps the most complete investigation on the conductivities of marbles is that of Peirce and Wilson.[1]

The properties of the "Alabama White" marble tested correspond very closely to the marble tested by the above authors, called "American White." The values determined in this investigation on marble and neat cement as well as those given by Peirce and Wilson for "American White" marble are given below:

	"Alabama White" Marble.	"American White" Marble.	"Neat" Cement.
Density	2.71	2.72	1.83
Specific heat	.213	.214	.278
Conductivity	.00614	.00596	.0014
Diffusivity	.0106	.0102	.0275

A marked decrease in the conductivity of the marble with increasing temperature between 50 and 200° C. was found. For the neat cement no pronounced change in the conductivity with the temperature was found. The values obtained for the conductivities at different temperature ranges are given below. Neither Peirce and Wilson nor others so far as we have found

	50° C. to 100° C.	100° C. to 150° C.	150° C. to 200° C.
Marble	.00614	.00524	.00415
"Neat" Cement	.00140	.00165	.00146

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[1] Proc. Am. Acad., Vol. 34 and 36, 1898 and 1900.

in the literature, give values of the thermal conductivity of marble for temperatures over 100° C. We were unable to determine the conductivity at temperatures above 200° C. owing to the fact that the cylinders cracked at higher temperatures.

The above determinations were made in connection with an extended investigation by Professor A. P. Carman and the writer on the thermal conductivity and diffusivity of concrete mixtures, to be published shortly by the University of Illinois Engineering Experiment Station.

Laboratory of Physics,
University of Illinois,
March, 1921.

THE EFFECT OF PRESSURE ON THE THERMAL CONDUCTIVITY OF METALS.

By P. W. Bridgman.

Two methods of measurement have been employed; the first is a radial flow method in which the heat is put in at the axis of a cylinder of the metal, and the difference of temperature determined at two different radial distances. The second method is a longitudinal flow method, in which the correction for the lateral loss of heat to the medium which transmits pressure is kept low by cutting down the dimensions of the specimens.

Measurements to 12,000 kg. per sq. cm. have been made on the following metals: Lead, tin, cadmium, iron, copper, nickel, silver, platinum, bismuth, antimony and zinc.

Within the limits of error the effect is linear with pressure. Of the eleven metals above, the thermal conductivity of four is increased. These four are: lead, tin, cadmium, and zinc. The thermal conductivity of the other seven metals decreases under pressure; the decrease of iron is least and that of bismuth the greatest. The increase of thermal conductivity of lead and tin is greater than the increase of electrical conductivity, so that the pressure coefficient of the Wiedemann-Franz ratio of these two metals is positive. The pressure coefficient of the Wiedemann-Franz ratio of the other nine metals is negative.

Harvard University.

THE NEW FIXED POINT ON THE THERMOMETRIC SCALE.

By Frederick Bates and F. P. Phelps.

When crystalline quartz is heated, recrystallization occurs in the neighborhood of 570° C.–575° C. In studying the rotation of the plane of polarization at high temperatures in this material, it was discovered that the energy changes occurred with great suddenness at the critical point when the crystal was absorbing energy. Careful exploration of this region with a precision thermocouple showed that the absorption of energy by a plate of quartz as small as 10 mm. in diameter by 1.5 mm. in thickness could readily be detected with the thermocouple junction placed in the immediate vicinity of the plate. The heating and cooling curves of larger plates were obtained by drilling an opening in the plate sufficiently large to insert the thermocouple. It was found that the inversion begins with great suddenness at 573°.3 C. Repeated determinations at this point showed a maximum variation of but 0.2° C. for a large number of determinations, and the experimental error for repeated determinations, using the same plate of quartz, was found to be of equal magnitude with that obtained using quartz from various parts of the world.

In order to standardize a thermocouple for this portion of the temperature scale, it is only necessary to take any piece of clear crystalline quartz, insert the thermocouple in the small opening therein, and heat slowly. Upon plotting voltage against time, an extremely sharp break at 573°.3 C. will be found.

In view of the attempts of previous investigators to definitely determine the temperature of inversion from alpha to beta quartz, it can now be definitely stated that the beginning of the change, as shown by the heating and cooling curves, are the only definitely reproducible points in the transformation.

BUREAU OF STANDARDS.

A DISAPPEARING FILAMENT OPTICAL PYROMETER FREE FROM DIFFRACTION EFFECTS AT THE FILAMENT.

BY C. O. FAIRCHILD.

THE writer has been using for the past two years an optical pyrometer in which the filament of the pyrometer lamp could be made to disappear perfectly at any magnification and resolving power of the eyepiece. With this instrument the precision of photometric measurement has been somewhat increased over that of any other form of photometer.

In order to obtain these results it was necessary to use such apertures in the telescope as would eliminate so far as the eye could detect the effects of diffraction at the pyrometer lamp filament. Proper attention to the laws of diffraction and the nature of optical images, together with experiment have made it possible to state definitely what the relation of these apertures to one another should be. The conclusion arrived at by Worthing and Forsythe[1] that in general the brightness of the pyrometer filament is not equal to the brightness of the background image when a photometric match is obtained is found applicable to special cases only. These writers made a study of the effects of diffraction at the pyrometer filament but evidently confused the effects of the filament and the diaphragms. Limitations which they placed on the design and use of the pyrometer are in nearly all cases removed.

In Fig. 1 stops *A* and *B* subtend equal angles at the center of the image *I*. In order that this image should be of uniform brightness geometrical optics would require that the aperture be not smaller than *C*. From diffraction considerations this is not, however, a sufficient condition, if an obstacle partly screens *I*. The latter conclusion can be derived directly from the classic

[1] Worthing and Forsythe, PHYS. REV., 4, No. 3, Sept., 1914.

diffraction theory of optical images and the effects of small circular apertures. The line *AB* in Fig. 2 bounds the shadow of the lamp filament *F*. Light is diffracted into this shadow. This light is supplied by that coming from a few zones *BD* contiguous to *B* and to a negligible amount by the light entering the central portion of the aperture *BC*. If the exit stop reaches close to the point *A* interference fringes will be visible. But if this exit aperture is sufficiently small the fringes near *A* will be screened from view. That is, the edges of the pyrometer filament will be sharply outlined against the background.[1]

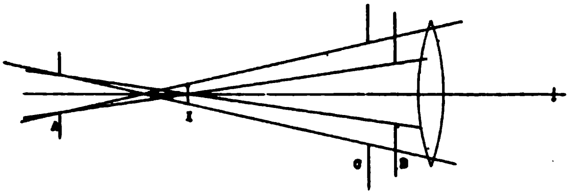

Fig. 1.

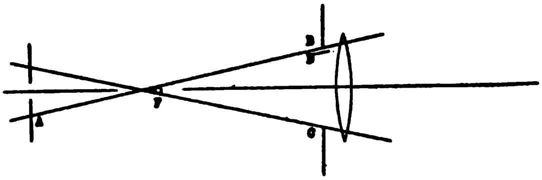

Fig. 2.

Within the shadow of the filament, light passing the two edges will interfere and for small filaments (1 to 4 mils) a single bright fringe on the axis is visible when the filament is unheated and the background image is brilliant. This fringe is most intense for small apertures and is only eliminated by relatively large apertures. It does not however effect the brightness match if the magnification is sufficient and the resolution good enough to prevent the observer from averaging this brightness with the remainder of the filament. With small apertures this fringe is broadened and can be made to cover the entire filament. In no case is it visible when the filament is matched in brightness with the background.

It follows then that the entrance angle of the pyrometer telescope must be so much larger than the exit angle, that diffraction caused by it shall produce fringes totally screened from view by the stop of the exit. In other words the image of the background, itself a diffraction pattern, must be sufficiently complete that larger entrance angles, *i.e.*, the addition of zones at the aperture periphery will produce a negligible change in this pattern.

A pyrometer with aperture selected as above can be used for sighting on small objects such as lamp filaments or openings of experimental "black body" radiators, without undue magnification, for it is found that the size of the image plays no part in the above treatment until it is nearly completely hidden by the filament of the pyrometer lamp.

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[1] This aperture must be large enough to allow good resolution and will depend somewhat on the magnification. Photographs have been made to illustrate this.

Brightness measurements have been made proving that within experimental error the image and pyrometer lamp filament can be matched when they are truly of equal brightness, and this through wide ranges of magnification and aperture values.

The proper selection of apertures is accomplished with ease by using a good eyepiece and an iris diaphragm in the telescope objective. The entrance angle is increased until the filament can be made to disappear. In one case this occurred at an angle of .03 radians with an exit angle of .013 radians. This proportion does not, of course, persist.

A mathematical treatment of this subject would be very difficult. Mathematicians have usually limited their solutions of diffraction problems to plane waves, point sources, or to very small apertures.[1] Observations in experiment have likewise been so limited.[2] Little attention seems to have been paid to the phenomena as modified by converging rays and by screening off a portion of an image by a screen in the focal plane.

Basu found the interference fringes in his experiments depending on the diameter of the filament. He ascribed these variations to the interference of the light reflected from the obstacle with the direct grazing rays. He observed as many as twenty fringes, using a point source. The writer has counted twelve fringes using illumination by converging rays from a fairly large source. These fringes are very sharp and very close to the filament.

That such fringes may be screened from view on the axis of the optical arrangement is apparently a new observation.

BUREAU OF STANDARDS, WASHINGTON, D. C.,
April 5, 1921.

THE ORGAN PIPE AS A COUPLED SYSTEM.

BY ARTHUR TABER JONES.

THE various changes in the pitch of an organ pipe as the pressure is gradually increased have never been fully explained. Lunn[3] has considered the small change in pitch when the pressure is near that for which the pipe is voiced. The changes in pitch have been examined experimentally by various observers, notably by Wachsmuth[4] and his pupils. Our understanding of the reed pipe has recently been considerably advanced by Vogel,[5] who has treated the reed pipe as a coupled system. The reed alone has a certain natural frequency, the tube above it has a series of natural frequencies, and the tones from the reed and the tube react upon each other. It seemed possible that the flue pipe might be regarded in a similar manner. The tube alone has a series of natural frequencies, the wind striking the upper lip of the pipe produces an edge tone, and the tones from the tube and the upper lip react upon each other. The pitch of the edge tone depends upon the blowing pressure, and various observers[1] have found that this pitch rises with increasing pressure, that the pitch is usually discontinuous at certain points, and that similar statements hold for organ pipes.

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[1] See for example works of Sommerfeld, Lommel, Hargreaves, Lord Rayleigh.

[2] On the Diffraction of Light by Cylinders of Large Radius, Nalinimohan Basu, Phil. Mag., 35, p. 79, Jan., 1918.

[3] PHYS. REV., (2), 15, p. 446, 1920.

[4] Ann. der Physik, (4), 14, p. 469, 1904.

[5] Ann. der Physik, (4), 62, p. 247, 1920.

If a flue pipe may be regarded as a coupled system we should expect the frequency of the sound from the pipe to be in the neighborhood of the natural frequency of that component which is the less damped. Now although the radiation of the sound from the pipe causes a rather large damping of the component from the tube, it seems likely that the edge tone is much more strongly damped. We should therefore expect the frequency of the sound from the pipe to be not far from the natural frequency of the tube, and we should also expect it to deviate from that frequency most widely when the natural frequency of the edge tone is near that of the tube. A preliminary study appears to substantiate this expectation.

The pipe used is a wooden open diapason built by the Austin Organ Company. The tube is 67.6 cm. x 9.6 cm. x 7.1 cm. inside. The pipe is voiced for a pressure of 3 in. [= 7.6 cm.] of water, and the upper lip is 19 mm. above the slit. All pitches were determined by comparison with a sonometer. The natural frequency of the tube was found by sending a puff of air quickly toward the mouth of the pipe from outside, and catching the pitch as the sound died away. The natural frequency of the edge tone was found by stuffing the upper part of the pipe with cotton batting, and then blowing the pipe in the usual manner. In Fig. 1 the frequency of the note from the pipe when no batting was used is plotted as a function of the corresponding frequency of the edge tone. For the curves *A* and *B* the pitch of the edge tone was so far below the natural pitch of the tube that the octave of the edge tone, which could usually be detected with a resonator, has been used instead of the funda-

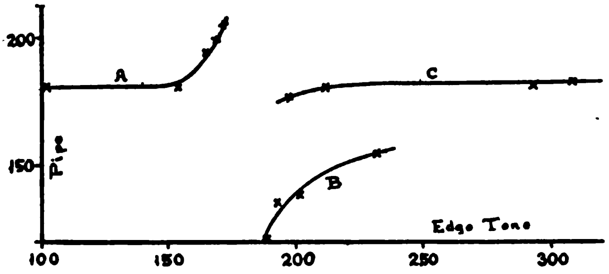

Fig. 1.

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[1] See Krüger, Ann. der Physik, (4), 62, p. 673, 1920.

mental. The ranges of pressure for the curves are as follows: *A*, 3.1 mm. of water to 4.9 mm.; *B*, 8.9 mm. to 11.1 mm.; *C*, above 23 mm. The curve *C* slopes slowly upward to the right to the highest pressure that was available (20 cm. of water). This pressure was not sufficient to make the pipe jump to the octave.

The natural frequency of the tube is about 185 vd. It is seen that if the pipe may be treated as a coupled system, and if the coupling is rather close, the curves are of the general form that is to be expected.

SMITH COLLEGE.

ON THE MEASUREMENT OF THE ACOUSTIC OUTPUT AND EFFICIENCY OF FOG-ALARM APPARATUS.

BY LOUIS V. KING.

AN account is given of fog-alarm tests carried out at Father Point, Que., in September, 1917.

When sound is being generated by compressed air, there is a considerable drop in temperature represented by the mechanical work propagated away as sound.

A small diaphone was installed and operated to give a continuous blast, so that this temperature drop, which is used in measuring acoustic output, could be studied. The results of the 1913 tests were confirmed, and it was found possible to obtain the acoustic characteristics of the apparatus by the use of two thermo-elements, one of which was inserted on the high-pressure side, the other on the low pressure side of the diaphone.

Combining these results with the harmonic analysis of the waves by Professor D. C. Miller it is now possible to state the amount of power contained in the fundamental and in each overtone.

Some evidence was obtained which showed that the harmonics were gradually killed out by propagation in the free atmosphere, the fundamental alone surviving at a distance of three miles or more.

MCGILL UNIVERSITY,
MONTREAL.

A SIMPLE APPARATUS FOR TESTING PITCH CONTROL.

BY PRESTON EDWARDS.

THE unit used in this apparatus is a cylindrical resonator to the face of which is attached a light torsional vibrator, tuned to the same pitch as the resonator. The vane of the vibrator is placed just in front of the mouth of the resonator, so as to be subjected to the reinforced vibrations of the air in that region. To the vibrator is attached a light concave mirror, and the resonator is so mounted as to permit of vertical and horizontal adjustment of the image. A number of these units are used, tuned at intervals of about one eighth of a tone, so that five will cover a semitone or nine a whole tone. The images are brought into a line on the screen, in the order of pitch, and form a good means of testing, by the broadening of one or more of them, whether a singer or performer can hold true pitch. It can also be used, by taking oscillograms, to determine the true nature of trills, vibratos, tremolos, etc.

JOHNS HOPKINS UNIVERSITY.

SOME NEW CONSEQUENCES OF THE THEORY OF ULTIMATE RATIONAL UNITS.

BY GILBERT N. LEWIS.

LEWIS AND ADAMS[1] stated the theory of ultimate rational units. From this new theory they were able to obtain exact expressions for the constant of Stefan's law and for the Planck constant *h*. Later experimental determinations of these quantities have given results in remarkable agreement with their calculations.

With the centimeter as unit of length, a unit of time may be chosen which makes the velocity of light unity. If desired, this unit may also be called the centimeter, and we may say with Minkowski that 1 sec. = 2.999×10^{10} cm. Adopting this system, we still have two independent units, length, and mass or energy. According to Lewis and Adams, there is a fundamental constant in nature which has the dimensions of energy × length. It is $(4\pi\epsilon)^2$, where ϵ is the electron charge in electrostatic units. By choosing the unit of energy so that this constant becomes unity and calling this new unit the reciprocal centimeter, we may write 1 erg = 2.778×10^{16} (cm.)$^{-1}$, or, in other words, the erg-centimeter is equal to 2.778×10^{16} ultimate rational units. It is in such units that the constant of Stefan's law (energy-density of radiation proportional to T^4) becomes unity.

According to an equation of Sackur, the entropy of a monatomic gas is given by the expression

$$S = R \ln T^{3/2} V M^{3/2} + K_1 = R \ln K_2 T^{3/2} V M^{3/2},$$

where T is the absolute temperature, V the volume and M the molecular weight. $K_1 = R \ln K_2$ is a constant of nature. According to the theory of ultimate rational units, if M is the mass of one molecule and V is the volume per molecule, then in the new units $K_1 = 0$ and $K_2 = 1$. Working back from this equation, the entropy of a monatomic gas can be calculated with a hundred-fold greater accuracy than it can be measured. The results are in complete agreement with existing data within the limits of their experimental error.

UNIVERSITY OF CALIFORNIA.

PRESENT STATUS OF THE ELECTRIC AND MAGNETIC UNITS.

BY J. H. DELLINGER.

IT is becoming more generally recognized that the units used in practically all electric and magnetic measurements are those of the international system.

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[1] PHYS. REV., 3, 92, 1914.

Table I.

The International System of Electric and Magnetic Units.

Quantity.	Symbol.	Unit.	Defining Equation.	Dimensional Formula.
Resistance	R	Ohm		$[R]$
Current	I	Ampere		$[I]$
Electromotive force	E	Volt	$E = RI$	$[E] = [RI]$
Quantity of electricity	Q	Coulomb	$Q = \int Idt$	$[Q] = [IT]$
Electrostatic capacity	C	Farad	$C = \frac{Q}{E}$	$[C] = \left[\frac{T}{R}\right]$
Dielectric constant[1]	K		$C = \frac{1.1124}{4\pi(10)^{12}} \frac{KS}{l}$	$[K] = \left[\frac{T}{LR}\right]$
Electric field intensity	$\mathfrak{E}$	Volt per cm.	$\mathbf{E} = \frac{dE}{ds}$	$[E] = \left[\frac{RI}{L}\right]$
Electric displacement	D	Coulomb per cm².	$D = \frac{1.1124}{4\pi(10)^{12}} K\mathbf{E}$	$[D] = \left[\frac{IT}{L^2}\right]$
Self-inductance[2]	$\mathfrak{L}$	Henry	$\mathbf{L} = \frac{E_1}{dI_1/dt}$	$[\mathbf{L}] = [\mathbf{M}] = [RT]$
Mutual inductance[3]	$\mathfrak{M}$	Henry	$\mathbf{M} = \frac{E_2}{dI_1/dt}$	
Magnetomotive force	$\mathfrak{F}$	Gilbert	$\mathbf{F} = \frac{4\pi}{10} NI$	$[\mathbf{F}] = [I]$
Magnetic field intensity (or magnetizing force)	H	Gilbert per cm.	$H = \frac{d\mathbf{F}}{ds}$	$[H] = \left[\frac{I}{L}\right]$
Magnetic flux[4]	ϕ	Maxwell	$\phi = 10^8 \int Edt$	$[\phi] = [RIT]$
Magnetic flux density	B	Gauss	$B = \frac{d\phi}{dS}$	$[B] = \left[\frac{RIT}{L^2}\right]$
Permeability	μ		$B = \mu H$	$[\mu] = \left[\frac{RT}{L}\right]$
Reluctance	$\mathfrak{R}$	Oersted	$\mathbf{R} = \frac{\mathbf{F}}{\phi}$	$[\mathbf{R}] = \left[\frac{1}{RT}\right]$
Intensity of magnetization	J	....	$B = B_0 + 4\pi J$	$[J] = \left[\frac{RIT}{L^2}\right]$
Magnetic susceptibility	κ	....	$J = \kappa H$	$[\kappa] = \left[\frac{RT}{L}\right]$
Magnetic pole strength[5]	m	....	$\phi = 4\pi m$	$[m] = [RIT]$

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[1] The defining equation is the formula for a parallel plate condenser of large S. This equation and other equations containing K differ from the corresponding equations used with c.g.s. units, by the factor $1.1124(10)^{-12}$; this factor is of historical origin, and equals $10^9/c^2$ where $c = 2.9982(10)^{10}$. In the author's paper, Bureau of Standards S292, of five years ago

Every complete system of electric units requires four fundamental units. In the international system of electric units these are the international ohm, the international ampere, the centimeter, and the second. This system is by no means the same as the so-called "practical" electromagnetic system. A few of the "practical" electromagnetic units were the historical basis from which the corresponding international units were developed, but the common statement that electric measurements are ordinarily made in the "practical" electromagnetic units, is erroneous. Many of the units in the "practical" electromagnetic system are of entirely inconvenient magnitude and are never used; for instance, the "practical" unit of magnetic flux density is 10^{-10} of the gauss. The international system, on the other hand, has been built up in such a way that the units are of convenient size and are the ones actually used.

Three of the four fundamental units of the international system have been defined by international agreement in terms of concrete standards. These are the international ohm, ampere and centimeter. The standards, in terms of which these units are defined, are maintained by the national standardizing laboratories and the International Bureau of Weights and Measures. The fourth fundamental unit, the second, of course makes use of the rotating earth as a standard.

Starting from these four fundamental units a complete system of electric and magnetic units is built up, using the equations of ordinary electric and magnetic practice, particularly those used in establishing standards. The resulting units are all of convenient magnitude and are the units ordinarily used, except of course, that the farad is too large for most purposes, and instead the microfarad and micromicrofarad are usually used. The units, defining equations, and dimensional formulas in terms of R, I, L and T, are shown in the preceding table.

The unit of energy in this system is the joule and the unit of power the watt. This system has the great advantage that both K and μ of empty space $= 1$. No other system of electric units has this advantage except the Gaussian and the Heaviside c.g.s. systems, which suffer from the disadvantage that every equation containing both electric and magnetic units contains the factor c, which $= 2.9982(10)^{10}$.

The following units have been definitely adopted and defined by international

a unit of K was defined which eliminated this factor. Practice, however, has favored the unit and equations here presented.

[2] In the definition of self-inductance, E_1 is the e.m.f. created in a circuit by the variation of the current I_1 in the same circuit.

[3] In the definition of mutual inductance, E_2 is the e.m.f. induced in a circuit by the variation of the current I_1 in another circuit.

[4] Φ in the equation is the change of flux in the time t giving rise to the e.m.f. E in a closed circuit linking with the flux.

[5] Φ in this equation is the total magnetic flux emanating from or associated with magnetic poles of aggregate pole strength m; it is strictly equal to the resultant flux through the magnet only when the magnet is very long and thin.

agreement: the international ohm, ampere, volt, coulomb, farad, and henry. The units of the strictly magnetic quantities do not as yet have such definite and official status. Those given in the table are the units adopted and used by the American Institute of Electrical Engineers for many years past. Only one has actually been adopted by international action, the maxwell, adopted by the 1900 Paris electrical congress. The A. I. E. E. Standards Committee has just this month decided to press for international adoption of this complete set of magnetic units.

The international units as given in Table I are supplemented in engineering practice by an alternative set of magnetic units in which the gilbert is replaced by the ampere-turn. These units are most convenient for magnetic engineering calculations, and at some future time it may become desirable to give them international status as a set of units supplementary to the ordinary international units. Those of the ampere-turn units which differ from the ordinary units are shown in Table II. below.

TABLE II.

The Ampere-Turn Units.

Quantity.	Symbol	Unit.	Defining Equation.	Ordinary Units in one Ampere-turn Unit.
Magnetomotive force.........	$\mathfrak{F}$	Ampere-turn	$\mathfrak{F} = NI$	$\frac{4\pi}{10}$
Magnetic field intensity (or magnetizing force)........	H	Ampere-turn per cm.	$H = \frac{d\mathfrak{F}}{ds}$	$\frac{4\pi}{10}$
Reluctance.....	$\mathfrak{R}$	Ampere-turn per maxwell	$\mathfrak{R} = \frac{\mathfrak{F}}{\phi}$	$\frac{4\pi}{10}$
Intensity of magnetization	J	Maxwell per cm^2.	$B = B_0 + J$	$\frac{1}{4\pi}$
Magnetic susceptibility	κ	....	$J = \frac{4\pi}{10} \kappa H$	$\frac{1}{4\pi}$
Magnetic pole strength......	m	Maxwell	$\phi = m$	$\frac{1}{4\pi}$

The ampere-turn units are "rational" in the Heaviside sense, that is, the 4 π's are redistributed in the equations involving these units. Rationalization is attained without changing the unit of permeability, a vital superiority of the ampere-turn units over the various systems of rationalized systems that have been provided except the Heaviside system. The ampere-turn units are moreover superior to the Heaviside in that the 4 π's are more advantageously distributed and the units are the ones in actual use in electrical and magnetic measurements. It is remarkable that a set of units which has been evolved in ordinary practice is more satisfactorily rationalized than any of the many systems that have been constructed from theoretical considerations with that very purpose in view.

Bureau of Standards,
Washington, D. C.

Discussion of a Kinetic Theory of Gravitation II., and Some New Experiments in Gravitation.

By Charles F. Brush.

Kinetic Theory of Gravitation.—The author's original (1910) theory is briefly reviewed, viz., that gravitation is due to the intrinsic energy of the ether, which is assumed to exist in wave form or energy flux of some kind propagated with high velocity in all conceivable directions so that the ethereal energy is isotropic, and is uniformly distributed everywhere except as modified by the presence of matter. The interblending energy shadows between gravitating bodies constitute a region of less than normal energy density into which the bodies are pushed by the superior energy density beyond them. *It is again insisted* that the energy acquired by a falling body is derived from the ether.

Nature of Ethereal Energy.—The hypothesis is proposed that *the ether is abstract energy* unassociated with anything. If the quantum theory of energy is tenable, we may perhaps regard the ether as *a vast atmosphere of energy quanta in violent agitation.*

Diamagnetism May Modify the Gravitational Field of a Diamagnetic Substance such as Bismuth.—The very slight *negative* magnetic permeability of bismuth *may cause a slight reflection or scattering of the gravitational energy flux.* This thought prompted the first of the series of experiments following.

1. *Gravitational Attractions of a Kg. each of Bismuth and of Zinc for a Small Nearby Ball of Silver Are Not the Same.*—Apparatus such as used by C. V. Boys in finding the gravitational constant and mean density of the earth was employed with slight modifications. The bismuth and zinc were in the form of cylinders of same size and shape. The torsional deflection in each case was divided by the weight of metal causing it, so as to reduce both to a common basis of attraction per Kg. *Taking the attraction of zinc as* 100, *that of bismuth is only about* 72.

2. *A Gravity Pendulum With a Bismuth Bob Has a Shorter Period Than a Similar Pendulum of Exactly the Same Oscillation Length Having a Zinc Bob.*—Many carefully designed and conducted experiments appear to demonstrate this.

3. *A Torsion Pendulum Having Equal Horizontal Arms Loaded With Equal Weights of Bismuth Has a Shorter Period Than When The Arms are Loaded With Equal Weights of Zinc of The Same Weight as The Bismuth Loads*; *The Radii of Gyration Being Exactly The Same in Both Cases.*—Many careful experiments appear to demonstrate this.

The author does not claim that diamagnetism is the cause of the remarkable behavior of bismuth, but thinks it probable: And he hesitates to draw general conclusions until after further experimentation on different lines now in progress. It seems fairly clear however that *weight* and *mass* are not so simply related as heretofore supposed.

CLEVELAND, OHIO,
April 4, 1921.

DEFECTS IN SYMMETRIC OPTICAL SYSTEMS.

BY IRWIN ROMAN.

AT the St. Louis meeting[1] it was shown that for a lens system composed of quadrics of revolution, the curvature of the field, the relative error in zonal magnification and the angular coma as seen from the diaphragm are independent of the eccentricity of the generating conic, while an explicit formula was given for the longitudinal aberration.

Since then it has been shown that if we neglect terms of orders beyond the second, every surface of revolution can be replaced by an equivalent quadric surface. Suppose that the surface is generated by revolving the curve

$$x = \chi^0 + \psi_1 y^2 + \psi_2 y^4 + \cdots, \qquad \psi_1 \neq \infty,$$

around the x-axis. The normal at the point whose optical height is h will then cut the axis at a point a distance r from the axial point of the surface and a distance s measured along the normal, where, neglecting terms of higher order n h, we have

$$r = s = \frac{1}{2\psi_1} + B\psi_1 h^2.$$

The quantity $B = 1 - \psi_2/\psi_1^3$ may be called a "coefficient of form."

For a sphere the value of B is zero, so that B is a measure of the departure from sphericity. If we generate the surface by revolving a conic around that axis which contains the foci, we get $B = e^2$. If we revolve the conic around that axis which does not contain the foci, we get $B = e^2/(e^2 - 1)$. If B is positive, there thus always exists an equivalent quadric of the former type, while if B is negative, or if it exceeds unity, there always exists an equivalent quadric of the latter type. The revised formula for longitudinal aberration merely replaces e^2 of the previous formula[1] by B.

THE TRANSMISSION OF WAVES THROUGH A SYMMETRIC OPTICAL INSTRUMENT.

BY IRWIN ROMAN.

IN the passage of a wave through an optical instrument, each point of the wave traverses a path which establishes a correspondence, point by point, between all the surfaces of the system, both lens and wave surfaces being considered as in the system. If each surface is expressed in terms of its own Gaussian parameters, such as are used in differential geometry, the parameters for every surface can be expressed, at least theoretically, in terms of the parameters for an arbitrary incident wave.

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[1] See this journal, March, 1920, p. 221.

If the system is one of revolution about a common axis, the azimuth around this axis may be selected as one of the two Gaussian parameters, so that the problem contains essentially only one parameter. This is the case considered in detail in the present work. The rectangular coördinates of each point of a surface are considered as expressed by power series in terms of the parameters selected, each surface having one parameter and its own set of coefficients in the series. The parameters are all reduced to terms of that for a particular incident wave. Certain invariant combinations are found among the coefficients of pairs of surfaces, these invariants furnishing the refracted surface for a particular incident wave and refracting surface, as well as the wave after propagation within a single medium.

While the treatment is one involving power series, the solution is complete in the sense that recursion formulas make it possible to retain terms of all orders, although explicit forms have been calculated for only the first four orders. As an application, the longitudinal aberration of a symmetric wave is calculated. For a particular instrument with four refracting surfaces the results obtained by this method and by the usual method, agree remarkably well. For the case of a plane wave refracted through a single spherical surface, the results agree with those found in the literature.

The method is applicable to non-spherical surfaces as easily as to spherical ones, provided that all surfaces of the system are symmetric with respect to the same axis. The detailed analysis for the unsymmetrical case is contemplated for the near future, but has not yet been started.

NORTHWESTERN UNIVERSITY.

A DIRECT READING SPECTROPHOTOMETER FOR MEASURING THE TRANSMISSIVITY OF LIQUIDS.

BY IRWIN G. PRIEST.

THIS spectrophotometer consists essentially of a constant deviation spectrometer (Hilger) in combination with the author's "exponential" or "variation of thickness" photometer first exhibited at the Convention of the American Oil Chemists' Society, New Orleans, May 11, 1920.[1] The essential features of the latter are:

(*a*) Two beams of light proceed horizontally, one above the other from a uniformly illuminated vertical surface.

(*b*) A rotating sector disk of known transmission can be interposed in the upper beam.

(*c*) By means of two totally reflecting partially immersed rhombs, the lower beam is diverted through a variable thickness of liquid determined by the distance between the rhombs, one of which is carried by a slide on a track parallel to the beams of light. The liquid is contained in a horizontal trough with open top into which the rhombs dip.

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[1] Priest, Cotton Oil Press, July, 1920, p. 96.

(*d*) The two beams are brought into juxtaposition in the photometric field by means of an arrangement of biprisms.

(*e*) The thickness of liquid in the lower beam is varied until its transmittance equals the transmission of the sector disk as judged by equality of brightness in the photometric field.

(*f*) A suitable scale provides for direct reading of transmissivity or the logarithm of transmissivity.

Two methods of obtaining the divided photometric field have been used, namely:

(*a*) A biprism at the spectrometer slit gives rise to two juxtaposed spectra which are viewed by a positive ocular;

(*b*) A biprism over the objective of the spectrometer telescope gives a divided field viewed without an ocular.

The air-glass surfaces of the totally reflecting immersion rhombs are protected by glass covers fused to the rhombs by the method of Fairchild.[1] The author is greatly indebted to Mr. Fairchild for the preparation of these rhombs.

This instrument has been developed to meet the needs of *rapid* as well as accurate work. It is direct-reading in transmissivity, logarithm of transmissivity and wave-length. The instrument does its own computing automatically and a spectral curve may be determined and plotted in a few minutes. It is thought it will be especially useful in the technologic examination of oils, dye solutions, sugar solutions, etc.

The commercial manufacture of this instrument is being undertaken by Keuffel and Esser, Hoboken, N. J. A detailed description will be published later.

NATIONAL BUREAU OF STANDARDS,
April 6, 1921.

THE EXCITATION OF ENHANCED SPECTRA IN LOW VOLTAGE ARCS.

BY PAUL D. FOOTE, W. F. MEGGERS AND F. L. MOHLER.

SOMMERFELD has shown there is evidence that enhanced spectra arise from the removal of more than one electron from the normal atom. In the case of the alkali earths the simplest type of enhanced spectra would be produced by the removal of both valence electrons followed by the return of one of these to its equilibrium position in successive interorbital transitions. We have to deal accordingly with a core of net charge $+ 2e$. The outer structure of the atom is then similar to the simply ionized element of next lower atomic number, with the exception that the net nuclear charge is $2e$ instead of e. Hence the enhanced spectra should be doublets, characteristic of the "arc" spectrum of the alkali metals, but with $4N$ instead of N appearing in the simple spectral series formulæ. Also the separation of the doublets in the enhanced spectra of an alkali earth should be approximately four times that for the corresponding alkali. Sommerfeld has shown that these conditions are well confirmed by existing data—not exactly to be sure, on account of the modifying effects arising from the mutual repulsion of the electrons forming the atom, but sufficiently close to indicate that the simple enhanced spectra of the alkali earths arise from the doubly charged atom.

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[1] Fairchild, Jour. Op. Soc. Am., 4, pp. 496–503, Nov., 1920.

The highest convergence frequency in the arc spectrum (nuclear charge $1e$) of an alkali earth is $1.5S$, corresponding by the quantum relation $h\nu = eV$ to the amount of work required to remove one valence electron. Similarly the frequency for the simple enhanced spectrum is $1.5\,\mathfrak{S}$ corresponding to the amount of work required to remove the second valence electron after the first has been removed. Some of these latter frequencies have been summarized by Fues.[1]

For magnesium as a particular example we accordingly have the following critical voltages.

This shows that on the basis of Sommerfeld's theory the enhanced spectrum of magnesium should be excited in a low voltage arc by a 15-volt impact—a remarkable conclusion when one considers that these spectra have been in general studied with a capacity discharge at ten to twenty thousand volts. Our experimental work has verified this theory. While only the ordinary "arc" spectrum appears at 8 volts practically every enhanced line listed by Fues is excited in a twenty volt arc. The reason these lines do not appear prominently in an ordinary arc as used by spectroscopists is that electrons of twenty volts velocity are infrequently produced even though the applied potential exceed this. The mean free path is so small that the velocity loss of 7.61 volts, resulting in ionization and the "arc" spectrum, maintains the velocity of most of the electrons at approximately this value or less.

In the present work we have employed a 3-electrode tube with a grid very close to a hot tungsten filament and a plate at a considerable distance, in metallic contact with the grid, the whole being immersed in the vapor at a low pressure. Hence it is possible to develop any desired velocity in so small a distance that collision with atoms is infrequent and the electrons may then pass into the force free space between the grid and plate where collision and emission of radiation take place.

This method of producing enhanced spectra opens a new field whereby the truly enhanced lines may be determined immediately upon examination of the photographic plates, since the enhanced spectrum is not produced below the second ionization point. In ordinary arcs, on the other hand, some electrons are accelerated, without collision, for several mean free paths, far enough to accumulate velocity sufficient for the production of the second type of ionization, with radiation characteristic of the spark. This is probably one reason why so little is known of the enhanced spectrum of mercury. We have photographs for mercury also but have not as yet determined the values of the higher critical voltages.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

[1] Fues, Ann. d. Physik, 63, p. 1, 1920.

ON THE PHOTOELECTRIC EFFECT OF ALKALI VAPORS.

BY JAKOB KUNZ.

ABOUT a year ago E. H. Williams and the writer have shown that cæsium vapor can not be ionized by light of a wave-length shorter than that determined by the equation $hn = eV$, where V is the ionization potential of cæsium vapor. In an article on atomic theory and low voltage arcs in cæsium vapor[1] P. D. Foote has shown that the resonance potential of 1.45 volts gives rise to the single line spectrum, the doublet $1.5s - 2p\frac{1}{2}$. Thus only two inelastic impacts of electrons are known in cæsium vapor, corresponding to 1.45 and 3.88 volts; the former producing the first doublet of the principal series, the latter the entire series. On the basis of Bohr's theory one should expect one line after the other to appear as the potential is increased from 1.45 to 3.88 volts. This successive appearance of lines has indeed been found in mercury vapor by J. Franck.[2] The vapors of mercury and cæsium seem to have different properties, if the results of both physicists are correct.

Low voltage arcs in metallic vapors and in helium have been explained by K. Compton as an absorption of light due to the resonance potential, through neighboring atoms, whereby an electron is lifted from the 1.5S level of energy to the $2p\frac{1}{2}$ level; where it requires a smaller amount of kinetic energy of a colliding electron in order to be removed from the atom. This point of view has been confirmed through the study of the effect of fluorescence on the ionization potential of iodine vapor.[3]

Mr. P. D. Foote draws also the conclusion that electrons revolving not in the $1.5S$ but in the $2p$ ring, should give rise to absorption of the first and second subordinate series, which usually are not absorption lines. We can raise the electron from the stable ring $1.5S$ to the less stable ring $2p$ either by absorption of the corresponding line or by colliding electrons. From the new position the electron may be ejected either by absorption of radiant energy or by incident electrons of proper velocity. If therefore in the photoelectric effect the alkali-vapor is under the influence of the first doublet of the principal series, a smaller frequency of light will be required to ionize the vapor than that of 3.88 volts. So it should be possible to drive an electron from an atom of cæsium vapor by a variety of ways, corresponding to the different absorption lines of the principal series. We may lift the electron from the normal stable orbit to the surface in two or more steps, but always so that $h(v_1 + v_2 + \cdots) = hr = eV$, where $V = 3.88$.

The metal cæsium is sensitive already for infra-red light, while the vapor requires ultra-violet light for the emission of electrons. The vicinity of the atoms in the metallic state seems to loosen the electrons on the surface of the atoms. Hence when the density of the vapor becomes very great, the ionization potentials may become smaller. The ionizing potentials may also apparently decrease when high electronic currents from very hot filaments are used where a measurable fraction of electrons leave the metal with a high velocity. The experiments on the photoelectric effect of alkali vapors are continued by H. E. Williams at the University of Illinois.

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[1] Phil. Mag., Vol. XI., p. 80, 1920.

[2] Zeitschrift für Physik, II., p. 1 , 1920.

[3] PHYS. REV., Vol. XVI., p. 501, 1920.

University of Illinois.

Photo-Electric Potentials from the Retina.

By E. L. Chaffee and W. T. Bovie.

The photo-electric currents obtained, when the retina of the eye is illuminated, have been investigated by several experimenters, using as an indicating instrument the capillary electrometer or Einthoven galvanometer. When such current-operated instruments are used, the deflections depend not only upon the electromotive force produced in the eye by photo-electric action, but also upon the resistance of the eye and of the rest of the circuit. In the present work, the study of the photo-electric potentials from the retina has been made by the use of a two-stage vacuum-tube amplifying system connected to operate an Einthoven galvanometer. The use of the amplifiers gives greater sensitivity to the complete measuring system, and what is of greater importance, gives deflections which are proportional only to the potentials produced in the retina instead of depending upon the resistance of the circuit.

Various curves recorded photographically are shown, giving the effects upon the photo-electric potential produced in the retina as the age of the eye after removal increases, as the intensity of the light is varied, and as the time of exposure is changed from very short flashes to exposures of a few seconds. Curves are also presented, giving the photo-electric responses for extremely low ntensities of illumination. The curves are discussed and a tentative theory for the action given.

Harvard University,
April 6, 1921.

The Variation of Metallic Conductivity with Electrostatic Charge.

By Henry A. Perkins.

A simple conception of metallic conduction based upon moving electrons seems to justify the assumption that a negative charge should increase the conductivity of a circuit, and a positive charge should decrease it. This has been tested as follows: A primary coil of about 80 turns was wound upon a glass cylinder inside of which fitted a smaller cylinder similarly wound as a secondary. The primary was excited from a 60-cycle commercial circuit. The secondary circuit, carefully insulated throughout included a moving coil galvanometer and was given an alternating charge from one terminal of a high tension transformer giving about 5,700 volts, the other terminal being grounded. The two coils wound upon glass acted as a condenser, so that a fairly large charge could be communicated to the insulated system.

In operation the galvanometer was not affected by the charging potential, but the induced alternating current caused a slight vibration of the "spot" on the scale and if it reached too high a value (say 3 milliamperes) produced a small deflection always in the same direction, and due, probably, to constriction of the helical portion of the suspension system. If, however, the induced current and the charging potential were both present, a deflection as high as 30 scale divisions was obtained, which indicated a direct current of about half a microampere. This phenomenon behaved in a perfectly regular and definite manner, and reversed if the phase of the charging E.M.F. were reversed by interchanging the terminals of the transformer. A change of phase less than 180° produced proportional changes in the reading of the instrument.

A simple theory of this effect may be worked out by neglecting the reactance of the circuit, which proved to be very small compared to its resistance. If the charging E.M.F. and induced current are of the same frequency and in the same or opposite phase, the conductance, during one half cycle varies sinusoidally from the normal value to a slightly higher one, and then returns to normal again, while during the other half cycle, it falls to a minimum and returns to normal in the same manner. This causes a pulsating unidirectional current in conjunction with the induced E.M.F. in the secondary circuit. whose value $I = Ey$ when y is the effective change in the conductivity of the circuit. The quantity, y, is proportional to the number, n, of electrons added to or subtracted from the secondary circuit. This number of electrons is equal to EC/e when E is the effective charging potential, C is the capacity of the secondary system and e is the charge of an electron. Similarly the ordinary conductance $Y = 1/R$ is proportional to the effective number of electrons concerned in carrying the current. Calling this quantity Nc, we may assume

$$\frac{Nc}{n} = \frac{Y}{y} = \frac{E}{IR}$$

and substituting for n its value EC/e we obtain

$$Nc = \frac{E\mathcal{E}C}{eIR}.$$

To allow for a change in the phase relation between E and $\mathcal{E}$, this must be multiplied by the cosine of the phase difference ϕ, so that the formula becomes

$$Nc = \frac{E\mathcal{E}C\cos\phi}{eIR}.$$

The values of the variables involved were measured and the resulting calculation gives $Nc = 1.7 \times 10^{13}$. The total mass of copper in the circuit was about 140 grams, so that assuming 29 free electrons (atomic number) for copper and taking Avogadro's constant as 7×10^{23} atoms in 63.6 grams of the wire, we may estimate the total number N of free electrons as 4.4×10^{25} thus giving the ratio $Nc/N = 4 \times 10^{-9}$ which means in round numbers that in this particular circuit only one electron in two hundred million is effective as a carrier of the current.

The size of wire used in the galvanometer is probably No. 36 B. & S. gauge. This would mean a length of 115,000 cm. The current caused by the added electrons, n, is given by $I = nve/l$, where v is their effective velocity and l is the length of the circuit. Calculating n as before from $n = EC/e$, we obtain in the case of a current of 4.4×10^{-7} amperes an effective velocity equal to 8,800 cm. per second in this particular circuit.

Trinity College,
March 31, 1921.

The Distributed Capacity of Inductance Coils.

By G. Breit.

Between any two portions of an inductance coil there is a certain amount of capacity. This capacity is distributed along the coil and is called the distributed capacity of the coil.

The distributed capacity of the coil gives rise to numerous phenomena, as *e.g.*, non-uniform distribution of current in the coil, a large increase in the resistance of the coil, a change in its apparent inductance, etc.

Some of these phenomena may be explained in terms of a fictitious capacity —called the "effective capacity" of the coil. The definition of effective is given in terms of resonance experiments as follows.

If a condenser of capacity C is connected across the coil terminals the resulting circuit resonates to a certain frequency $\omega/2\pi$. Experiments show that in most cases it is possible to find such numbers L, C_0 that

$$L(C + C_0) = \frac{1}{\omega^2}.$$

The constant C_0, if it exists, is called the "effective capacity" of the coil.

A formula has been derived for the calculation of the effective capacity of a coil. This formula is

$$C_0 = \int_{x_1}^{x_2} \frac{M(x)}{L} \left\{ \int_{1}^{x} \frac{\alpha(x)}{L}\, dx \right\} dx,$$

where x is an arbitrary parameter along the wire

$$\alpha(x) = \frac{Q(x)}{\left(\frac{di_1}{dt}\right)}$$

where $Q(x)dx$ is the charge between x and $x + dx$, where i_1 is the current at the terminal where L is the self inductance, where $M(x)dx$ is the mutual inductance of the element dx to the rest of the coil.

The formula has been applied to special cases and verified in these cases experimentally. These cases with corresponding formulas are:

(I.) Short single layer solenoid one turn of which has the perimeter l, or pancake of small depth used in elliptical shield of major semi-axis a and minor semi-axis β and insulated from shield the space between coil and shield having dielectric constant K.

$$C_0 = 0.06952\, Kl\frac{\alpha}{\beta}\ \mu\mu f = \frac{Kl}{16}\frac{\alpha}{\beta}\ \text{c.g.s. electrostatic units.}$$

(II.) Same coil connected to shield at terminal

$$C_1 = 0.06952\, Kl\left[\frac{\alpha}{\beta} + \frac{2}{\log\sqrt{\dfrac{\alpha+\beta}{\alpha-\beta}}}\right]\mu\mu f$$

$$= \frac{Kl}{16}\left[\frac{\alpha}{\beta} + \frac{2}{\log\sqrt{\dfrac{\alpha+\beta}{\alpha-\beta}}}\right]\ \text{c.g.s. electrostatic units.}$$

(III.) Same coil used outside shield insulated

$$C_0 = 0.06952\, Kl\ \mu\mu f = \frac{Kl}{16}\ \text{c.g.s. electrostatic units.}$$

(IV.) Same coil used outside shield and grounded

$$C_0 = 1.1124\left[\frac{1}{16} + \frac{1}{8\log\left(\dfrac{16R}{a}\right)}\right] Kl\ \mu\mu f$$

$$= \left[\frac{1}{16} + \frac{1}{8\log\dfrac{16R}{a}}\right] Kl\ \text{c.g.s. electrostatic units.}$$

(V.) Pancake coil of radius a wound to center

(*a*) when grounded at center

$$C_0 = 0.568\, Ka\ \mu\mu f = \frac{8}{5\pi} Ka\ \text{c.g.s. electrostatic units.}$$

(*b*) When grounded on the outside

$$C_0 = 0.330\, Ka\ \mu\mu f = \frac{14}{15\pi} Ka\ \text{c.g.s. electrostatic units.}$$

(*c*) When coil is insulated

$$C_0 = 0.251\, Ka\ \mu\mu f = \frac{32}{45\pi} Ka\ \text{c.g.s. electrostatic units.}$$

(VI.) Multilayer coil of large diameter with circular cross section, a single turn having perimeter l when insulated

$$C_0 = (0.08453\, K_e + 0.08230\, K_o)l\ \ \mu\mu f$$

where K_e is the effective dielectric constant of the medium inside the coil, and K_o is the dielectric constant of the material outside.

(VII.) Multilayer coil of large diameter with square cross section, a single turn having perimeter l when insulated

$$C_0 = (0.0886\,K_e + 0.1050\,K_o)l\ \mu\mu f.$$

The last two formulas have not been tested experimentally.

Bureau of Standards,
Washington, D. C.

Some Effects of the Distributed Capacity of Coils to Ground.

By G. Breit.

When two condensers connected in series with common terminal grounded are connected across the terminals of an inductance coil as in the figure, their effective capacity in series is not equal to their effective capacity so far as resonance of the coil is concerned. If the capacity of one condenser is C_1 and of the other C_2, the quantity $C_1C_2/(C_1 + C_2)$ is the effective capacity of the two condensers in series. As stated it does not stay constant when C_1 is changed arbitrarily and C_2 is readjusted for resonance.

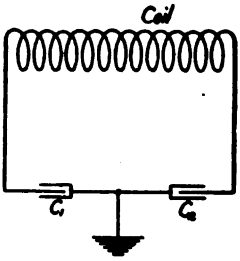

Fig. 1.

A mathematical calculation shows, however, that provided the coil is symmetrical with respect to its two terminals the quantity $C_1C_2/(C_1 + C_2)$ is linearly related to $1/(C_1 + C_2)$.

Experimental verification of this fact has been obtained.

The connections used are shown in Fig. 1 attached.

Bureau of Standards,
Washington, D. C.

The High-Frequency Resistance of Inductance Coils.

By G. Breit.

The meaning of the term "resistance," so clear for direct current and for low-frequency alternating current, needs careful consideration in the case of high-frequency alternating currents. It is found that in the case of an inductance coil the resistance may assume varying values depending on the particular point in the inductance coil with respect to which the resistance is measured. This is due to the non-uniform distribution of current in the coil.

From a knowledge of the resistance with respect to all points of an inductance coil the current at any place in the coil may be computed for a given distribution of e.m.f.'s in the coil. These results were experimentally verified in the case of coil aerials.

Further, from a knowledge of current distribution and from calculations on skin effect the resistance itself may be calculated. The results of these calculations were tested by experiment.

Bureau of Standards,
Washington, D. C.

METHOD OF MEASURING COIL CAPACITIES AND STANDARDIZING WAVEMETERS.

BY G. BREIT.

A METHOD has been worked out for the adjustment of two frequencies to an accurately known ratio.

The method is based on the fact that an electron tube detector distorts the wave form of the E.M.F. impressed on it and thus produces harmonics of that E.M.F. in its output.

The harmonics produced by a circuit of adjustable frequency are made to give beats with the fundamental of a circuit of fixed frequency. The beats are rectified in an amplifier and are heard as a musical note.

When the frequency of the beats is zero the ratio of the frequencies is exactly a whole number. The whole number may be made very large as, *e.g.*, 100.

There is generally little difficulty in ascertaining the whole number. Thus the method gives frequencies in accurately known ratios.

A knowledge of such ratios is essential to the determination of coil capacities and to the standardization of wavemeters.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

TABLES FOR THE CALCULATION OF THE INDUCTANCE OF CIRCULAR COILS OF RECTANGULAR CROSS SECTION.

BY FREDERICK W. GROVER.

WITH the advance of radio communication, the calculation of inductance has become increasingly important. In the case of the more complicated circuits, the inductance is obtained by first imagining the system to be subdivided into simpler elements for which formulas are already available. By combining the calculated inductances of the elements according to the well known laws for the combination of self and mutual inductances, the solution for the whole circuit is built up.

Perhaps the most important single formulas for the calculation of inductance are, from this standpoint, those for the mutual inductance of parallel, coaxial, circular currents. Formulas have been developed by various investigators which cover, with great accuracy, all the possible cases. The author of the present paper has calculated a table[1] which materially shortens and systematizes the calculation of the mutual inductance of any pair of such circular currents.

Scarcely less basic are the formulas for the inductance of circular coils of rectangular cross section. This is the form of coil usually employed when a larger inductance is required than can readily be obtained from a single layer solenoid. It is true that for higher frequencies, the larger capacity of such a coil is objectionable, but this can be reduced to the order of magnitude of that of a single layer coil by employing a "banked" winding, or a "honeycomb" winding, or by the use of insulating material to increase the distance between successive layers of the winding. The inductance of all these forms of coil, and of flat spiral coils as well, may be obtained from the formulas for the inductance of circular coils of rectangular cross section. The relatively small correction to take into account the fact, that the current in an actual coil is not uniformly distributed over the cross section, may be obtained by known formulas for windings of round wire or of strip.[1] It must, however, be noted that the dimensions of the cross section of the equivalent coil of rectangular cross section are not the same as the overall dimensions of the coil. The axial dimension of the cross section of the equivalent coil is to be taken as equal to the product of the number of turns in the layer by the distance between centers of adjacent wires in the layer, while the radial dimension is to be taken as the product of the number of layers by the distance between the centers of corresponding wires in consecutive layers.

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[1] Bureau of Standards, Circ. 74, p. 286.

In addition to the direct application to multiple layer coils there must be mentioned the possibility of calculating the mutual inductance of coaxial coils of rectangular cross section from a knowledge of the self inductance of such coils. In certain cases the mutual inductance of two coils may be calculated as the difference of the self inductance of certain related coils.[2] This is of especial importance in cases where the coils are nearly in contact, and other methods fail. Yet another application of this basic type of circuit is that of the approximate calculation of the leakage reactance of transformer coils whose dimension along the core is small compared with the dimension of section measured perpendicular to this direction.

Formulas for the inductance of circular coils of small rectangular cross section were long ago developed by Maxwell,[3] Weinstein[4] and Stefan.[5] In recent years Stefan's formula has been extended to apply to coils of larger cross section by Lyle,[6] who prepared tables of coefficients to aid in numerical calculations. Still more recently, the case where the cross sectional dimensions are relatively very large has been covered by the formulas of Butterworth[7] and Dwight,[8] so that it is now possible to treat with accuracy, any coil of this type whatever. Calculations are, however, at best somewhat laborious, and where frequent computations are required, very tedious. Furthermore, without the benefit of considerable experience with these formulas, the computer labors under considerable confusion as to the choice of the formula best suited for the case in hand.

To meet these difficulties, Coursey[9] has prepared curves to simplify the calculations. As reproduced by the printer, values of the inductance may be determined by graphical interpolation from these curves to one per cent. or perhaps a little better. In computing the curves the method of geometric mean distance was used. This is inadequate to give results with the accuracy just stated in every case when the thickness of the winding is greater than about three tenths of the mean diameter of the coil, the error being greater the thicker and longer the coil, that is, the greater the relative dimensions of the cross section. In cases of coils with very thick windings the error may easily reach ten to twenty per cent. In addition, Coursey's curves do not cover the case of coils whose radial dimension exceeds the axial (pancake coils).

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[1] Bureau of Standards, Sci. Paper 169, p. 140; Circ. 74, p. 261.

[2] Bureau of Standards, Sci. Paper 169, p. 41.

[3] Electr. and Mag., Vol. II, § 706.

[4] Wied. Ann., 21, p. 329, 1884.

[5] Wied. Ann., 22, p. 113, 1884.

[6] Phil. Trans., 213 A, p. 421, 1914. Bureau of Standards, Sci. Paper 320, p. 569, 1918.

[7] Phil. Mag., 29, p. 578, 1915.

[8] Proc. A.I.E.E., 1919.

[9] Proc. Phys. Soc. London, 31, p. 155, 1919.

To make rapid calculations possible with an accuracy better than one tenth of one per cent., the author of the present paper has calculated for the Bureau of Standards, constants which cover the whole range of possible coils. On account of the difficulties in the reproduction of curves to give the accuracy desired, it has seemed advisable to present the results in the form of tables. This does not, of course, prevent the use of curves drawn, as desired, from the data of the tables.

The calculation of the inductance using these tables is based on the use of two formulas which may be written as follows:

$$L = 0.002\pi^2\left(\frac{2a}{b}\right)n^2aK' = 0.002\pi^2\left(\frac{2a}{b}\right)n^2a(K - dk)\text{microhenrys}, \quad \text{(A)}$$

$$L = 0.001n^2aP' = 0.001n^2aPf \text{ microhenrys}, \quad \text{(B)}$$

where L = inductance with uniform distribution of the current,

a = the mean radius of the coil,

b = the axial dimension of the cross section,

c = the radial dimension of the cross section,

n = the total number of turns.

The quantities K', P', dk and f are functions of the two parameters

$$c/2a = \frac{\text{thickness}}{\text{mean diameter}}$$

and b/c or c/b, the ratio of the two dimensions of the cross section. The values in the tables are given for values of b/c or c/b in steps of 0.1, and for $c/2a$ in steps of 0.1 with the added values of 0.025 and 0.05.

Formula (A) is especially useful for relatively long coils, while formula (B) applies best to pancake coils, although both may be used in any case, and thus provide the means of checking the calculations.

When the given case may be treated without interpolation in the tables, the quantities K' and P' should be used. Otherwise, greater accuracy is attained by interpolating dk or f, and using the alternative forms of the formulas. The quantity K is the value of K' for the single layer coil of the same length and mean diameter as the actual coil. A table of this quantity was published by

Nagaoka.[1] The factor P is the value of P' applicable to a disk coil of the same mean diameter and radial dimension c as the coil in question. A table of values of P for values of $c/2a$ in steps of 0.01 has been prepared by the author using the formulas of Rayleigh and Niven, Lyle and Spielrein.[2] Interpolation of P may be very accurately and easily accomplished from this table.

All the values in the tables have been calculated from the various formulas mentioned above, that formula being chosen in each case whose convergence is most satisfactory. In certain cases where two overlapping formulas could be employed, both have been used as a check on the result.

Electrical Engineering Dept.,
Union University,
April 4, 1921.

On Some New Formulæ for the Direct Numerical Calculation of the Coefficient of Mutual Induction of Coaxial Circles.

By Louis V. King.

If we start with the positive numbers (a_0, b_0), the arithmetical operations given below are easily carried out on a modern calculating machine.

$$\begin{array}{lll} a_0 & b_0 & \\ a_1 = \tfrac{1}{2}(a_0 + b_0) & b_1 = \sqrt{a_0 b_0} & c_1 = \tfrac{1}{2}(a_0 - b_0) \\ a_2 = \tfrac{1}{2}(a_1 + b_1) & b_2 = \sqrt{a_1 b_1} & c_2 = \tfrac{1}{2}(a_1 - b_1) \\ \cdots & \cdots & \cdots \\ a_n = \tfrac{1}{2}(a_{n-1} + b_{n-1}) & b_n = \sqrt{a_{n-1} b_{n-1}} & c_n = \tfrac{1}{2}(a_{n-1} - b_{n-1}) \\ \cdots & \cdots & \cdots \end{array} \tag{1}$$

The a's and b's converge with great rapidity to the same limit, which may be denoted by a_n.

The array of numbers given above originated with Lagrange (1784–85) and was employed by Legendre (1786) in computing his well-known tables of elliptic integrals. The use of the above process for the numerical evaluation of elliptic integrals was discussed independently by Gauss (1818), and was called by him the "algorithm of the arithmetico-geometrical mean."

Denoting by K and E the complete elliptic integrals of the first and second kinds, and by k and k' the modulus and its complement, it may be proved that

$$K = \tfrac{1}{2}\pi/a_n \quad \text{and} \quad (K - E)/K = \tfrac{1}{2}(c_0^2 + 2c_1^2 + 4c_2^2 + \cdots + 2^n c_n^2 + \cdots) \tag{2}$$

both well-known results.

It does not appear to have been generally remarked hitherto that $2^{-n} \log (a_n/c_n)$ converges to the limit $\tfrac{1}{2}\pi K'/K$. The writer has been unable to find this result explicitly stated by Gauss or by any subsequent writer, although the following formula

$$\tfrac{1}{2}\pi K'/K = \tfrac{1}{2} \log (4a_1/c_1) - \sum_{1}^{\infty} (\tfrac{1}{2})^n \log (a_n/a_{n+1}) \tag{3}$$

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[1] Jour. Coll. Sci. Tokyo, 27, art. 6, p. 18, 1909.

[2] Arch. f. Elekt., 3, p. 187, 1915.

derived from it was found among a number of formulæ in Gauss' posthumous papers, stated without proof. An equivalent result is given however, by Legendre.

The use of these *arithmetico-geometrical scales* for the direct numerical computation of formulæ involving elliptic integrals is now made possible by the modern calculating machine on which square-root operations are easily carried out.

Let A and a be the radii of two co-axial circles and d the distance between their planes. If we compute $r_1^2 = (A + a)^2 + d^2$ and $r_2^2 = (A - a)^2 + d^2$, and the arithmetico-geometrical scale ($a_0 = r_1$, $b_0 = r_2$) is then derived, we find that Maxwell's well-known formulæ for the mutual inductance of the two circles may be written

$$M = (2\pi^2/a_n)(c_1^2 + 2c_2^2 + 4c_2^2 + \cdots + 2^{n-1}C_n^2 + \cdots). \qquad (4)$$

This formula is extremely convergent and convenient to use in *all circumstances*, three terms being usually sufficient to give seven-figure accuracy.

When the coils are very close together, ($r_2/r_1 > 0, 1$) there is some advantage in computing the complementary scale, ($a_0' = r_1 + r_2$, $b_0' = r_1 - r_2$), which is then extremely convergent, a_2' and b_2' being the same to seven figures. We then find that

$$M = 2\pi a_n'$$

$$\left[\frac{(r_1^2 - 2c_2'^2 - 4c_3'^2 - \cdots)}{a_n'^2}\left(\log\frac{4a_3'}{r_2} - \tfrac{1}{2}\log\frac{a_2'}{a_3'}, -\tfrac{1}{4}\log\frac{a_3'}{a_4'} - \cdots\right) - 2\right] \qquad (5)$$

The new formulæ (4) and (5) when applied to numerical examples show a decided advantage over existing methods of computing M.

Formulæ of this type, making use of trigonometrical recurrence-formulæ combined with the A.G.M. scales have been derived for the direct numerical calculation of the self and mutual inductances of single layer coils.

McGILL UNIVERSITY,
MONTREAL, April 6, 1921.

NOTE ON THE EARTH'S MAGNETIC FIELD.

BY W. F. G. SWANN.

H. LAMB has shown that if the conductivity of the material of the interior of the earth were as great as that of copper, a system of currents such as would account for the earth's magnetic field would if once started within the earth, take ten million years to die to $1/e$ of their original values even though no external electromotive forces were present to maintain them. This result is due to the very great influence of self induction in the case of a body of the size of the earth.

In the present note it is shown that if the earth had been permanently magnetized at some time, and if the permanent magnetism were suddenly destroyed, the induced currents set up would perpetuate the original magnetic field to such an extent that, for a conductivity equal to that of copper, several million years would elapse before the field had diminished to $1/e$ of the value corresponding to the original magnetization.

A further extension of this idea shows that if the earth's magnetism be pictured as due to the superposition of two uniform magnetizations, one parallel to the geographic axis and one perpendicular thereto, and if the secular rotation of the pole be pictured as a uniform rotation of the second named magnetization relative to the earth with a period corresponding to one complete rotation in 500 years, then we are unable to admit more than a certain limiting conductivity for the earth's interior, as otherwise the induced currents set up by the rotation would be too large to be consistent with the rotation postulated. The limiting conductivity in question is of an order of magnitude certainly less than 1/5,000 that of copper.

UNIVERSITY OF MINNESOTA.

THE ELECTRICAL RESISTANCE OF THE HUMAN BODY.

BY F. WENNER, JAS. S. MARTIN, NYNA FORMAN.

FROM a study of the results that have been published on the measurements of the electrical resistance of the human body, we find that the values obtained differ widely depending upon the methods of measurements and the conditions under which the measurements are made.

About two years ago, Professor Gildemeister[1] made an extensive investigation of the resistance of the human body. In summing up the results of his measurements, we find that he obtained values for the electrical resistance of substantially the same parts of the body varying from 148,000 ohms to 68 ohms, depending upon the duration of current flow, the voltage, and the frequency. On increasing the frequency by a factor of 430 he found the resistance to decrease by a factor of 15. He discusses his data fully and reaches the conclusion that in the case of accident, when the skin is burned, the resistance falls to a very low value.

Dowse and Iredell[2] obtained much more consistent data and it is interesting to note that while they changed the frequency by a factor of 1,000 they found the impedance from hand to hand to change by a factor less than two. They further conclude that the change in resistance is less than the change in impedance. The results cannot therefore be considered to be in accord with those obtained by Professor Gildemeister.

The area of the electrodes, condition of the skin where the test current enters and leaves the body, and electrolytic and electrostatic actions taking place at or in the vicinity of the electrodes is a factor which apparently affected all the values previously obtained. We, therefore, thought that it might be of interest to take a few measurements in such a way that this factor could have no effect upon the values obtained.

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[1] Elektrotechnische Zeitschrift, 1919, Vol. 40, p. 463.

[2] Archives of Radiology and Electrotherapy, 1920, Vol. 25, p. 33.

An essential feature of the plan adopted for attacking the problem is the use of four connections to the body; two of these serve to lead the test current to and from the body and the other two serve to bring the potential difference, developed in a certain part of the body by the test current, to a place where it can be measured. This is a method used for the purpose of eliminating the effects of contact resistances where we wish to determine the resistance of short and thick metallic conductors. Other features of the plan are the use of an alternating test current, a method of measurement which requires the passage of little or no current through the potential terminals, and a detector which responds to alternating current only. It was thought that possibly this plan would give results sufficiently consistent to show definite differences between individuals or changes in the same individual.

During July, 1920, a series of measurements was made using the left hand and foot as current terminals and the right hand and foot as potential terminals. The arrangement was such that the magnitude of the test current could readily be changed by a factor of two, and its frequency changed from 25 to 100 cycles.

No change in resistance was observed on changing either the frequency or the voltage, though the sensitivity was such that changes of a few per cent. could have been detected. Consequently in a second series of measurements made in November, no provision was made for changing the frequency of the test current and to avoid the operation of a special machine 60 cycles was used.

The resistance of the trunk was found to vary with the position of the individual or to be larger with the muscles tense, than relaxed, and measurements on the same individuals made at different times give slightly different values for the resistance of the trunk of the body. For most individuals the resistance of the trunk was found to be between 25 and 30 ohms.

These we believe to be the first measurements giving any definite idea as to the resistance of the "vital" parts of the body.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

NEW METHODS FOR MAINTAINING CONSTANT FREQUENCY IN HIGH-FREQUENCY CIRCUITS.

BY W. G. CADY.

As stated in a former communication,[1] a plate properly cut from a piezoelectric crystal and provided with metallic coatings vibrates mechanically when the coatings are connected to an E.M.F. of high frequency, provided that the frequency is sufficiently near the natural frequency of mechanical vibration. The absorption of energy gives the plate an effective series resistance which is a maximum at the resonant frequency. The effective parallel capacity of the plate is a maximum at a frequency slightly below resonance, and a minimum at a frequency slightly above resonance, passing through the normal value at resonance. The total range in capacity is greater, the less the vibrations are damped, and in the case of quartz it may be more than twenty times as great as the normal capacity of the plate. The accompanying change in frequency may be a very small fraction of a per cent.

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[1] PHYS. REV., April, 1921.

This dependence of capacity upon frequency may be utilized as a frequency stabilizer, by simply connecting the piezo-electric plate in parallel with the tuning condenser of an ordinary vacuum tube oscillating circuit. When the frequency is slightly above that at which the plate is in mechanical resonance, an increase in the capacity of the variable condenser is accompanied by a nearly equal decrease in the effective capacity of the plate, so that the frequency remains constant within exceedingly narrow limits. If the condenser reading is increased beyond an amount corresponding to the maximum plate capacity, the plate suddenly stops vibrating, and the frequency abruptly decreases by a large amount. A similar process, in the reverse sense, takes place on decreasing the capacity of the variable condenser.

For example, a certain quartz plate 3.9 cm. long vibrated at a frequency of about 69,700. A change in the variable condenser which, when the quartz plate was removed, altered the frequency by 3 per cent., varied it by less than one part in 20,000 when the quartz was in circuit. This plate had coatings so small that its normal capacity was only about 0.67 micro-micro-farads; yet when vibrating, its effective capacity varied from 10 to − 9 M.M.F.

A similar stabilizing effect can be obtained for any frequency over a very wide range by selecting a plate of the right length.

Two other methods of connecting a quartz plate may be mentioned, which under favorable conditions have been found to exert an even more marked stabilizing influence upon the frequency. In one of these, the plate is provided with two pairs of coatings, one pair being connected in parallel with the tickler coil in the anode circuit, the other in parallel with the grid condenser and leak. If the circuit without the plate is on the verge of oscillating, then when the plate is introduced, the vibrations of the quartz furnish the "feed-back," and the circuit oscillates at a frequency determined solely by the plate itself. By a proper choice of circuit constants, the plate can be made to vibrate at either the fundamental frequency or the second or third harmonic.

In the third method, the plate has but one pair of coatings, connected in parallel with the grid condenser and leak. Here again, by a suitable choice of circuit constants, the electrical frequency will be found to be determined by the vibrations of the plate. In this, as in the other methods, the frequency is practically unaffected by small changes in capacity or self-inductance of the circuit.

Wesleyan University,
Middletown, Conn.

Magnetic Rotatory Dispersion in Transparent Liquids.

By R. A. Castleman, Jr., and E. O. Hulburt.

Theoretical.—From the electron theory of H. A. Lorentz a theoretical formula was derived which expressed the relation between the angle of magnetic rotation of the plane of polarization of plane polarized light produced by the magnetized medium, the wave-length of the light, the strength of the magnetic field, and other physical quantities. Absorption of light in the medium was neglected in the theory.

Experimental.—The magnetic rotation angle of a number of transparent liquids (carbon disulphide, benzene, nitrobenzene, etc.) for a series of wave-lengths in the visible spectrum was measured. The apparatus consisted of a grating monochromatic illuminator, two nicol prisms, and a Ruhmkorff magnet between whose poles was placed a cell containing the liquid under examination.

Comparison of Theory and Experiment.—The magnetic rotation angles for various wave-lengths were calculated from the theoretical formula and compared with the observed ones. It was found that the theory agreed with the observations for the longer wave-lengths but that for the shorter ones the theoretical rotations were greater than the observed, the discrepancy increasing with decrease in wave-length. It is pointed out that if the effect of absorption were incorporated in the theory the above discrepancy would be lessened. Lack of experimental data precluded consideration of absorption.

Value of e/m.—The values of the ratio of the charge to the mass of the dispersion electrons were calculated from the observations by means of the theoretical formula, and they were found to lie between 0.49×10^7 and 1.12×10^7, when the electronic charge is expressed in c.g.s. electro-magnetic units.

JOHNS HOPKINS UNIVERSITY.

TOTAL EMISSIVE POWERS AND RESISTIVITIES OF TUNGSTEN AT INCANDESCENCE.

BY A. G. WORTHING AND W. E. FORSYTHE.

TOTAL emissive power and resistivity relations of metals at incandescent temperatures are readily obtained from current, voltage and temperature measurements on filaments mounted in well evacuated lamp bulbs. The process is considerably simplified when the cooling effects of leads are eliminated by means of fine potential wires of the same material attached to the filaments at considerable distances from the cooling leads.

In this work, lamp A–33, containing a tungsten filament, 0.257 mm. in diameter and about 30 cm. in length with potential leads so attached as to permit of voltage measurements across the central third (10.85 cm.) of its length, has been taken as a standard with which others have been compared. Certain data on this lamp have been discussed elsewhere.[1] Due to an accidental error in platting, the possibilities of the data were not appreciated at that time. Since then, however, the conclusions to be drawn from data on this lamp for the same temperature range, as well as predictions for greatly increased temperature ranges, have been verified by many tests on other lamps. Moreover, so far as individual variations between lamps are concerned, A–33 seems to represent a fair average.

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[1] Worthing, PHYS. REV., 10, p. 377, 1917.

To well within the accuracy of the potentiometer and temperature measurements, the resistance-temperature and emission-temperature relations for this lamp are given by

$$\text{(I.)} \qquad \frac{R}{R_0} = \left(\frac{T}{T_0}\right)^{1.200},$$

and

$$\text{(II.)} \qquad \log E = 3.680\,(\log T - 3.3) - \frac{1040}{T} + 1.900,$$

where E is expressed in watts/cm^2 and T in ° K. Deviations from fulfillment for the latter equation are not greater than those due to temperature errors of 0.5° K.

How well the resistance-temperature relation is fulfilled is shown in the accompanying plat in which data on other lamps are included for temperatures ranging from 1400° to 2700° K. That the relation in each case is given by equation I. is evident. It is equally evident that the exponent changes as we pass from one lamp filament to another. This probably depends upon whether the filaments are thoriated or not, and also certainly upon the history of the filament in the lamp. On this account, the actual exponents corresponding to T–110 less T–111 and T–104 less T–101 may not be as significant as in the other instances. Tests show some deviation from equation I. as room temperatures are approached. On the other hand, tests carried up to 3200° K. indicate that the equation holds perfectly for increasing temperatures. Absolute resistivities may be obtained by taking 59.2×10^{-6} ohm cm. as the absolute value at 2000° K.

How well equation (II.) holds is shown in Fig. 4 of the paper referred to above. Data on various other lamps which were not broken up for filament dimensions show agreement with the curve presented there so far as variation with temperature is concerned. A recent attempt to extend that curve to noticeably higher temperatures has yielded results in accord with it and with equation (II.) up to 3200° K.

Combining equation (II.) with the Stefan-Boltzmann constant yields the total emissive power of tungsten as a function of temperature. Results based on 5.72×10^{-12} watts/cm^2 deg^4 for that constant and the temperature scale[1] for tungsten adopted by the General Electric Company are given in the table. Starting with low values at low temperature, there is seen to be a gradual increase of the exponential type as though there were a limiting value in the neighborhood of 0.40, as might be expected from a knowledge of emissive powers in the visible and the ultra-violet.

An attempt to compute the total emissive powers from the resistivities at low temperatures as was done by Foote[2] for platinum using the equation

$$e_t = 0.5736\,\sqrt{\rho T} - 0.1769\rho T$$

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[1] Gen. Elec. Rev., 20, p. 819, 1917.

[2] Bull. Bur. Stds., 11, p. 607, 1915.

yielded values considerably lower than the true values, indicating, as was found by Davison and Weeks[1] for platinum, that there were considerable deviations from the cosine law of emission for the total radiation of tungsten. Similar computations, using reflectivities of tungsten at room temperature as determined by Coblentz[2] and the variation with temperature as determined by Weniger and Pfund,[3] show a similar lack of fulfillment of the cosine law. Using Weniger and Pfund's data at room temperature (unpublished) in place of Coblentz' showed still greater variations. At 2540° K. the operating temperature of the ordinary vacuum tungsten lamp, the two values thus computed were found too low by 7½ per cent. and 13 per cent. respectively. It is hoped that a test to show the cosine law variation for the total radiation may be carried out soon at our laboratory.

Total Emissive Powers of Tungsten.

T in °K.	E (tungsten) in $\frac{\text{watts}}{\text{cm}^2}$	E (black body) in $\frac{\text{watts}}{\text{cm}^2}$	e_t
1000	0.57	5.72	0.100
1500	5.64	29.0	.194
2000	24.2	91.5	.264
2500	69.8	223.	.313
3000	161.	463.	.347
3500	317.	858.	.369

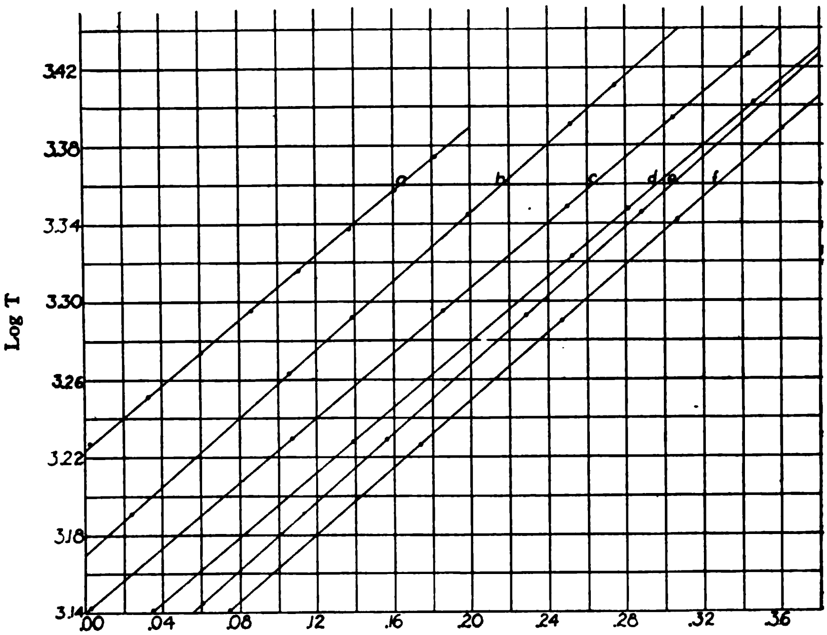

Fig. 1.

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[1] PHYS. REV., 17, p. 261, 1921.

[2] Bull. Bur. Stds., 14, p. 312, 1918.

[3] PHYS. REV., 14, p. 427, 1919.

Temperature Resistance Relations for Various Tungsten Lamps.

Curve	Lamp.	Exponent in Equation (I.).
a	T–1	1.210
b	T–98	1.135
c	A–33	1.200
d	A–30	1.187
e	T–110 less T–111	1.128
f	T–104 less T–101	1.160

NELA RESEARCH LABORATORIES,
NELA PARK, CLEVELAND, OHIO,
April 7, 1921.

COLOR TEMPERATURE OF HIGH EFFICIENCY LAMPS.

BY W. E. FORSYTHE.

THE efficiency and life of incandescent lamps depend upon the temperatures at which they are operated, the efficiency being greater for high temperatures and the life shorter. Lamps are therefore operated at temperatures which will give the desired life. The integral color of the light given by the lamp also depends upon the temperature, the light being bluer for the higher temperatures.

It has been found by experiment that the light from incandescent lamps can be matched in color with that from a black body at some particular temperature which temperature is called the color temperature. Making use of this fact the quality of the light given by the different lamps can be given by its color temperature.

Several different kinds and types of incandescent lamps operated at certain efficiencies have been color-matched with a black body and the results are given in Table I.

Lamp.	W. P. S. C.	Temperatures Degrees K.	Color Temperature.
50-watt Carbon	3.76	2115[1]	2156
50-watt Gem	3.1	2180[1]	2195
50-watt Tantalum	2.56	2160[2]	2260
40-watt Mazda B	1.36	2410	2460
100-watt Mazda C	1.00	2745	2740
500-watt Mazda C	0.72	2835	2880
1000-watt Mazda C	0.62	3010	2985
1000-watt Stereopticon	0.52	3185	3175
900-watt Movie	0.46	3290	3220

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[1] True temperature calculated from brightness temperature, assuming an emissive power of 80 per cent.

[2] True temperature calculated from brightness temperature, assuming an emissive power of 48 per cent.

These temperatures are based upon the assumption of Wien's equation with c_2 taken as 14350 μ deg. and upon the melting point of gold taken as 1336° K. On this scale the melting point of palladium has been found to be 1828° K. For convenience in the calibration of optical pyrometers, a black body held at the melting point of palladium is used as the point of reference.

NELA RESEARCH LABORATORIES,
CLEVELAND, OHIO,
April, 1921.

THE DETECTING EFFICIENCY OF THE RESISTANCE-CAPACITY COUPLED ELECTRON TUBE AMPLIFIER.

BY E. O. HULBURT.

Definition of Detecting Efficiency.—The detecting efficiency is defined by the ratio b_0/A^2, where b_0 is the rectified high frequency component of the output plate circuit produced by a high frequency voltage of magnitude A impressed on the grid of the input tube.

Derivation of Theoretical Formula.—A theoretical formula is derived for the detecting efficiency of an amplifier consisting of two tubes coupled by resistances and capacities.

Apparatus.—Measurements of the detecting efficiency were carried out by means of a *condenser potential divider*, and a sensitive *galvanometer* connected in the plate circuit of the detector tube.

Comparison of Theory with Experiment.—The detecting efficiency of the two tube amplifier for various frequencies and coupling capacities was measured and found to agree with the theoretical formula with a fair accuracy. The detecting efficiency increased with the wave-length throughout the range from 400 to 1,600 meters. The *current* and *sound intensity* amplification were determined.

Comparison with Other Amplifiers.—Measurements of the detecting efficiency for various wave-lengths of a two and three tube resistance-capacity coupled amplifier, of a three-tube transformer coupled amplifier, and of two two-tube transformer coupled amplifiers were obtained which gave the relative merits of the various amplifiers as far as their detecting efficiency for an unmodulated signal was concerned.

JOHNS HOPKINS UNIVERSITY.

RADIO SIGNAL FADING PHENOMENA.

BY J. H. DELLINGER AND L. E. WHITTEMORE.

A STUDY has been made of the phenomena exhibited by variations of intensity of received radio signals. The following four functions characteristic of radio transmission were included in the study: intensity of signals, fading of signals, direction of waves, strays. Particular attention was given to the results of some recent tests of short wave radio fading conducted with the coöperation of the American Radio Relay League. It is concluded that the sources or causes of fading or of strays are in the atmosphere between the earth's surface and the Heaviside surface which is a conducting surface some sixty miles above the earth. Daytime transmission is largely carried on by means of waves moving along the ground, while night transmission, especially for great distances and short waves, is by means of waves transmitted along the Heaviside surface. Waves at night are thus free from the absorption encountered in the daytime but are subject to great variations caused by irregularities of the ionized air at or near the Heaviside surface. These variations probably account for fading.

Bureau of Standards,
Washington, D. C.

Some Physical Problems of Aircraft Radio.

By L. E. Whittemore.

Among the lines in which the physicist can be of service to industry and commerce is the solution of problems of radio communication, of which an important example is the development of the use of radio for communication with aircraft.

A statement is given of several problems involved in the reception of radio signals on aircraft and on the ground from aircraft and in the transmission of signals from aircraft and from the ground to aircraft. The progress made to date on several of these problems is summarized.

It is to be noted that these questions are of interest to both the mathematical and the experimental physicist. Their solution will not only make aerial navigation safer and transportation by aircraft more useful, but will also aid greatly in developing the methods of radio communication for other purposes.

Bureau of Standards,
Washington, D. C.

The Design of a Standard Wavemeter.

By R. T. Cox.

It is explained that the increasing demand for more precise wave-length measurement is making the design of wavemeters more and more a problem for the physicist rather than the engineer. The mechanical and electrical requirements which the necessity of precise measurement imposes on each part of a wavemeter are stated. The way in which these requirements have been met in the latest wavemeter of the Bureau of Standards is described. Possible modifications of the design in the interest of economy, where the requirements are not so exacting, are indicated.

Bureau of Standards,
Washington, D. C.

THE HIGH-FREQUENCY IMPEDANCE OF RADIO TELEPHONE RECEIVERS.

BY C. T. ZAHN.

IN order to study the behavior of certain radio circuits where telephones are used, it is necessary to know the radio-frequency impedance, as well as the audio-frequency impedance and the acoustical properties. On account of the capacities between the closely spaced leads used with the receivers the impedance is very different from that of the receivers alone.

It was the purpose of the work here described to measure the impedance at various radio-frequencies of different types of head-set in use. The measurements were made by measuring the resistance of a simple tuned circuit by the resistance-variation method, and then connecting the telephones across the condenser and returning and measuring the resistance again, and computing the impedance from these measurements.

The head-sets have a critical frequency where the reactance changes sign and near which the resistance is maximum. This critical frequency for the receivers tested varies in the approximate range of 8,000–50,000 cycles per second, the maximum values of resistance and reactance being of the order of 100,000 ohms.

BUREAU OF STANDARDS,
WASHINGTON, D. C.

A RECORDER OF RADIO TIME SIGNALS.

BY E. A. ECKHARDT AND J. C. KARCHER.

A RADIO time signal recorder was developed by the authors which permits simultaneous recording of radio time signals or other radio signals and local signals such as the breaks of a chronometer, or signals made by the pressing of a key by an observer. The recording of all signals is accomplished by a single chronographic pen and the circuits are so arranged that the lags are due to differences in the raction times of the relays involved instead of the reaction times themselves. The apparatus provides a means for obtaining simultaneous time indices in connection with recording operations going on simultaneously at different points without the necessity of wire connections between them. Various velocity determinations are thus made possible, as, for example, of earth waves, sound, etc.

Radio signals from Lyons, France (3,800 miles air line) have been recorded with regularity. In such recording the receiving-recording apparatus contained only one electron tube of the ordinary receiving type in most cases, and never more than two. The antenna used was a 50-foot 6-wire flat-top antenna, 8 feet above the roof of a building 57 feet high.

The receiving equipment involves a regenerative electron tube circuit the plate circuit of which contains the windings of a telegraph relay. The grid potential of the tube is adjustable by means of a potentiometer. Considering the regenerative circuit alone there is a critical grid potential at which the circuit will become self-oscillatory as this potential is approached from more negative values. The passing of the circuit from the non-oscillating to the oscillating state is accompanied by a large rise in the mean plate current. In the circuits we are using this rise is from a small fraction of a milliampere to several milliamperes. The relay in the plate circuit is so adjusted that it will be released against an outer contact at the smaller value and pulled against an inner contact at the larger one. The opening of the outer contact results in an excursion of the recording pen from the datum line. The closing of the inner contact short circuits a low resistance coil of a few turns placed between the regenerating coils and thus stops the oscillations. The opening of this contact does not involve any readjustments in the regenerative circuit and consequently permits working very close to the critical potential. In this manner a greater sensitivity has been obtained than any now on record.

If the regenerative circuit be suitably related to a radio receiving antenna the rise in potential due to radio signals may be used to raise the grid potential above the critical value. The operation is then as described above.

In order to record local signals as well as the radio signals the pen controlling magnet has a double winding. The closing of the local circuit then magnetizes or demagnetizes the pen controlling magnet according as the other circuit is open or closed. In this way the two kinds of records may be obtained simultaneously without mutual interference.

Bureau of Standards,
Washington, D. C.,
April 6, 1921.

An International Series of Radio Audibility and Direction Measurements.

By Carl Kinsley.

In August, 1919, a series of measurements were made simultaneously in England, France, Italy and the United States. In England two stations were used, in France there were also two and in the United States four stations were employed which were widely separated in Maine, Arizona, Texas and the District of Columbia. These experiments consisted of audibility and direction measurements upon the signals sent out by the Annapolis arc transmitter.

A comparison of the observations has been made and it is found that the variations of audibility at the different stations does not occur simultaneously. During the course of the experiments the "fading" of the signals was particularly marked at times other than the twilight hours. The most pronounced variations of direction occurred after midnight. These did not take place simultaneously at the different stations. These results confirm previous experiments which were made at stations less widely separated.

There seems to be no reason for attributing the variations to the transmitting station. The variations in audibility and in direction do not in general occur at the same time and the largest change of either is noticeably accompanied by stability in the other. It is probable that the causes may be different—possibly absorption alone is responsible for the audibility variations. A variation in direction of thirty degrees was noted in these experiments and it seems probable that a reflection of the electromagnetic wave at a point not distant from the observer has produced a distorted resultant wave at the receiving station.

DETECTION COEFFICIENT OF THERMIONIC VALVES.

BY E. L. CHAFFEE.

THE detection coefficient of a thermionic valve, when used without a stoppage condenser, is ordinarily derived considering only the curvature of the plate-current grid-voltage curve. Such methods neglect the current taken by the grid, the drop in voltage in the grid circuit as a result of this grid current, and the consequent rectified plate-current. In this paper, an expression is derived theoretically for the detection coefficient, taking into account the grid circuit effects, as well as the plate circuit rectification. The expression is given below.

$$D = \frac{\text{Rectified Plate Current}}{(\text{Applied Grid Voltage})^2} = A\frac{K_c^2}{(K_c + K_g)^2}\left\{\frac{\partial\sigma}{\partial E_g} - \frac{\mu K_p}{K_c + K_g}\left(\frac{\partial K_g}{\partial E_g}\right)\right\}.$$

Where A is a constant, K_c is the equivalent conductivity of the grid circuit exterior to the tubes,

$$K_g = \frac{\partial I_g}{\partial E_g}$$

is the conductivity of the grid to filament space,

$$K_p = \frac{\partial I_p}{\partial E_p}$$

is the conductance of the plate to filament space with grid voltage constant, μ is the ordinary amplification factor, and

$$\sigma = \frac{\partial I_p}{\partial E_g}$$

is the mutual conductance.

It is shown that the second term, which is due to the grid current, may be in actual practice a very large factor in determining the sensitivity of a tube when used as a detector. Experimental curves are given both on hard tubes and tubes containing a trace of gas, and the experimental results compared with the theoretical expression.

CRUFT LABORATORY,
HARVARD UNIVERSITY,
April 6, 1921.

THE RADIO RESEARCH FIELD.

BY J. H. DELLINGER AND L. E. WHITTEMORE.

RADIO communication is a rapidly growing subject both technically and commercially. It was given a great stimulus by the war and is now becoming widely used for government, commercial and private communication. Its place will undoubtedly be that of supplementing the wire telegraph and telephone service rather than supplanting it, the linkage between the two methods of communication being shown by recent work on wire-radio systems and by several demonstrations where part of the distance was covered by wire lines and part by radio. The broadcasting by radio telephony of information which has been collected by wire is becoming more common and offers almost unlimited possibilities.

Many schools and universities have installed courses in radio communication, and research work is being done by them as well as by many private experimenters and the large commercial companies which are engaged in operating radio stations and selling radio apparatus.

Much has been learned about the principles of radio communication and the behavior of radio circuits since the use of the electron tube has made precision measurements possible. The Radio Laboratory of the Bureau of Standards has been attempting to serve in this connection by the development of methods of measurement and the study of fundamental principles as well as by encouraging the use of radio communication and radio methods for many purposes.

During the course of work on this subject suggestions continually arise for further research and it is obviously impossible for a limited staff to conduct work on all of them. The Bureau has coöperated with other investigators in the past by suggesting suitable research subjects, and has prepared a classified list of radio research subjects with the hope that it may be suggestive to many who are anxious to contribute to the development of this field of knowledge. Radio methods offer a convenient avenue of approach to many problems in electric waves outside the strictly radio field. The instruments and methods of radio research are so diversified as to offer a broad training to the research worker.

The list of research problems has been classified in accordance with an extension of the Dewey decimal classification applied to radio, which has been developed by the Bureau, in order that related subjects may be conveniently grouped near one another in the list. The classification is useful in classifying other radio material such as reference abstracts, drawings, books, reports, correspondence, etc.

The classification by itself serves to indicate the wide scope of the radio research field, and the detailed statements under the several subjects emphasize the large number which await solution. The outline given is of course not complete nor is it uniformly detailed. But it is hoped that it will serve to show the opportunity and need for a great deal of work in the radio research field. Fortunately, many of the experimental researches may be conducted with relatively cheap and simple apparatus.

Bureau of Standards,
Washington, D. C.

ON EMERGENT β RAYS FROM MATERIALS OF DIFFERENT ATOMIC WEIGHT AND DIFFERENT THICKNESS PRODUCED BY γ RAYS: STUDIED BY THE STATISTICAL METHOD.

BY ALOIS F. KOVARIK.

IN the study of the number of γ rays from a gram of radium per second it seemed necessary to perform some experiments on the emergent β rays produced by γ rays in passing through matter, inasmuch as the method of observation was the statistical method of counting individual β particles or groups of β particles simultaneously produced, while the published results all deal with the ionization method. In connection with the above problem two things are of importance, namely, the variation of the emergent β radiation with the thickness of material used and the variation with the material. Eve has given some results on both of these problems using the ionization method in his observation. Eve and Bragg and Madsen obtained curves in general agreement with the equation obtained on the supposition that the emergent rays are due to absorbed γ ray energy and that both radiations are absorbed in passing through matter. Eve and Hackett have shown that the emergent radiation does not increase regularly with atomic weight, but decreases from Al to Zn and then increases again for higher atomic weights. In the present experiments we are dealing with the number of the β particles and not their ionization. The counting chambers were of thin material but either of different thickness of the same material or of different materials. The source of radiation was radium. Because of very marked scattering of γ rays and production of secondary radiation in air and objects about the room, this scattered and secondary radiation was also measured by interposing in the direct path of the γ rays a large amount of lead at places found by experiment to give the optimum effect of this radiation.[1] The count due to both, less the latter, gives the effect of the direct radiation. For the increasing small thickness the count increased to a maximum and then for greater thickness decreased again with the law of absorption for γ rays.

With materials of different atomic weights the count decreased from C but began to increase with Cu and increased for higher atomic weights. The minimum in the curve comes at somewhat smaller atomic weight than that reported by Hackett using the ionization method. In general the rising portion of the curve was not as steep. This may be expected when the number of β particles is counted instead of measuring their ionization since the velocity of the emerging β particles is affected by the material through which they pass and ionization increases with the decrease of the velocity.

SHEFFIELD SCIENTIFIC SCHOOL,
YALE UNIVERSITY.

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[1] Similar to Lawson's method, Akad. Wiss. Wien, 125, 1, 1916.

THE HIGH-FREQUENCY LIMITS OF X-RAY SPECTRA AT DIFFERENT ANGLES FROM THE CATHODE STREAM.

BY DAVID L. WEBSTER.

IN a previous paper[1] evidence was collected from various sources on the distribution of x-rays relative to the cathode stream. It appeared that there were phenomena related to the Doppler effect, in such a way as to indicate: (1) that the general radiation, or continuous spectrum, is emitted by electrons most of which are moving at high speeds while emitting; (2) that the frequency of emission of each electron is determined by the energy it emits, apparently without regard to its surroundings, even though it may move through many atoms during the process of emission. This means that the electron must carry its whole emitting mechanism in itself, as a part of its own structure.

The data were not very definite, and the most doubtful point in the reasoning was a deduction from the experiments, to the effect that the high-frequency limit of the spectrum must be the same in all directions from the tube. This has since been confirmed for voltages up to 10 kilovolts by Wagner,[2] but it is denied for 80 kilovolts by Zecher,[3] who finds a difference of 11 per cent. in the high-frequency limits at angles of 60 and 150 degrees from the cathode rays. I have nearly completed apparatus for the study of certain phases of the distribution problem, and am able now to report preliminary results on this point, for tungsten, but unfortunately only up to 67 kilovolts because of some gas that has appeared in the tube. At this voltage, as well as at lower ones that have been tested, down to 38 kv., *no difference between the high-frequency limits has been found.* The accuracy of measurements to date is about one per cent., and the angles used are 55 and 145 degrees, nearly the same as Zecher's.

I think the explanation of Zecher's result lies partly in the fact that he used a photographic spectrum, in which the limit is always slightly indefinite, and even more in the fact that he used an induction coil to drive his tube. The present work was done by ionization, with a D.C. outfit of the kenotron type whose voltage fluctuation was a very small fraction of one per cent.

I hope soon to improve the accuracy of the measurements and extend their range.

STANFORD UNIVERSITY,
CALIFORNIA.

A GENERALIZATION OF THE ELECTRODYNAMIC EQUATIONS WITH APPLICATIONS TO THE STRUCTURE OF THE ELECTRON AND TO RADIATIONLESS ORBITS.

BY LEIGH PAGE.

A YEAR ago the author published[4] a generalization of the electrodynamic equations which took into account the possibility of rotation of electric fields.

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[1] PHYS. REV., 13, 303, 1919.

[2] P. Z., 21, 621, 1920.

[3] Ann. der Phys., 63, 28, 1920.

[4] Proc. Nat. Acad. Sci., 6, p. 115, 1920.

A further generalization considers the possibility both of rotating electric and rotating magnetic fields. Each element of charge is supposed to be the origin of equal electric and magnetic fields. If the electron is of the Lorentz type, sufficient rotation of the electric fields of each element of charge about the normal to the surface annuls the intrinsic magnetic field outside the electron's surface, and the magnetic field inside the surface is responsible for an inward stress which just counterbalances the outward stress due to electrostatic repulsion, and accounts for the *stable* equilibrium of the electron.

If the fields of the elements of charge on the surface of an accelerated electron have a small additional rotation parallel to the acceleration, the radiation field vanishes, and radiationless orbits are possible. Details are contained in a more extended paper which has been submitted to the PHYSICAL REVIEW for publication.

YALE UNIVERSITY.

THE RESISTANCE AND INDUCTANCE OF A THREE-PHASE CABLE.

BY HARVEY L. CURTIS.

AN integration method of computing alternating current resistance and inductance was presented before the Physical Society[1] at its meeting in April, 1920. This method has now been applied to the case of a three-phase cable of cylindrical wires. The resistance and inductance of one of the wires have been computed when balanced three-phase currents are flowing in the cable. The results show that the resistance is more and the inductance less than in a similar single-phase cable.

In the case of a single-phase cable, the change of resistance with frequency is approximately given by the formula

$$\frac{R}{R_0} = \frac{1 + \frac{\lambda^2}{6} + \frac{\lambda^4}{480} + \cdots}{1 + \frac{\lambda^2}{12} + \frac{\lambda^4}{1440} + \cdots} + \frac{\lambda^2 a^2}{s^2}\,\frac{1 + \frac{\lambda^2}{24} + \frac{\lambda^4}{8640} + \cdots}{1 + \frac{\lambda^2}{2} + \frac{\lambda^4}{96} + \cdots}$$

$$+ \text{ terms in } \frac{a^4}{s^4},\ \frac{a^6}{s^6},\ \text{etc.}$$

Where R_0 = the direct current resistance.
R = the alternating current resistance at the frequency f.
$\lambda = \frac{2\pi^2 f a^2}{\sigma}$
σ = the resistivity of the material of which the cable is made.
a = the radius of one of the wires.
s = the distance between the centers of the two wires of the cable.

The first term of the second member is the change of resistance in one wire when the return is at infinity. The second term gives the effect of the adjacent wire and may be called the proximity term since it depends upon the distance between the wires. The terms a^4/s^4, a^6/s^6, etc., never affect the result by as much as 10 per cent, and hence can be neglected for approximate calculations.

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[1] PHYS. REV., 2d Series, XV., p. 528. A more complete discussion is given in Scientific Paper No. 374 of the Bureau of Standards.

In a three-phase cable, the formula for change of resistance is approximately

$$\frac{R}{R_0} = \frac{1 + \frac{\lambda^2}{6} + \frac{\lambda^4}{480} \cdots}{1 + \frac{\lambda^2}{12} + \frac{\lambda^4}{1440} \cdots} + \frac{3\lambda^2 a^2}{2s^2} \frac{1 + \frac{\lambda^2}{24} + \frac{\lambda^4}{8640} \cdots}{1 + \frac{\lambda^2}{2} + \frac{\lambda^4}{96} \cdots}$$

$$+ \text{ terms in } \frac{a^4}{s^4}, \frac{a^6}{s^6}, \text{ etc.}$$

The symbols have the same meaning as in the preceding case, except that s is now the length of a side of the equilateral traingle which is formed by joining the centers of the three wires of the cable. The first term of the right hand member is the same as in the single-phase cable, but the proximity term is 50 per cent. greater.

The inductance is also very similar for the two cases. For the single-phase cable the approximate formula is

$$L = 2l \log \frac{s}{a} + \frac{l}{2} \frac{1 + \frac{\lambda^2}{24} + \frac{\lambda^4}{4320} + \cdots}{1 + \frac{\lambda^2}{12} + \frac{\lambda^4}{1440} + \cdots}$$

$$- \frac{2l\lambda^2 a^2}{3s^2} \frac{1 + \frac{\lambda^2}{40} + \frac{\lambda^4}{10080} + \cdots}{1 + \frac{\lambda^2}{2} + \frac{\lambda^4}{96} + \cdots}$$

$$- \text{ terms in } \frac{a^4}{s^4}, \frac{a^6}{s^6}, \text{ etc.}$$

where L is the inductance of one wire of the cable, and l is the length of the cable. For the three-phase cable the last or proximity term is 50 per cent. greater than in the above formula.

The above formulas show that the proximity effect is approximately 50 per cent. greater in the three-phase cable than in the single-phase cable. However, this is not exact, since the coefficients of the terms in a^4/s^4, a^6/s^6, etc., have a different numerical ratio than the term in a^2/s^2. For exact computations, the complete formulas must be consulted.

Bureau of Standards,
Washington, D. C.,
April 6, 1921.

Thickness of Stratified Soap Films.

By P. V. Wells.

Newton noticed two black spots in the soap film, each layer of uniform thickness with sharp boundaries. Kelvin showed that such stratification indicated the range of molecular forces. Johonnot noticed at least five different strata, but Perrin, using the microscope with reflected light, as in metallography, observed hundreds of such "plages" increasing in thickness by uniform steps, each plage a multiple of the unit plage in thickness.

Measuring the thickness photometrically and colorimetrically, Perrin's multiple "plage" law is verified, the unit plage being 4×10^{-7} cm. thick, the same as the black spot. The results, considered in relation to the recent work on the spreading of oil films on water, suggest that the elementary layer is a bimolecular film of oleic acid, the molecules being bound together by their active groups, their inactive tails forming the external surfaces of the film. Such a double layer of molecules is of the proper thickness, is stable, and would show many of the phenomena of stratified films. For instance discs of unit thickness are sometimes observed floating about upon a plage with brownian motion in equilibrium, instead of pulling together into a globule as would be expected. A detailed study of the life history of stratified films adduce much evidence supporting this structure.

If the black spot (plage 1) is bimolecular, and plages of greater thickness (plages 2, 3, 4, $\cdots$ n, etc.) consist of superposed strata each two molecules thick, such a theory may be of interest and importance in the problem of lubrication at low speeds and high pressures. Besides the monomolecular film of oleic acid firmly attached to the metal surfaces, there may be strata of bimolecular thickness, forming inactive lamina between the bearing surface, and behaving quite differently from the lubricant in bulk. Recent progress in lubrication renders such concepts plausible.

BUREAU OF STANDARDS.

THE TEMPERATURE OF AN ELECTRICALLY HEATED FILAMENT.

BY CHESTER SNOW.

THE temperature distribution of a fine wire, electrically heated by a current passing along it, and surrounded by a poorly conducting medium, has been studied for the temperature interval 0 to about 600° C. The temperature-coefficient of resistance and the thermal conductivity of the wire may be considered constant over this range of temperatures. The wires were connected to massive terminals which were practically unheated by the current.

It was found by experiments on relatively long wires surrounded by air at various pressures, and by finely powdered material such as flour, that the effect of the surrounding medium upon the wire temperature could be accounted for by assuming the heat flow from wire to medium proportional to the difference in temperature between the wire and the nearest layer of the medium. Such a temperature difference existed, at least in a statistical sense.

Using this fact and the known conductivity of the wire, expressions have been obtained mathematically for the wire temperature at any point. The following cases were considered:

1. Thermal equilibrium when the filament formed part of a circuit in which the current was maintained constant.

2. Thermal equilibrium when the filament formed part of a circuit in which the electromotive force was maintained constant.

3. Constant current. Time rate of heating of the wire and the medium.

In case 1, formulas have been derived by which the lateral heat flow to the medium, and the temperature of the wire and medium, may be computed from observations of wire resistance as a function of the current. The variations of lateral heat loss with diameter of wire were found by experiment to be in accordance with the theory. Convenient formulas have been obtained to correct for end conduction in long or short wires.

In case 2, the current obtained depended upon the temperature of the filament. Graphical methods were devised by which temperature of the middle of the wire was plotted as function of the dimensions or of the constants of the material of the wire. The dimensions for maximum wire temperature with a given electromotive force were derived. The effect of varying the material of the wires, upon the equilibrium temperatures, was also considered. The effect of varying the thermal contact between the wire and medium, upon the equilibrium temperature of each was also studied. This factor was found to be just as important as the electrical and thermal constants of the wire, in determining the final wire temperature produced by a given electromotive force. The sensitiveness of the resistance variation with current in high vacua suggests the possibility of using this device as an electric vacuum gauge.

In case 3, the time rates of rise of wire and medium were considered. It was assumed that the above law of interchange of heat between wire and medium held even when the temperatures were varying. The heat flow in the medium was considered to be practically radial. The equations of heat conduction for the two mutually influencing thermal bodies (wire and medium) led to an integral equation, the kernel of which was obtained as a Fourier-Bessel expansion. It was found that a single term, of exponential type, could be chosen which would represent this kernel with sufficient approximation and which led to a simple solution. By this means it was found that the wire temperature $U(t)$ at any point, at time t, and the corresponding temperature of the adjacent medium $V(t)$ were given by

$$U(t) = \frac{U(\infty)}{b(\alpha_1 - \alpha_2)}\left[\alpha_2(\alpha_1 - b)(1 - e^{-\alpha_1 t}) + \alpha_1(b - \alpha_2)(1 - e^{-\alpha_2 t})\right]$$

and

$$V(t) = \frac{V(\infty)}{\alpha_1 - \alpha_2}\left[\alpha_1(1 - e^{-\alpha_2 t}) - \alpha_2(1 - e^{-\alpha_1 t})\right],$$

where t was the time measured from the instant of closing the circuit and where $U(\alpha)$ and $V(\alpha)$ were the final temperatures of wire and medium, and α_1, α_2, and b were given functions of the thermal and electrical constants of the wire and of the thermal contact of the wire and its surrounding medium. All but two of these constants can be obtained from steady-state measurements. The remaining two can be obtained from oscillograph measurements, of current and voltage, from which the temperature-time curve of the wire may be computed. The formulas indicate that the wire temperature rises with an appreciable initial slope to a limiting value, $U(\infty)$, without passing through any maximum value. The medium rises more slowly with an initial slope of zero, to a final temperature lower shan the wire.

BUREAU OF STANDARDS,
WASHINGTON, D. C.,
April 18, 1921.

STANDARD WAVE-LENGTHS AND CONSTANT FREQUENCY DIFFERENCES IN THE SPECTRA OF INERT GASES.

BY W. F. MEGGERS.

THE spectra of the inert gases (helium, neon, argon, krypton and xenon) are known to consist of sharp lines which represent wave-lengths of a very high degree of homogeneity.[1] In fact, the spectral lines obtained from the heavier gases are the narrowest regions of spectral emission known at present. This makes them susceptible of extreme precision in wave-length comparisons by means of interferometers. Precision, reproducibility and convenience are the properties of these lines which recommend them for use as standards of wave-length, or ultimately as fundamental standards of length. The Bureau of Standards has, from time to time, carefully compared wave-lengths in the spectra of the inert gases with the wave-length of the red radiation from cadmium which is the international primary standard of wave-length. The results from 21 lines in the He spectrum (2945–7281A) and 55 lines in the spectrum of neon (3369–8495A) have already been published.[2] Work has recently been completed on 51 lines (3948 to 8521A) in the argon spectrum, 18 lines in krypton (4723–7601A), and 12 lines in the spectrum of xenon (4500–4923A). All these wave-length determinations were made by the interferometer method devised by Fabry and Perot,[3] which consists in accurately obtaining, from circular interference fringes, the number of standard and approximately known wave-lengths contained in a fixed distance between two parallel interferometer plates. The probable error in the final wave-lengths is about one or two thousandths of an angstrom unit. In other words, the wave-lengths are correct to one part in several millions.

Frequency differences (*i.e.*, differences in the number of waves per cm. vacuum) are very interesting in the light of these accurate measurements. In the case of neon it was shown that 37 of the standard lines were associated in quadruplets having frequency differences[4] which were constant within the accuracy of the wave-length measurements, and leading to the following frequency relations between members of quadruplets:

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[1] Fabry and Buisson, J. de Phys., (5), 2, p. 442, 1912.

[2] B. S. Bulletin 14, p. 159, 1917, and 14, p. 765, 1918.

[3] Fabry and Perot, Ann. d. Chim., 25, p. 110, 1902.

[4] These regularities in the neon spectrum have recently been arranged in spectral series by Meissner (Ann. d. Phys., 58, p. 333, 1919), and more completely by Paschen (Ann. d. Phys., 60, p. 405, 1919).

$$\nu_2 = \nu_1 + 1070.077,$$
$$\nu_3 = \nu_2 + \ \ 359.956,$$
$$\nu_4 = \nu_3 + \ \ 417.449.$$

Similar quadruplets[1] in which 28 of the standard argon wave-lengths find places show that the frequency differences are again constant within 1 part in several millions and the following frequency relations are derived:

$$\nu_2 = \nu_1 + 846.162,$$
$$\nu_3 = \nu_2 + 803.075,$$
$$\nu_4 = \nu_3 + 606.837.$$

Eight of the standard krypton lines are found to constitute pairs whose mean frequency difference is 945.028, and no value deviates from the mean by more than one part in 9 millions.

Fainter lines in the xenon spectrum must be measured with the interferometer before the constancy of frequency differences[2] in its spectrum can be tested.

Bureau of Standards,
April 18, 1921.

Note on the Diurnal Variation of the Atmospheric-Electric Potential-Gradient.

By S. J. Mauchly.

Since the summer of 1915 observations have been made aboard the *Carnegie* to determine the nature of the diurnal variation of the atmospheric-electric elements over the oceans. For the potential-gradient the general procedure is to make a set of 20 observations during each of 24 consecutive hours. The observations for such a set require about 20 minutes and their mean value is referred to the mean time for the set. From reductions based on the observations made prior to April, 1916, it appeared that the diurnal variation of the potential-gradient over the oceans probably did not differ much from that which has been found at many land stations; that is, they indicated two rather pronounced maxima and two minima during a 24-hour period. However, very few data were available from oceans other than the Pacific, and a large percentage was derived from series of observations which were terminated by the advent of unfavorable weather.

The largely increased amount of material which has accumulated now makes it possible to reject all data corresponding to less than a 24-hour run and still have 45 complete 24-hour runs available. The data for each run, therefore, corresponds to an actually occurring sequence of phenomena, and the mean results are free from the errors which would result from combining the results of partial series of observations.

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[1] Rydberg, Astroph. J., 6, p. 345, 1897. Paulson, Physik. Z., 15, p. 831, 1914. Meissner, Physik. Z., 17, p. 551, 1916.

[2] Paulson, Astroph. J., 40, p. 298, 1914.

Of the 45 diurnal-variation runs referred to, 30 were made in the Pacific, 5 in the Atlantic, and 10 in the Indian Ocean. The means corresponding to the separate oceans, as derived from 39 runs, are represented in Fig. 1. They show: (1) That the mean diurnal-variation curves for the Pacific, Atlantic and Indian oceans are similar in form; (2) that the principal component of the variation consists of a 24-hour "wave"; and (3) that the phase of this wave differs for the several oceans by amounts which correspond approximately to the differences between the mean longitudes of the points at which the observations were made.

Since the curves of Fig. 1 suggest the simultaneous occurrence of maximum phase over all three oceans, it was decided to refer the results of each run to Greenwich Mean Time (civil), and re-compute the means for the separate oceans on this basis. The results are shown in Fig. 2, together with a curve which includes the data from 6 new runs. The differences between the several curves of Fig. 2 are of course not to be thought of as representative of separate characteristics, since the smoothness of the respective curves is seen to be directly related to the number of component runs.

The curves of Fig. 2 show a decided similarity to land results for high latitudes and also to many of the winter curves obtained in ordinary latitudes. If differences in local mean time are taken into account it appears that, in addition to their similarity of form, many of these land curves are *simultaneously in the same phase as the general ocean curve* of Fig. 2. For the summer, however, as is well known, most land stations show, in addition to the above 24-hour wave, a decided secondary wave which seems to occur in general at about the same local mean time at different stations.

The results of the 45 complete runs representing a general distribution over the accessible oceans indicate, therefore, as a preliminary result, that *the chief component of the diurnal variation of the potential-gradient over the major portion of the Earth (especially the oceans) is a "wave" of 24-hour period which occurs simultaneously in all localities.* While the preliminary investigations show that for the oceans this general statement holds for all latitudes and seasons, there are evidences of secondary variations which are being investigated, and whose nature will become more certain as the results of additional observations become available.

DEPARTMENT OF TERRESTRIAL MAGNETISM,
WASHINGTON, D. C., April, 1921.

PECULIAR ICE FORMATIONS.

BY HERBERT GROVE DORSEY.

THE first of these formations of ice occurred on the night of December 15, 1916. The night was clear and calm and some water had been left in a pan placed on a chair on a small uncovered porch. The temperature the following morning was 22° Fahrenheit.

The pan was an ordinary tin coated iron pan, 17 inches in diameter at the top, 12 inches at the bottom and six inches deep. About one inch of unboiled hydrant water was in the pan, and when it froze a thin irregular column of ice was formed near the side of the pan next to the back of the chair.

Fig. 1 shows a front view of the formation, Fig. 2 a side view of it and Fig. 3 is a larger photograph to show the details. The photographic dark slide was placed back of it to give contrast. The steel scale was in the plane of the edge of the ice column and melted the ice so that it was resting on the bottom. It appears nearer than the column because the ice is melted in front of the scale.

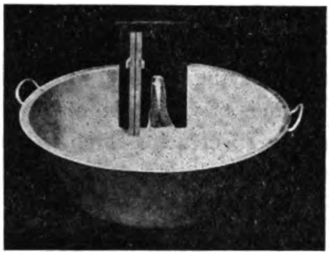

Fig. 1.

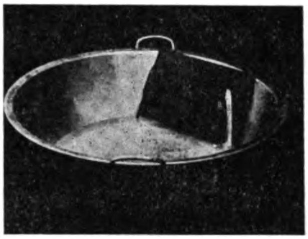

Fig. 2.

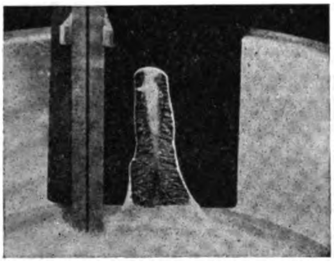

Fig. 3.

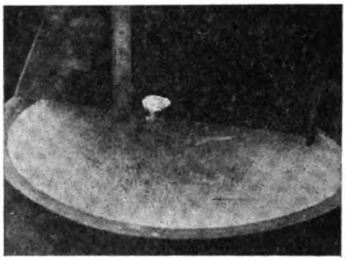

Fig. 4.

The maximum height of the ice column was four and one half inches and the breadth at the bottom was three inches, narrowing irregularly to one and three eighths inches near the middle and to one inch at the top. The average thickness was about one half inch, and instead of being vertical it leaned towards the center of the pan. It will be noticed that while the air bubbles in the pan were in vertical lines, they arose near the middle of the column and then separated out laterally into almost horizontal lines. Near the top the air bubbles were grouped together so closely that it gave the appearance of white ice or snow; the rest of the ice in the pan and in the formation was transparent except for the bubbles.

Water had been frozen in this same pan several nights previously and after the unusual formation occurred, it was tried under nearly similar conditions many times that winter and the following winter, but the only approach to anything so unusual occurred four nights later in a small wooden chopping bowl about ten inches in diameter. As shown in Fig. 4 the formation in this case was almost egg-shaped, the maximum length of the egg being seven eighths of an inch. The bowl was cracked when this occurred.

GLOUCESTER, MASS.,
April 1, 1921.

ON THE EXPLODED-WIRE SPECTRUM OF CALCIUM.

BY R. A. SAWYER AND A. L. BECKER.

THE spectra of exploded wires of calcium and several other elements have been photographed.

By expending constant power upon wires of varying mass, or varying power upon wires of constant mass, a series of explosions of varying temperature has been obtained. It has been possible to obtain spectra in which the relative intensities of the three Ca lines, H, K and λ 4227 vary as they vary in the spectra of the different classes of stars.

These three Ca lines also appear more prominently in the spectra of exploded wires of other metals than the spectra of the metals themselves. The amount of calcium present must be very minute. It is therefore probable that the calcium lines particularly H and K are tremendously enhanced by the very high temperature of the explosion. The intense Ca lines of the solar flash spectra have seemingly required considerable quantities of the heavier Ca to be above the hydrogen layers of the solar atmosphere. The present results explain this anomaly as primarily a temperature effect.

UNIVERSITY OF MICHIGAN.

ON THE RADIATION OF ENERGY FROM COILS.

BY ARTHUR GORDON WEBSTER.

INSTEAD of the usual approximations, exact formulæ are developed for the radiation from a coil in which the density of current is constant, both for rectangular and for circular coils. This paper will be published in the Proceedings of the National Academy of Sciences.

CLARK UNIVERSITY.

ON THE VIBRATIONS OF GUN-BARRELS.

BY ARTHUR GORDON WEBSTER.

THE theory of the propagation of the couple that is developed by the kick of the gun is considered, and an estimate of the time taken to arrive at the end of the barrel is obtained. The method of finding the error of sighting due to the elastic wave is indicated. This paper will be published in the Proceedings of the National Academy of Sciences.

CLARK UNIVERSITY.

Second Series *September, 1921* *Vol. XVIII, No. 3*

THE

PHYSICAL REVIEW.

THE DETECTING EFFICIENCY OF THE RESISTANCE-CAPACITY COUPLED ELECTRON TUBE AMPLIFIER.

BY E. O. HULBURT.

SYNOPSIS.

Detecting efficiency of the resistance-capacity coupled, two tube amplifier.—A theoretical formula is derived and compared with the results of *measurements* made by using a condenser potential divider to vary the high frequency input voltage A and a sensitive galvanometer to measure the rectified high frequency component of the output plate current b_0, the detecting efficiency being defined as b_0/A^2. Satisfactory agreement was found for the *various frequencies and coupling capacities* tried. The *effect of a third tube* was determined experimentally. For convenience of *comparison with transformer-coupled amplifiers*, curves are given showing the detecting efficiencies of the various amplifiers as a function of the frequency and hence their relative merits in detecting unmodulated signals.

Current amplification of the above amplifier as a function of the frequency was also determined.

1. INTRODUCTORY.

THE detecting efficiency of the electron tube amplifier means, in general words, the efficiency of the amplifier to make weak signals intelligible. From general considerations it can be seen that the detecting efficiency depends upon the relation between the input grid voltage change, and the resulting change in the output plate current. This has already been discussed in two papers,[1] one of which dealt with experimental measurements of the detecting efficiency of a three tube high-frequency transformer-coupled amplifier, and the other with the detecting efficiency of the single electron tube; this last considered the question from both a theoretical and an experimental standpoint. In the case of the high-frequency transformer-coupled amplifier the detecting efficiency was defined by the relation $\lim_{A \doteq 0} b_0/A^2$ where A and b_0 are the amplitudes, respectively, of the input grid potential and of the rectified component of the output plate current. In the present paper is described an investigation, both theoretical and experimental, of the detecting efficiency of the high frequency resistance-capacity coupled amplifier. A theoretical analysis, in which certain simplifying assumptions were made, has been carried out, which showed how the detecting efficiency depended upon the constants of the coupling circuits. Experiments were then performed to obtain data on this type of amplifier, and to test the theoretical relations.

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[1] PHYS. REV., 16, 274, 1920; 16, 408, 1920.

2. THEORETICAL.

Consider the high frequency amplifier of two tubes with resistance-capacity coupling as shown in Fig. 1. This amplifier may be used to receive modulated radio-frequency signals. There are other types of resistance-capacity coupling than the one shown in Fig. 1. For example, if separate plate batteries are used for the two tubes, the capacity C_4 and resistance r_3 may be omitted. Such an amplifier will, in general, possess different characteristics from those of the amplifier of Fig. 1. However, the connections shown in Fig. 1 are commonly used and it is this type of connection which was chosen for this investigation.

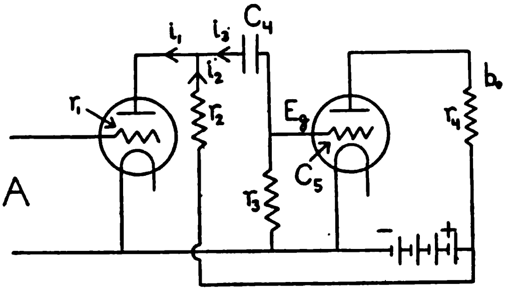

Fig. 1.

Let A be the amplitude of the radio frequency voltage impressed on the grid of the first tube, and let b_0 be the rectified component of the resulting radio frequency current in the plate circuit of the second tube. We now consider the action of the amplifier in the following way. Due to A, a high frequency component of the plate current of the first tube is produced. Call this i_1. This current divides into i_2 and i_3, as shown in Fig. 1. i_3 impresses a high frequency potential oscillation E_g on the grid of the second tube, which in turn causes the rectified high frequency component b_0 of the plate current of the second tube. Amplification is considered to take place in both tubes, but rectification to occur only in the second tube. If rectification occurs in the first tube, this may result in a shift of the operating point of the grid of the second tube, and hence a change in the characteristics of the amplifier. Experiment has shown, however, that for small input voltages the rectifying effect of the first tube is small, and so in the present treatment it is justifiable to ignore this effect.

We proceed to derive the relation between A and b_0 in terms of the constants of the circuits and of the tubes. Let r_1 be the internal resistance between the filament and the plate of the first tube. Let C_5 be the filament-grid capacity of the second tube. Resistances r_2 and r_3, and capacity C_4 are as shown in Fig. 1. Let $\omega/2\pi$ be the frequency of the impressed voltage.

Let

$$\begin{aligned} \frac{1}{r_1} &= g_1, & \omega C_4 &= x_4, \\ \frac{1}{r_2} &= g_2, & \omega C_5 &= x_5, \\ \frac{1}{r_3} &= g_3. \end{aligned} \tag{1}$$

Consider the first tube. If the grid-plate capacity of this tube is neglected, it has been shown[1] that the relation between the impressed grid voltage A and the resulting high-frequency component i_1 of the plate current is given by the relation

$$i_1 = \frac{kA}{\frac{1}{g_1} + Z_p}, \tag{2}$$

k is the amplification constant of the tube, and Z_p the impedance of the plate circuit external to the tube. The amplification constant k and the internal resistance $1/g_1$ of tube are real quantities, and not complex, in the case of zero grid-filament capacity.

Considering the circuit shown in Fig. 1, the value of Z_p is found to be

$$Z_p = \frac{1}{g_2 + \frac{1}{\frac{1}{jx_4} + \frac{1}{g_3 + jx_5}}}, \tag{3}$$

where $j = \sqrt{-1}$. Substituting (3) in (2) gives

$$i_1 = kAg_1 \frac{g_2g_3 - x_4x_5 + j[x_4(g_2 + g_3) + x_5g_2]}{g_2g_3 - x_4x_5 + g_1g_3 + j[x_4(g_1 + g_2 + g_3) + x_5(g_1 + g_2)]}.$$

The modulus of this is

$$i_1 = kAg_1 \sqrt{\frac{(g_2g_3 - x_4x_5)^2 + [x_4(g_2 + g_3) + x_5g_2]^2}{(g_2g_3 - x_4x_5 + g_1g_3)^2 + [x_4(g_1 + g_2 + g_3) + x_5(g_1 + g_2)]^2}}, \tag{4}$$

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[1] G. Breit, Phys. Rev., 16, 388, 1920. Others have given similar formulas.

where i_1 now stands for the root-mean square value of the high frequency plate current of the first tube. It should be noticed that in the above treatment conductance through the coupling condenser C_4 has been neglected. The measured direct current value of this resistance was over 50 megohms. Terms expressing this condenser leakage were introduced into formula (4), but were found to be negligible.

Let us turn our attention to what takes place in the coupling circuit and find the relation between i_1 and E_g, the high frequency voltage impressed on the grid of the second tube. From Kirchhoff's laws

$$i_1 = i_2 + i_3,$$

$$\frac{i_2}{g_2} = i_3\left(\frac{1}{jx_4} + \frac{1}{g_3 + jx_5}\right),$$

$$E_g = \frac{i_3}{g_3 + jx_5}.$$

It is assumed that the grid-filament conduction current of the second tube is zero. Then

$$E_g = \frac{i_1}{(g_3 + jx_5)\left\{1 + g_2\left(\frac{1}{jx_4} + \frac{1}{g_3 + jx_5}\right)\right\}}.$$

The modulus of this is

$$E_g = \frac{i_1}{\left\{\left(g_2 + g_3 + \frac{x_5}{x_4}g_2\right)^2 + \left(x_5 - \frac{g_2g_3}{x_4}\right)^2\right\}^{1/2}}. \tag{5}$$

In this expression E_g stands for the root-mean-square value of the grid voltage of the second tube.

Consider now the second tube, in which amplification and rectification both occur. It has been shown experimentally in the previous papers[1] that the relation

$$\frac{b_0}{E_g^2} = n \tag{6}$$

is approximately true, where n is a quantity which is independent of the magnitude of E_g. It is assumed that in the present case n did not depend upon the frequency of the impressed voltage, because the impedance of the plate circuit of the second tube was independent of the frequency. This assumption was subsequently justified by experiment.

Introducing (4) and (5) into (6) we have

$$\frac{b_0}{A^2} = nk^2g_1^2 \frac{\dfrac{(g_2g_3 - x_4x_5)^2 + [x_4(g_2 + g_3) + x_5g_2]^2}{(g_2g_3 - x_4x_5 + g_1g_3)^2 + [x_4(g_1 + g_2 + g_3) + x_5(g_1 + g_2)]^2}}{\left(g_2 + g_3 + \frac{x_5}{x_4}g_2\right)^2 + \left(x_5 - \frac{g_2g_3}{x_4}\right)^2}. \tag{7}$$

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[1] *Loc. cit.*

This expression gives the desired relation between the detecting efficiency b_0/A^2 and the constants of the tubes and of the circuits. In the former papers[1] the detecting efficiency has been defined by the relation $\lim_{A\to 0} b_0/A^2$. Although this is theoretically the proper definition (because it avoids any possibility of rectification in the first tube), we have been content in the present instance to measure the value of b_0/A^2 for finite values of A, and to call this the detecting efficiency.

3. Apparatus.

The apparatus consisted of a condenser potential divider, the amplifier, and a D'Arsonval galvanometer connected in the plate circuit of the last tube. These will be described in the order named. The arrangement is shown schematically in Fig. 2. The condenser potential divider consisting of the three condensers c_1, c_2, c_3, the inductance coil L, and the thermogalvanometer T, were the same as described in a previous paper.[2] By coupling L to a suitable electron tube generating set, unmodulated high frequency voltage of small known amplitude and frequency was impressed on the grid of the first electron tube. The high resistance leak r_0 (1.2×10^6 ohms) was connected across c_2 to ensure a definite value of the grid potential during the experiment. The effect of r_0 upon the impedance of c_2 was negligible because c_2 was large (about 0.05 μFδ) and the frequencies used were of the order of 3×10^5.

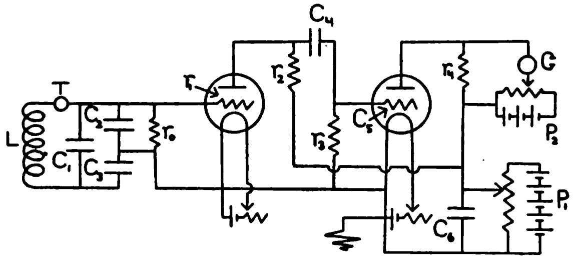

Fig. 2.

The amplifier was a two-tube one with resistance-capacity coupling. The tubes were Western Electric Company tubes, Type VTI; they were used with the filament current always 1.10 amperes and the plate voltage always 23.7 volts. Separate storage cells supplied each filament; the plate voltage supply was common to all the tubes. The plate battery was shunted by a 2 μF condenser c_6. The resistance r_2 was 27.6×10^3 ohms and r_3 was 393×10^3 ohms. The resistances r_0, r_2 and r_3, were non-inductive. A resistance unit was made by painting both sides of a strip of cardboard about 4 mm. wide with black drawing ink. Two sturdy brass clips were pinched on to each end of the strip, so that the length of the strip between the jaws was about 0.5 mm. This was sealed in a small glass tube with sealing wax, the connecting wires being soldered to the brass clips and brought out through the wax seals. Such a resistance when well made, was found to change in value by not more than ten per cent. in two months. The resistance of 30,000 ohm units was found to be appreciably constant for direct currents varying from 10^{-4} to 10^{-6} amperes. The currents which they carried in the amplifier never exceeded these values. The high frequency resistance of a unit was assumed to be the same as the resistance found by a direct current measurement.

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[1] *Loc. cit.*

[2] *Loc. cit.*

The change in the value of the rectified high frequency component of the plate current of the second tube of the amplifier (which we have designated by b_0) was measured by a D'Arsonval galvanometer, G, Fig. 2, connected across a resistance r_4, placed in the plate circuit. r_4 was 60,000 ohms. The galvanometer had a resistance of 16 ohms and a sensibility of 1.7×10^{-9} amperes per millimeter deflection at a scale distance of 120 cm. P_1 and P_2, Fig. 2, were potential dividers, P_1 serving to keep the plate voltage at a standard value, and P_2 to compensate for the potential drop in the resistance r_4, so that the galvanometer rested approximately at zero. When the input grid voltage was changed, a deflection of the galvanometer resulted which was proportional to the change in the rectified high frequency component of the output plate current.

It was important that the filament voltage of the last tube and the voltages of P_1 and P_2 be constant. When a slow drift of the galvanometer occurred, the error was eliminated by averaging deflections. To insure that high frequency currents were not causing unknown disturbances in the galvanometer the following test was made. The deflection was observed with a certain voltage impressed on the amplifier. A 2 μF condenser was then connected across the galvanometer terminals, and it was found that the deflection remained unchanged. This showed that the high frequency currents did not appreciably affect the galvanometer.

4. THE TWO-TUBE AMPLIFIER.

(*a*) *The Variation of the Detecting efficiency with the Coupling Capacity.*—With the arrangement of apparatus as shown in Fig. 2, the resistances r_2 and r_3 were varied until the value of b_0 for a specified A at a frequency of 3 × 10^5 was approximately a maximum. This adjustment was not critical, and the values of r_2 and r_3 finally chosen were 27.6 × 10^3 and 393 × 10^3 ohms, respectively. After this was done the values of b^0 were observed for a series of values of A and for a series of values of the coupling condenser C_4. The values of b_0, shown in the family of curves in Fig. 3, have been plotted as ordinates against the values of A^2 as abscissas. The frequency used was 3 × 10^5, corresponding to a wave-length of 1,000 m. It is seen that b_0 is approximately proportional to A^2, which is a first check on the correctness of formula (7). The fact that b_0 was approximately proportional to A^2 meant that the action

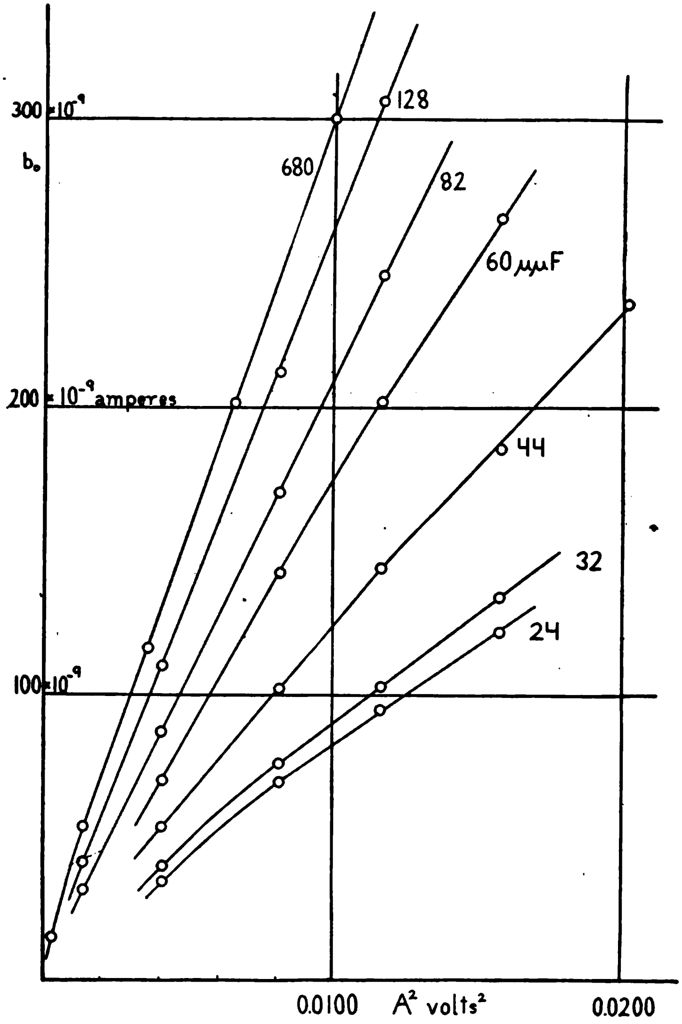

Fig. 3.

of the last electron tube was represented approximately by the expansion of the plate-current grid-voltage relation, by Taylor's theorem, in which derivatives of higher order than the second were neglected. The slight bends in the lines of Fig. 3 may perhaps be ascribed to rectification occurring in the first tube, as discussed in section 2.

To see further to what extent the assumptions underlying (7) were in agreement with experiment, the values of b_0/A^2 for each value of C_4 in Fig. 3, for $A^2 = 0.010$ volts², have been plotted as ordinates against C_4 as abscissas in the full line curve of Fig. 4. In all cases the ratio b_0/A^2 has been expressed in amperes divided by volts². The numerical values of the circuit constants were as follows: $r_1 = 65 \times 10^3$ ohms, $r_2 = 27.6 \times 10^3$ ohms, $r_3 = 393 \times 10^3$ ohms, $\omega = 6\pi \times 10^5$, $C_4 = 680\ \mu\mu\text{F}$, and $C_5 = 38\ \mu\mu\text{F}$. The value for r_1 had been previously determined for this electron tube by measurements of its direct current characteristics for the filament and plate voltages used in these experiments. The value of C_5, *i.e.*, 38 $\mu\mu$F, was found by a measurement of the grid-to-filament capacity *in situ*, and included the capacity of the tube socket and connecting wires. Introducing the above values of the quantities into equation (7), and using the observed value of b_0/A^2, *i.e.*, 30×10^{-6}, for $C_4 = 680\ \mu\mu\text{F}$, the values of b_0/A^2 were computed for a series of values of C_4 from 680 to 10 $\mu\mu$F. The theoretical curve is shown in the dotted line of Fig. 4. The two curves agree closely for values of C_4 from 680 $\mu\mu$F to 200 $\mu\mu$F. For C_4 below 200 $\mu\mu$F the observed values of the detecting efficiency are seen to be less than the theoretical values.

If the more exact formula for the detecting efficiency had been used, *i.e.*, $\lim_{A \doteq 0} b_0/A^2$, instead of the formula b_0/A^2 where $A^2 = 0.010$ volts², the agreement between the observed and the theoretical detecting efficiencies would perhaps have been better. The data of Fig. 3 indicate this. At all events the agreement is sufficiently good to show that the simple theory embodied in formula (7) is an approximate statement of the facts.

(*b*) *The Variation of the Detecting Efficiency with the Frequency.*—The detecting efficiency of the amplifier was measured for a number of frequencies of the input voltage A. The coupling condenser C_4 was kept at its largest value, 680 $\mu\mu$F, throughout this series of measurements. The other capacities and resistances of the amplifier were the same as before. The values of A^2 were between 0.005 and 0.012 volts². The values of b_0/A^2, computed from the observed values of b_0 and A^2, have been plotted against wave-length in Fig. 5 for a range of wave-lengths from 400 to 1,600 meters. The points are shown as small circles in the figure; a smooth line, curve 1, has been drawn through them. The theoretical curve, computed from formula (7), is shown in the dotted curve 2, Fig. 5. This curve has been made to agree with the observed one at the wave-length 1,000 meters. The agreement between the theoretical and observed curves is good, and shows that formula (7) represents the behavior of the amplifier with fair accuracy.

(*c*) *Amplification.*—To determine the amplification due to the use of the first tube the detecting efficiency of the second tube alone was measured. This was done by disconnecting the first tube entirely and impressing the input voltage directly on the grid of the second tube. The values thus found for each wave-length are plotted in Curve 3, Fig. 5. It is seen that the detecting efficiency of the second tube in this case was appreciably constant with wave-length. This bears out the assumption underlying formula (6). By dividing the ordinates of curve 1 by those of curve 2, Fig. 5, for the same wave-length, the amplification of the rectified high frequency component b_0 of the plate current of the second tube due to the use of the first tube with the resistance-capacity coupling was determined. Since the detecting efficiency of the second tube, when alone, was 10×10^{-6}, the curve 1, Fig. 5, also shows the amplification. The marginal numbers on the right of Fig. 5 refer to the amplification. These numbers give the *current* amplification. The

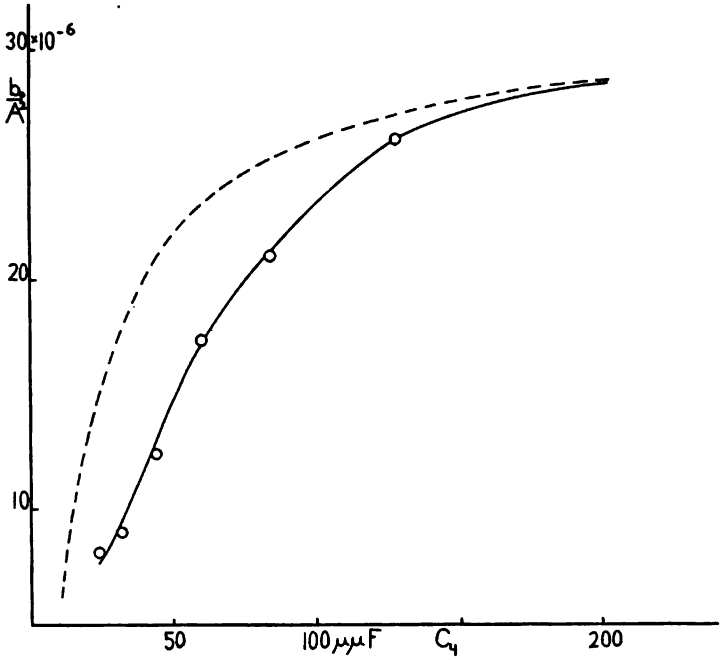

Fig. 4.

power amplification or, if telephones are used, the *sound intensity* amplification is proportional to the square of these numbers.

5. COMPARISON WITH OTHER AMPLIFIERS.

A third tube was coupled to the two-step amplifier by means of resistance-capacity coupling. The plate resistance of this coupling was 19.4×10^3 ohms, the grid resistance was 134×10^3 ohms, and the coupling capacity was 680 $\mu\mu$ F. These correspond to r_2, r_3 and C_4 of Fig. 2, respectively. The detecting efficiency of this three-tube amplifier was measured for the wave-length range from 400 to 1600 meters. The curve is shown in curve 2, Fig. 6. Curves 1 and 3, Fig. 6, are reproductions of curves 1 and 3, Fig. 5, and refer to the two and one tube amplifiers respectively; they are placed in Fig. 6 for the sake of comparison. It is seen that for wave-lengths below 500 meters the addition of one and two tubes with resistance capacity coupling to the single detector tube produced no increase in amplification.

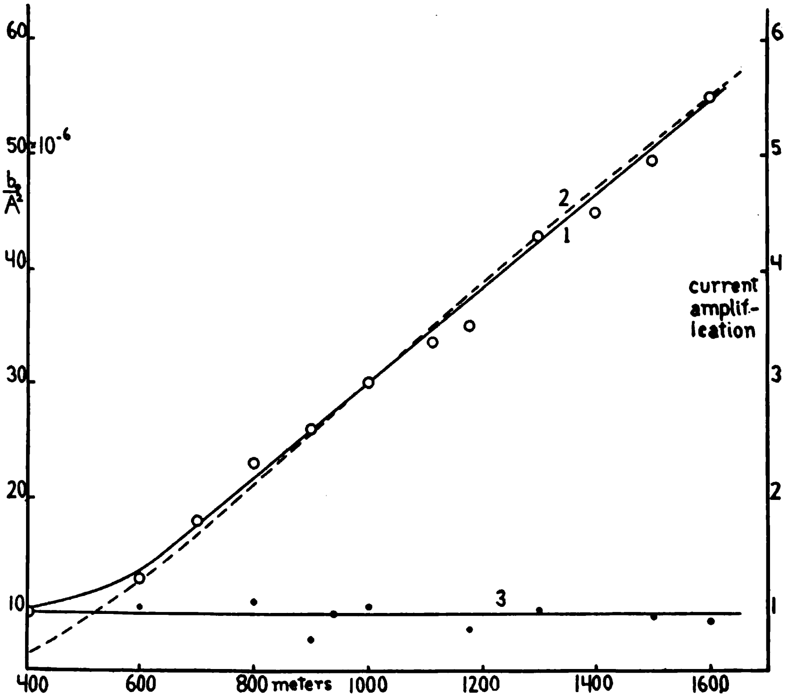

Fig. 5.

To show the comparison with a transformer-coupled amplifier data from a former paper[1] have been used and have been plotted in curves

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[1] PHYS. REV., 16, 274, 1920. It should be noted that in Fig. 3 of this paper the number 15 should be 1.5.

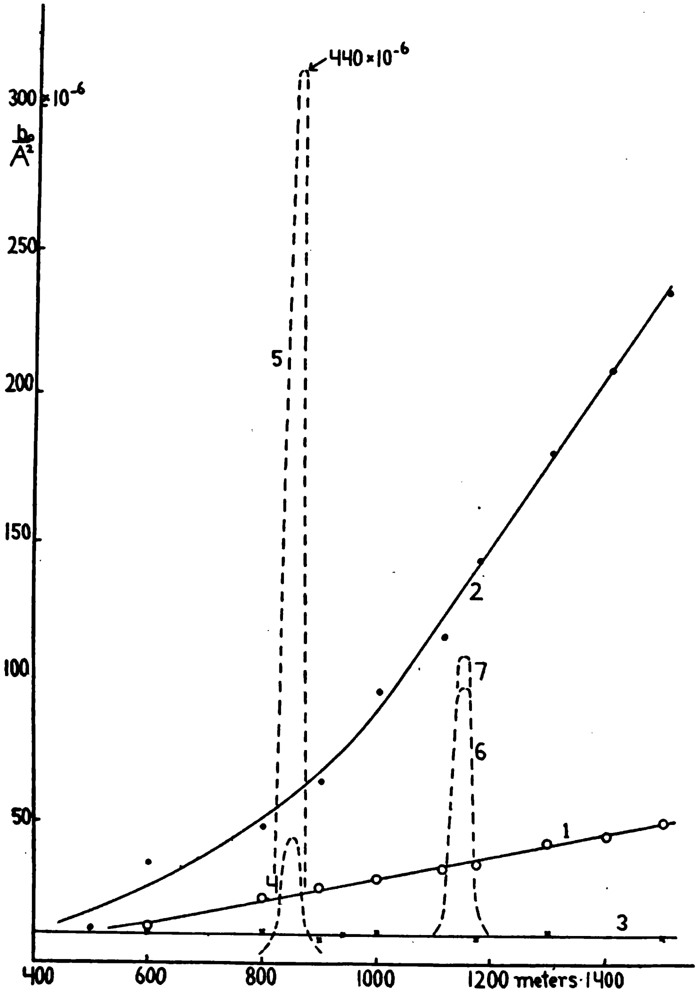

Fig. 6.

4 and 5, Fig. 6. These curves refer to a two-tube and a three-tube amplifier, respectively. The two-tube amplifier consisted of two tubes connected by a high frequency air (wood) core transformer tuned to a wave-length of 850 meters. The transformer was made of No. 36 silk-covered copper wire wound on paraffined wooden spools 3 cm. in diameter, 200 turns on the primary and 250 turns on the secondary. The detector tube of this amplifier was a different one from that of the two-tube resistance-capacity amplifier of curve 1, b_0/A^2 being 4.4×10^{-6}, whereas in the resistance capacity case b_0/A^2 was 10×10^{-6}; so that the curves 1 and 4 are merely an over-all comparison of the two ampli-

fiers. The three-tube amplifier of curve 5 was made by coupling a third tube to the two-tube amplifier by a similar transformer.

A more direct comparison of the performance of resistance-capacity coupling and transformer coupling was carried out in the following way. By means of two double-pole double-throw switches it was arranged so that either the resistance-capacity coupling or the transformer coupling could be used in turn. Fig. 7 shows the connections. When the resistance-capacity coupling was used the amplifier was the same as that used above for curve 1, Fig. 6. The transformer coupling consisted of a tuned air core transformer. This was made of No. 36 silk-covered copper wire wound on paraffined wooden spools 3 cm. in diameter, 225 turns on the primary and 275 turns on the secondary. The primary was tuned by a small air condenser, c_7, Fig. 7, of capacity 37 $\mu\mu$F, and the secondary by another condenser c_8, of capacity 32 $\mu\mu$F. The transformer was tuned to a wave-length 1,150 meters. The detecting efficiency of this amplifier for various wave-lengths is shown by curve 6, Fig. 6. In order to make this comparison the procedure was as follows: With no impressed voltage A on the grid of the first tube the potential divider P_2, Fig. 2, was adjusted so that the galvanometer rested approximately at zero. Then A was turned on and the deflection of the galvanometer noted. The transformer coupling was then thrown in by means of the double-throw switches. It was found necessary to readjust P_2 by as much as perhaps 0.05 volts, and when A was turned on, the deflection was in the *opposite* direction to what it had been before in the case of the resistance capacity coupling. This meant that the grid potential of the second tube was not the same for the two cases, and hence that the detector tube was operating upon two different points on the grid-plate characteristic curve, one point being on a concave portion of the curve and the other on a convex portion. As a result curves 6 and 1, Fig. 6, do not necessarily show the true comparison between the two methods of coupling. To obtain a better comparison a voltage was inserted at point a, Fig. 7, with the negative terminal connected to the

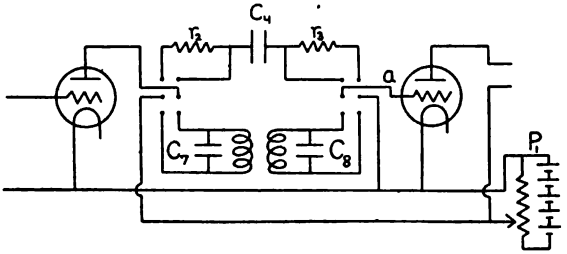

Fig. 7.

grid. Using the resistance-capacity coupling the voltage at *a* was adjusted until the galvanometer deflection, due to *A*, had the same absolute value as before, but in the opposite direction. The voltage at *a* was 1.10 volts. The deflection was now in the same direction as for the transformer coupling. Further, it was found that only a small readjustment of P_2 was required in changing from one coupling to the other. This meant that the detector was being used at approximately the same operating point in the two cases. The values of the detecting efficiency of the transformer-coupled amplifier were now found to be about ten per cent. greater than the values given in curve 6, Fig. 6, and are shown in curve 7, Fig. 6. Curves 7 and 1 exhibit the comparison between the two methods of coupling under approximately similar conditions. It should be noticed that Fig. 6 portrays in a clear manner the relative merits of a number of amplifiers as far as their detecting efficiency for an unmodulated voltage is concerned.

In conclusion the author finds pleasure in expressing his thanks to Mr. W. G. Brombacher for valuable assistance in carrying out the experimental arrangements.

Johns Hopkins University,
January, 1921.

AN ACOUSTICAL GALVANOMETER FOR THE MEASUREMENT OF SMALL ALTERNATING CURRENTS.

BY F. R. WATSON AND L. B. HAM.

SYNOPSIS.

Acoustic Alternating Current Galvanometer.—(1) *Description.* By sending the current through a telephone receiver, magnifying the emitted sound by means of a double resonator and measuring the intensity of the sound by means of a Rayleigh disk hung in the tube connecting the resonating chambers, a measure of the alternating current is obtained in terms of the rotation of the disk. (2) *Variation of the sensitivity with frequency* was investigated with the help of a hot wire ammeter. For the particular instrument studied maxima were found at frequencies of the resonator and telephone diaphragm. The sensitivity for 510 was about 5.7×10^{-8} amperes. Probably a much more sensitive instrument could be made by using sharper resonance and a more delicate quartz suspension.

Measurement of intensity of pure telephone sounds could be effected by means of the above instrument if the frequency used is known as well as the corresponding sensitivity.

DESIGN OF INSTRUMENT.

THE design of an acoustical galvanometer was suggested by an earlier investigation where a Rayleigh disc double resonator[1] was used to measure the faint sounds transmitted by plaster partitions.[2] It seemed likely that a response could be obtained if a telephone receiver screwed on the open end of the resonator were used as the source of sound. Fig. 1 pictures the mechanical details of the instrument. *A* and

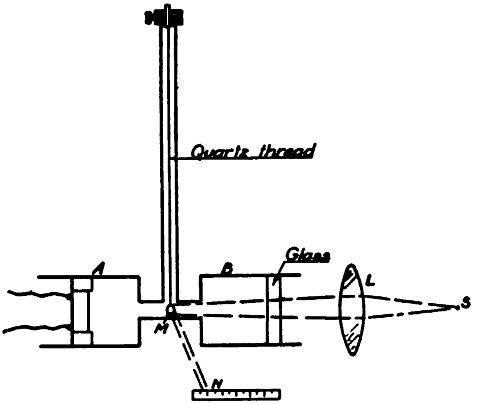

Fig. 1.
Acoustical Galvanometer.

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[1] Lord Rayleigh, "Theory of Sound," Vol. 2, Sec. 253b.

[2] Watson, PHYS. REV., Vol. 15, p. 231, 1920.

B are two chambers of a double resonator connected by a tube. A telephone receiver is screwed tightly to the outer end of chamber A and, by the movement of its plate when an electric current passes, sets up sound vibrations that are amplified by the resonator.[1] Proper proportioning of the parts gives maxima of response and furnishes a means of measuring the alternating currents that actuate the telephone.

Action of the Galvanometer.

The presence of the sound waves is made visible by means of a Rayleigh disc M. This is a thin, plane, circular glass plate which is suspended by a quartz thread in the axis of the connecting tube. Theory shows that a loop of the stationary waves may exist in the connecting tube so that if the disc is placed at an angle to the axis of the tube it will tend to turn flatwise to the alternating currents of air.[2]

Any rotation of the disc is made visible in a quantitative way by the deflection of a beam of light incident on the disc. The beam starts from the lamp S, passes through the lens L, the plate glass B and finally, after reflection from the disc M, to the scale N. These deflections furnish a measure of the alternating currents that actuate the telephone.

Theory.

The frequency of the resonator may be calculated approximately from the theory.[3] The formula for the frequency, N, becomes

$$N = \frac{a}{2\pi}\sqrt{c\left(\frac{1}{S}+\frac{1}{S'}\right)},$$

where a is the velocity of sound, S, the volume of chamber B, S' the volume of the loud speaking telephone, and C, the conductivity, which is equal in this case to the area of cross section of the connecting tube divided by its length. The presence of the Rayleigh disc interferes with the free flow of air and virtually reduces the cross-section of the neck.

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[1] The dimensions of the resonator are given as follows: The chamber B was a brass cylinder 5.9 cm. internal diameter and 2.9 cm. long, closed by a plate glass to allow passage of light. The connecting tube was 1.2 cm. internal diameter with a total length of 12.3 cm. while the chamber A consisted of the volume of a Western Electric No. 117W loud-speaking telephone. A quartz thread about 46 cm. long served to suspend the Rayleigh disc, the latter being a thin, plane, microscope cover glass about ¼ inch in diameter. The period of the suspended system was about 2.5 minutes. The pitch of natural resonance was 510. The instrument would give response for sounds of threshold audibility. It seems likely that a better tuning could be obtained if the chambers A and B were made adjustable, that is, have the telephone in A and the glass plate in B each adjustable as a piston. Good results have been obtained in another instrument with chambers A and B of equal volume.

[2] Rayleigh, *loc. cit.*

[3] Rayleigh, Phil. Mag., Vol. 36, p. 231, 1918.

The intensity of the sound I that rotates the disc is related to the deflection of the disc according to the relation:[1] $I = k\varphi \sin 2(\theta - \varphi)$, where k is a constant and θ is constant, equal for most sensitive arrangement to 45°, and φ is the angle of rotation of the disc.

Experimental Investigation.

The current necessary to operate the acoustical galvanometer was so small that ordinary alternating current ammeters gave no response. The following arrangement was then devised to test the instrument and calibrate it. An alternating current was passed in series through the resistance S one meter in length and a hot wire ammeter F. Current from S was shunted to actuate the acoustic galvanometer g. The construction of the ammeter F was suggested by Fleming's design[2] and consisted of two manganin wires each one meter long and .003 inches in diameter, mounted parallel to each other 2.5 mm. apart and connected in series, the total resistance being 201.4 ohms. Delicate springs at the center pulled vertically on the wires so that any expansion due to heating when the current passed resulted in an upward motion for one wire and downward for the other. Since the two wires were not in a vertical

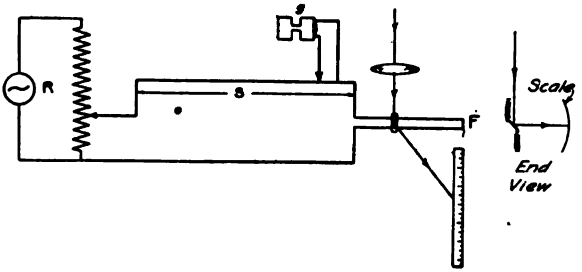

Fig. 2.

Arrangement of apparatus for testing and calibrating the acoustical galvanometer.

Table I.

Data for Calibration of Hot Wire Ammeter.

E.M.F. Applied.	Scale Reading.
2.1 volts	1.65 cm.
4.2	5.7
6.3	11.0
8.35	19.6

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[1] Watson, Phys. Rev., Vol. 7, p. 125, 1916.

[2] Phil. Mag., Vol. 7, p. 595, 1904.

plane, a tiny mirror mounted across the wires rotated slightly when the wires moved and gave a deflection of a reflected beam of light. The calibration of this ammeter gave the following results when a direct current was used. (See Table I.) The range of the instrument was found to be about 5 to 50 milliamperes.

By using alternating current instead of direct, the acoustical galvanometer could then be calibrated by comparison with the Fleming instrument.

Alternating current was obtained from two sources. In one case the current was generated by means of a tone variator in which a telephone receiver was mounted in the adjustable piston. When air was blown across the mouth of the instrument, sound vibrations were set up and produced varying pressures on the telephone plate, thus generating induced alternating currents. The tone variator was placed in a soundproof box and blown by air from a constant pressure tank. It had a range of about an octave. When confined to a single pitch it proved satisfactory but when the pitch was varied, the alternating currents were not uniform in intensity, particularly at the extremes of the range. The other source of alternating current was a Siemens and Halske high frequency machine which gave a wider range of pitches with better control of current.

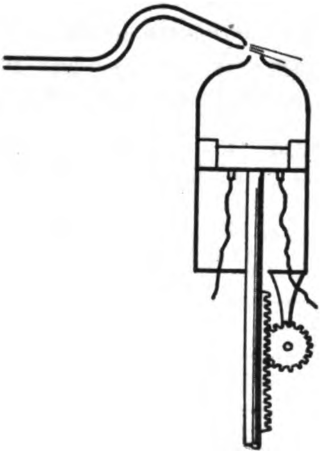

Fig. 3.

Source of Alternating Current.

Table II, gives tabulated results obtained with a constant current of varying frequency and Fig. 5 pictures these in a curve.

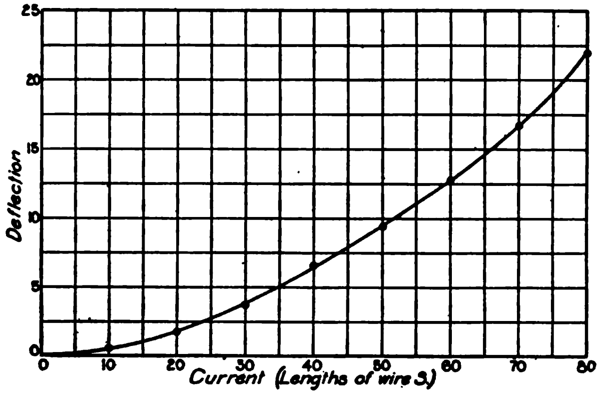

Fig. 4.

Plot Showing Variation of Galvanometer Deflections for Varying Current of Constant Frequency.

TABLE II.

Variation of Deflection with Pitch.

Pitch	150	175	200	225	235	250	256	288	350	400	426	450	480
Deflection	1.5	3.8	2.7	3.5	6.3	9.5	15.4	5.9	5.0	6.5	14.2	19.2	32.2

Pitch	500	512	520	530	540	550	605	645	700	750	775	800	810
Deflection	(Off scale)			31.2	24.2	20.2	8.3	6.5	5.7	6.0	8.5	12.7	14.5

Pitch	820	830	840	850	878	935	990	1080	1160
Deflection	17.5	20.7	21.3	17.8	6.8	0.9	0.4	0.2	0.0

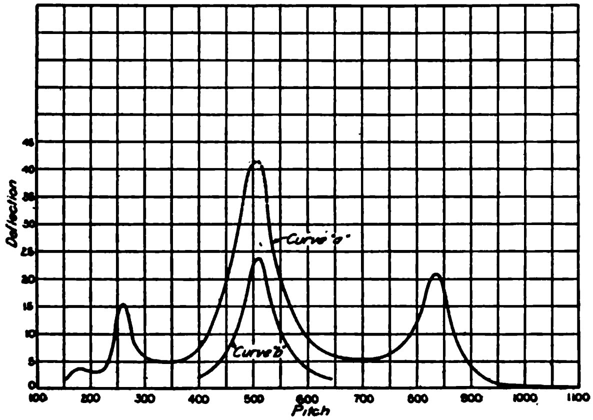

Fig. 5.

Plot Showing Variation of Galvanometer Deflections for Varying Frequency.

Curve *a* shows extreme deflections in the region of 510 frequency, so that a second set of readings was taken with a smaller current to determine the maximum. This data is recorded in Table III. and pictured in curve *b*, Fig. 5.

TABLE III.

Data Showing Resonance for Pitch 510.

Pitch	400	450	470	480	490	500	505	510	515	520
Deflection	2.0	6.7	10.0	15.6	18.2	22.0	23.0	24.0	23.3	22.4

Pitch	530	540	550	560	570	580	590	600	645
Deflection	17.0	13.7	10.7	8.3	6.9	5.1	4.5	4.3	1.7

Curve *a* shows three maxima. The maximum for the frequency 510 is due to the natural period of the double resonator as calculated approximately from the theory. The smaller maximum at 256 vibrations, having practically one half the frequency 510, is probably due to a stimulation of the natural vibration of the resonator. The maximum at 840 would appear due to the natural frequency of the telephone plate since such plates have fundamental frequencies in this region.

Calibration of the Galvanometer.

The calibration of the galvanometer was made for the frequency of 510 vibrations per second. One arrangement of apparatus for this purpose is shown in Fig. 2.

The portion of the resistance *S* spanned by the galvanometer shunt was .548 ohm and the shunt circuit resistance *g* 400 ohms, both measured with direct current. When an alternating current of 0.0195 amperes, measured by the hot wire ammeter, flowed in the main circuit, the acoustic galvanometer gave a deflection of 24 divisions, each division being 17.5 mm. wide. The current through the galvanometer was then calculated by the formula:

$$Ig = I\frac{s}{s+g} \text{ to be .00002656 amperes.}$$

For 1 mm. deflection with the scale 1 meter distant from the galvanometer, the current would be:

$$.00002656 \div (24 \times 17.5) = 5.7 \times 10^{-8} \text{ amperes.}$$

This calculation probably does not give the extreme sensitivity of the instrument since the effective resistance of the telephone receiver for alternating current would be greater than 400 ohms as determined by direct current. The calculated result, however, gives an idea of the order of magnitude of the actuating current. Smaller currents could be measured by adjustments of the relative sizes and shapes of the resonator chambers and the connecting neck. Further possibilities for greater sensitivity are suggested in the coordination of the frequencies of the telephone plate and the resonator, the tuning of the electric circuit with a condenser and in the use of a more delicate quartz thread.

SUGGESTED USES FOR THE INSTRUMENT.

In addition to the measurement of small alternating currents, the instrument may be employed for testing larger currents by using a more robust construction. A portable form of the instrument has been constructed in which the Rayleigh Disc is mounted on a light, stiff pin and the restoring force is furnished by a coiled hair spring attached to the pin. The galvanometer appears promising for use in telephone research where, in a limited sense, it would act as a substitute for the ear and thus measure the actual performance of telephone receivers with varying conditions of current, capacity, and other factors. It would also be useful in psychology tests where quantitative measurements are desired for comparison with the responses of the ear.

LABORATORY OF PHYSICS,
UNIVERSITY OF ILLINOIS,
January, 1921.

EVIDENCE FOR THE EXISTENCE OF HOMOGENEOUS GROUPS OF LARGE IONS.

By P. J. Nolan.

Synopsis.

Homogeneous groups of large ions in air.—The existence of such groups, which seemed proved by previous work reported by the author and others, has recently been questioned by the negative results obtained by Blackwood. Hence further experiments have been made using the Zeleny method adopted by Blackwood instead of the McClelland method previously employed. The thirty curves obtained nearly all show two or more peaks or breaks corresponding to definite groups. These peaks came at the same places whether the ions were produced by bubbling the air through alcohol or by passing it over phosphorus. *The mobilities of the groups* agree with the values previously reported within 10 per cent. or less: .050, .025, .018, .0075, .0042, .0025, .0013, .00067, .00033. Some evidence of intermediate groups was also obtained. *In explanation of Blackwood's negative results* it is suggested that since ions are constantly changing from group to group, definite peaks can be expected only if the time spent by each ion in the measuring chamber is relatively short. In Blackwood's experiments this time was from 9 to 150 seconds whereas it was only from 0.5 to 13 seconds in the experiment reported here.

In various papers[1] published between the years 1916 and 1919 the present writer and others have described experiments showing that, when large ions are produced in air in various ways, the ions can be divided into a number of groups, each group having a definite mobility. In a paper[2] recently published Blackwood presents evidence against the existence of such groups. He finds that the ions distribute themselves continuously over a wide range; in other words he finds "a continuous spectrum of mobilities and not a band spectrum." The present paper gives the results of a reëxamination of the question in the light of Blackwood's criticisms. The evidence here presented is, it is felt, quite convincing as to the existence of the group system and is on the whole a very complete confirmation of previous work.

A short account of the work on group ionization will show that the conclusion that this type of ionization exists was not rashly made. The first indication of groups was obtained in 1912 by J. J. Nolan while working at the electrification caused by spraying water and mention is made of the groups in a paper[3] published in 1914. Before the examination of these groups was completed J. A. McClelland and P. J. Nolan discovered groups in the ionization produced by bubbling air through mercury. When the mobilities were calculated and compared it was found that there was a considerable amount of numerical agreement between the values for the mobilities of various groups. The method of measuring mobilities used was the well-known McClelland method in which the ionized air is drawn along a tube containing an insulated central electrode and curves are plotted between the voltage on the outer tube and the currents to the central electrode. These curves were found to consist of a number of straight lines indicating the presence of distinct groups of ions. The mobilities of the groups were deduced by taking the voltages corresponding to the corners as the saturation voltages for the different groups. The obvious objections to the McClelland method as a method for separating out possible groups of ions of mobilities not widely different were fully realized, but careful consideration failed to account for the experimental results on any other hypothesis than that of the existence of separate groups. Experiments conducted with different methods of producing the air-blast through the mobility tube and also with mobility tubes of different dimensions confirmed the existence of groups. The results on the spray ions and on the ions due to mercury were published at the same time with a joint discussion.

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[1] J. J. Nolan, Proc. Roy. Irish Acad., Vol. 33, (A), 1916. J. A. McClelland and P. J. Nolan, Proc. Roy. Irish Acad., Vol. 33 (A), p. 24, 1916; Vol. 34 (A), p. 51, 1918; Vol. 35 (A), p. 1, 1919.

[2] Blackwood, Phys. Rev., Aug., 1920.

[3] J. J. Nolan, Proc. Roy. Soc. (A), Vol. 90, 1914.

Table I., column A, gives the mobilities of the ions obtained with water and columns B to E give the mobilities observed with mercury with dried and undried air and with long and short time-intervals between production and measurement. It was thought that the mobilities given in columns C, D and E represented groups which were gradually growing and that the final stable sizes were the ions given in columns A and B. The coincidence between the numbers in columns B and D, C and E which are set forth in the table were at the time thought to be accidental. The spray ions did not show any change with time. It was supposed that the ions from mercury took time to add on water vapor, the approach to stable conditions being delayed, in the case of the experiments with partially dried air, by the diminution of the amount of water vapor present.

Further work on the ionization produced by bubbling air through alcohol and by passing air over phosphorus revealed the presence of groups of ions in both cases. In the determination of the mobilities of the various groups the following method was adopted. A preliminary investigation of the current-voltage curve was made and a portion which contained only one corner was selected. This portion was explored very carefully to ensure that it was made up of two intersecting straight lines

Table I.

Water.	Mercury.				Alcohol.	Phosphorus.
	Long Time-Interval.		Short Time-Interval.			
	Undried.	Dried.	Undried.	Dried.		
A	B	C	D	E	F	G
1.09					1.10 ?	
.53					.50	
				.32	.31	
.24			.20		.22	.22
.12					.12	
				.092		.092
.046			.048	.043	.049	.053
		.024				.028
.013	.014		.02		.017	.018
		.0068		.0064	.0077	.0074
.0043	.0040		.0045		.0040	.0041
		.0021		.0022	.0023	.0024
.0010	.0013		.0013		.0014	.0012
		.00056			.00063	.00064
.00038	.00034				.00034	.00031
					.00015	.00015
						.000085
						.000053

and to determine the exact point of intersection. In all about 250 such curves were plotted during the course of the alcohol and phosphorus experiments. Table I., column F, gives the mobilities of the ions found in air which had bubbled through alcohol. The results with alcohol caused a modification of the views previously advanced. With alcohol both the series of groups due to water and due to mercury under dry conditions were obtained and no gradual change with time in the mobility of any group was observed in spite of the fact that a large number of observations was taken at a wide range of time-intervals and in the critical manner just described. The agreement of the mobility values as illustrated in Table I. was striking. We were thus driven to the conclusion that each group represented an ion of at least some degree of stability. The fact that the opposite result was expected when the work was undertaken gives all the more value to the conclusion arrived at. The results of the phosphorus investigation are given in column G; the groups given there were observed when dried air as well as undried air was passed over phosphorus. The phosphorus observations supported the previous conclusion that the mobilities were definite and did not change with time. The agreement between the mobility values gave still further confirmation of the group idea.

APPARATUS AND METHOD.

The work described in the present paper was commenced as soon as possible after the appearance of Blackwood's paper. As he had based his conclusion that there were no groups on results obtained with the Zeleny method, it was decided to use that method in trying to confirm our previous work. Blackwood examined the ionization produced by spraying water and also that formed when air is passed over a hot platinum wire. The writer has never investigated the ionization produced in the latter way but the experience of other workers in this laboratory is that it is difficult unless very special precautions are taken to obtain steady conditions when dealing with this type of ionization. The quantity of ionization obtained by bubbling air through alcohol or by passing air over phosphorus is much larger than that due to spraying water. Another important consideration is that in the water spraying experiments of J. J. Nolan, the results are not consistent in the region of the larger ions.[1] Variations are observed in the mobility values of two of the groups which are larger than would be expected from experimental errors. Two observations of a mobility which does not enter into the group system are recorded. Variations in the quantities of the different groups are noticed. This variation is especially noted in the largest ion and sometimes the quantity of this ion present is so small that it would be difficult to assert that it was not absent. These considerations led to the use of the alcohol and phosphorus methods of producing ionization in preference to the hot wire and water spray methods. This decision was also influenced by the fact that the writer is more familiar with the former methods.

The arrangements of the apparatus for bubbling air through alcohol and for passing air over phosphorus were essentially the same as those described in previous papers. The Zeleny method of measuring mobilities, as used by Blackwood, consists in drawing the ionized air through a tube containing two central electrodes separated by a short gap. The longer electrode is earthed and the shorter electrode, which is the farther from the entrance to the tube, is insulated and connected to an electrometer. Curves are drawn with the voltages on the outer tube as abscissæ and the currents to the short insulated terminal as ordinates. It is easy to show that if there are present ions of one mobility the curve will consist of one peak and that if several groups of ions are present there will be several peaks, one corresponding to each group.

Two Zeleny mobilities tubes were used during the course of the work; their dimensions are as follows:

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[1] P. R. I. A., Vol. 33 (A), p. 17, 1916. P. R. S., Vol. 94, p. 126, 1917.

Internal diameter of outer tube (in both tubes)	= 3 cm.
External diameter of inner electrode (in both tubes)	= .95 cm.
Length of earthed electrode	37.4 and 10 cm.
L_1 length of insulated electrode	12.3 and 5 cm.
L length of earthed electrode + insulated electrode + gap	49.8 and 15.1 cm.

One tube was more than three times as long as the other and the ratio L/L_1 was approximately four in the long tube and three in the short tube. The tube which Blackwood used was nearly three times as long as the longer tube and his ratio L/L_1 was four. This ratio is important because the bigger it is the more pronounced should be the peaks on the Zeleny curve. On the other hand, the larger the ratio the smaller will be the quantity of ionization for measurement. The shortening of the insulated electrode introduces another very serious difficulty, the nature of which will be referred to later.

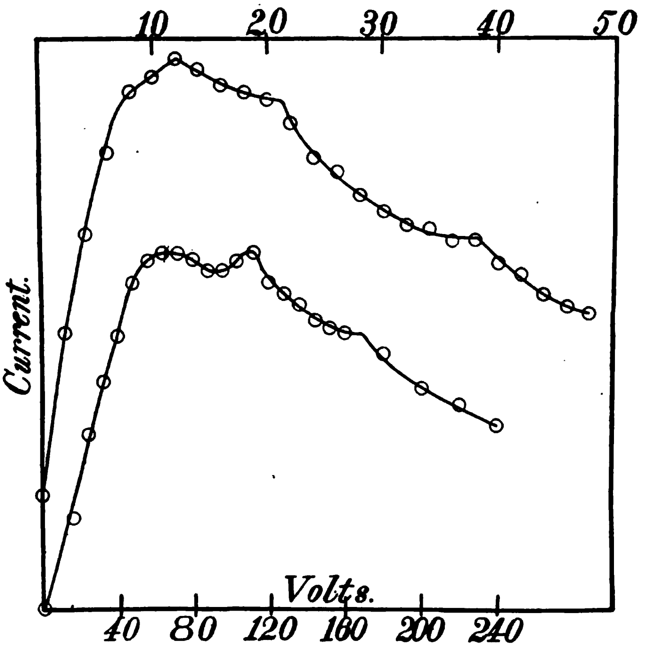

Fig. 1.

Results with Alcohol.

The ionization produced by bubbling air through alcohol was first tested using the long mobility tube. One of the first curves obtained is the upper curve given in Fig. 1. The voltages referring to this curve are shown along the upper horizontal line of the figure. In general outline it resembles the curves given by Blackwood. There is only one peak but there are two decided corners at 21 and 38 volts and perhaps a corner at 7 volts. If voltage steps twice as large had been used these corners would scarcely have been detected and a smooth curve would have been the result. Although these corners show clearly that there is not a continuous spectrum of mobilities, there is a wide difference between this curve and the theoretical curve given by Blackwood, the realization of which experimentally he looked for as a confirmation of the existence of groups. This theoretical curve is that due to three groups of ions of mobilities in the ratio 1, 2.5 and 11 present in equal quantities and it shows three well-defined peaks. Our experience shows that the groups are present in varying proportions which seldom approach equality. If a theoretical curve were drawn with one group present in large quantity and others in smaller quantities, the peaks would not be so well marked. Again with alcohol and phosphorus the ratio of mobilities is 1, 1.8, 3.4, etc. The ratio of successive mobilities has a considerable influence on the appearance of the Zeleny curve; the closer the mobilities the less defined are the peaks. We have given reasons why the spray ionization was not used and we considered that these reasons outweighed the advantage that the spray ions are in general more widely spaced than the ions due to alcohol or phosphorus. Making allowance for the inequality in the amounts of the different groups and the closeness of successive mobilities a theoretical curve would be drawn with peaks not so well marked as in Blackwood's theoretical curve. These two considerations, however, are not sufficient in themselves to explain the non-appearance of peaks in this curve. An explanation may be found by consideration of the fact that the ions are changing while passing along the mobility tube. This will cause a rounding-off of the peaks and a filling-up of the hollows in the theoretical curve as ions which grow during their passage through the tube will appear to have mobilities which are between their initial and final values in the tube. It is obvious that if a very long mobility tube and a very slow air-blast were used the ions would spend a long time in the tube and the effect of change might be so great as to obliterate any evidence whatsoever of groups. Accordingly we can interpret the corners on the curve as peaks of the theoretical curve which are flattened and almost obscured by the growth of the ions in the tube. If this interpretation is correct the curve indicates the presence of three groups (and a possible fourth) the mobilities of which can be deduced by taking the voltages at the points of discontinuity as saturation voltages for the various groups. Furthermore it should be possible to obtain curves more like the theoretical curve by reducing the time which the ions spend in passing along the mobility tube. Experiments were accordingly conducted with various air blasts faster than that used in the first experiments. Various volumes of tubing were inserted between the bubbling vessel and the mobility tube so that the time interval between the formation of the ions and the measurement of their mobilities might be equal to or in some cases greater than the time-interval used in the previous experiments. The longer this time-interval, the bigger will be the ions when they reach the tube and consequently the less will the ions change when passing along the tube. The time of the ions in the field can also be reduced by reducing the length of the tube and it was with this object that the shorter mobility tube was used in some experiments.

The lower curve in Fig. 1 was obtained using a faster blast than that used in the previous experiments. This curve shows two well marked humps and one corner. This curve is more like the theoretical curve; one peak is like the peaks in the theoretical curve but the other peak is much flatter. This curve places the existence of group of ions beyond any doubt. The two peaks are undoubtedly due to two distinct groups of ions and the corner indicates the presence of a third group.

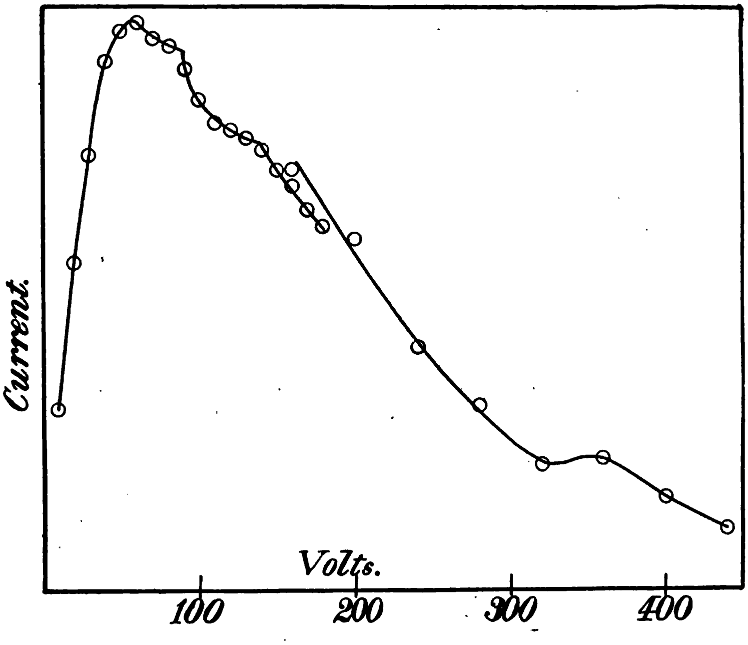

Fig. 2.

Fig. 2 gives another sample of the curves obtained with the alcohol ionization. Two parts are shown representing separate experiments in which the conditions were the same. In this curve there is a peak, two corners and a hump at a high voltage. The hump at 360 volts is convincing evidence of the existence of a separate group of mobility approximately equal to .00067 cm./sec. A preliminary consideration of the theoretical Zeleny curve, taking into account the various causes which might obscure peaks, led to the conclusion that peaks would be more easily observed on the right hand side of the maximum point of the curve. This conclusion was justified by the results obtained during the course of the work.

To obtain still better definition of the peaks experiments were performed with faster blasts and with the shorter mobility tube. Results so obtained will be discussed later.

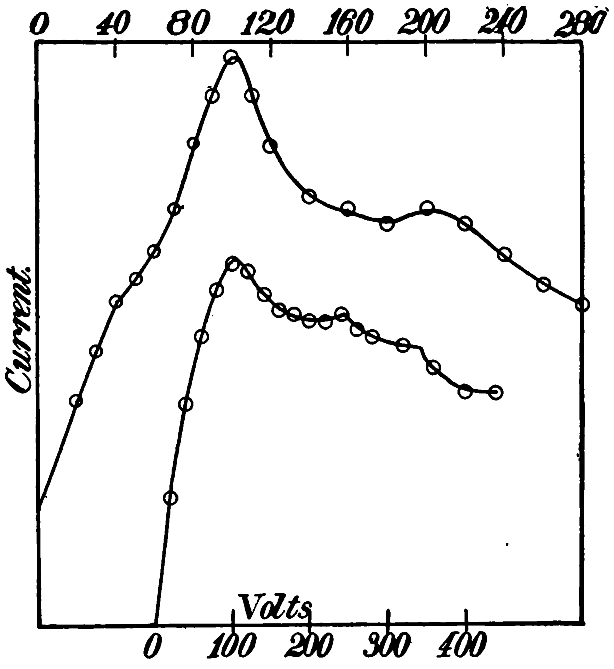

Fig. 3.

Results with Phosphorus.

The upper curve in Fig. 3 is due to the phosphorus ionization, the longer mobility tube being used. There are two well-marked peaks and one corner, showing clearly that groups exist in air passed over phosphorus. The lower curve in Fig. 3 gives another sample of the phosphorus curves; it shows the presence of three groups. Curve *A*, Fig. 4, is the only instance of a definite peak at a small voltage well removed from the maximum peak of the curve. It is plotted with the means of two series taken, one immediately after the other, which agreed very well. When

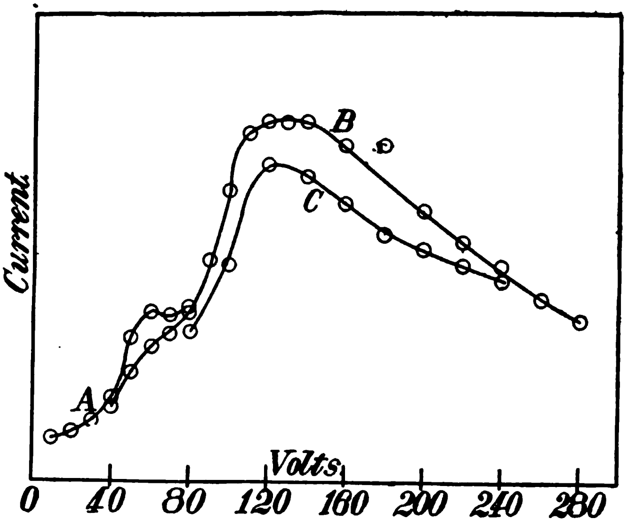
Current
Volts
A
B
C
0
40
80
120
160
200
240
280

Fig. 4.

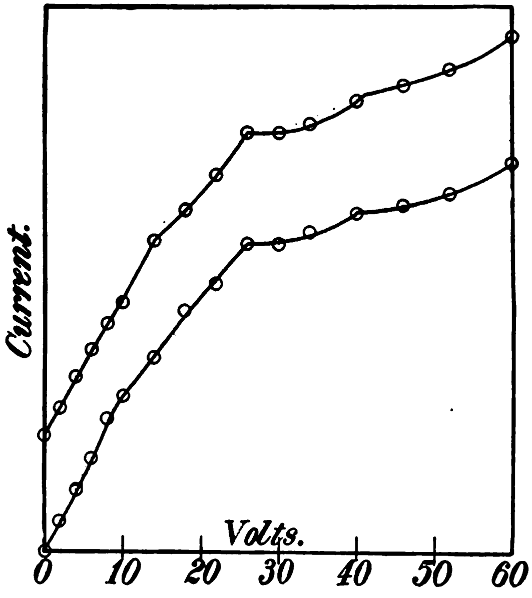
Current.
Volts.
0
10
20
30
40
50
60

Fig. 5.

the complete curve *B* was taken this peak at 60 volts had become flattened out but there still remained evidence of a group at this point. Curve *C* taken immediately after Curve *B* gives the same mobility for its highest point but its peak is sharper. It is an interesting fact that the sharpness of the peaks varies considerably and in this case the variation occurs during an immediate repetition of the numbers. In the previous paper on phosphorus changes in the nature of the ionization were recorded and also the fact that some of the groups of ions could not be observed on some occasions. The three curves in Fig. 4 also show a variation in the quantity of the ionization. This variation was often noticed when observations were repeated both in the case of the phosphorus and alcohol but it is thought that such variations do not affect the validity of the arguments from the whole series of curves. It may be noticed that the number for 180 volts in curve *B* is well above the curve. A peak is not drawn here because it would depend on only one number and it does not appear in curve *C*. The peaks and corners on which we rely depend on current observations for several successive voltages and are supported by other curves. A slight continuous variation of the quantity of ionization during an experiment such as sometimes occurred does not affect the general evidence of the curves. It may be remarked here that in the previous work by the McClelland method practically all the curves were drawn using the means of observations with ascending and descending voltages. As in those experiments a short part of the curve was examined, the experiment was performed in a short time and the variations which occurred during a series of observations were small.

Fig. 5 gives portion of a curve worked over in detail using the smaller mobility tube. One curve was taken immediately after the other. The difference between the two sets of numbers is very small. The curves are plotted apart with different zeros but as the zero for both curves is given it will be seen how good is the agreement between the readings for the various voltages. There is a well marked corner at 26 volts and a faint corner about 40 volts in both cases. It is interesting that the corner at 26 volts is so well marked as the mobility deduced from it (.033) cannot be fitted into the groups we have already found. There is some evidence of corners about 8 volts in one curve and about 14 volts in the other curve. However as this region was not fully explored there is no further evidence for or against groups at these points. A slight difference in the numbers about this region in the two curves causes the appearance of the corner at 8 volts in the one case and at 14 volts in the other. This lack of agreement also prevents us from making any decision.

This completes a selection of the curves obtained. In all about thirty curves were taken and only two of these curves showed no evidence of a peak or corner in addition to the main peak. The failure to obtain evidence of groups on these two occasions can be explained. One of these curves was taken with a slow blast and this as we have seen militates against the observation of groups. In the other curve only two numbers were taken on the left hand side of the maximum point of the curve and possibly a corner there escaped notice.

Intermediate Mobilities.

During the course of the alcohol investigations an attempt was made to get if possible even better defined peaks by shortening the time of the ions in the mobility tube. Fast air-blasts were used and the shorter mobility tube was also used with this object. The curves obtained did not show better defined peaks but they indicated the presence of some groups of intermediate mobilities. The mobilities calculated from the previous experiments agreed fairly well with the numbers set forth in Table I. The curves now obtained showed not only these mobilities but indicated that some groups of mobilities of intermediate values were present. This matter was not fully investigated and all that can be said at present is that indications of such intermediate groups were noticed. These groups may merely represent stages in the process of growth of the ions or in other words less stable aggregations which persist for a short time in the growth between two more stable ions. The percentage of such ions present will be smaller than the percentage of the more stable ions and consequently a good resolving method is necessary for their discovery. This agrees with the fact that indications of their presence were obtained when using the Zeleny method in the manner in which one might expect the best definition of peaks. The presence of such groups would explain why the main peaks were not better defined in previous experiments and might also account for the variation in the width of the peaks.

Agreement with Previous Results.

The general agreement between the mobility values obtained by the Zeleny method and those obtained by the McClelland method is shown in Table II. The present numbers are not so numerous or exhaustive as the previous ones. Mobilities of intermediate groups are not included.

TABLE II.

Al. (McC.)	.049		.017	.0077	.0040	.0023	.0014	.00063	.00034
Ph. (McC.)	.053	.028	.018	.0074	.0041	.0024	.0012	.00064	.00031
Al. (Ze.)	.049	.025	.018	.0075	.0045	.0026	.0014	.00067	
Ph. (Ze.)		.022		.0075	.0043	.0026	.0014	.00074	.00035

BLACKWOOD'S RESULTS.

It is possible to explain fairly satisfactorily the difference between the results obtained by the writer with the Zeleny apparatus and Blackwood's results with the same type of apparatus. We have already emphasized the importance of the length of time spent by the ions in the mobility tube and it is interesting to compare the researches in this respect. If we divide the volume of the tube by the volume of the blast per second we get the average time of the ions in the tube. The volume of the tube $= \pi(b^2 - a^2)L$, where b is the internal diameter of the outside tube, a is the external diameter of the electrode and L is the sum of the lengths of the earthed and insulated terminals and the small gap between them. In Blackwood's apparatus $L = 140$ cm. and the cross-section = 16.5 sq. cm.; in the author's apparatus $L = 49.8$ cm. and the cross-section = 6.4 sq. cm. The corresponding volumes are 2310 c.c. and 320 c.c. approx. Thus for the same air-blast the time of the ions in the tube was seven times longer in Blackwood's experiments than in ours.

From the summary of Blackwood's paper published in the Proceedings of the Nat. Acad. of Sciences, May, 1920, we see that in the experiment there quoted the ions of mobility .0004 cm. per sec. would be saturated at about 40 volts. This enables us to calculate the air-blast and we thus deduce that in this experiment the ions spent about 150 seconds in the mobility tube. Similarly from the curves given in Fig. 11 of Blackwood's paper in the PHYSICAL REVIEW it can be calculated that the corresponding time in the experiments with the hot wire was about 9 seconds. These experiments were conducted for the purpose of examining the relation between the mobility and the age of the ions and were apparently carried out with the fastest air-blast used. Thus it seems probable that the extreme values of the time of the ions in the tube were 150 and 9 seconds. In our work the extreme values for this time were 13 and .5 second. The times in our experiments were much shorter than Blackwood's and we think that this to a large extent explains why he failed to detect groups. It is possible that if he used smaller voltage-steps, especially in the experiments conducted with fast air-blasts, some discontinuities might have been observed. If intermediate groups are present even in small quantities they would have an obscuring effect on the peaks of the

Zeleny curve. Possibly this is a contributory cause of Blackwood's failure.

In examining spray ions by the McClelland method, Blackwood obtained curves showing breaks, indicating a ratio of about 2 for successive mobilities. Table I. shows that J. J. Nolan found the spray ions had a ratio of about 3.4 between successive mobilities. The ratio in the case of the phosphorus and alcohol ions is about 1.8. It is possible that in Blackwood's experiments the ions of mobilities such as .00064, .0023, .0075, etc., were present just as in the case of alcohol and phosphorus. (J. J. Nolan observed the group of mobility .00064 sometimes.) This would account for the ratio observed. We have never properly understood why there should be twice as many groups in ionization due to alcohol or phosphorus as in that due to water or mercury. It may be connected with the relative quantities of positive and negative ions. In the case of alcohol and phosphorus the number of ions of the two signs are always exactly equal; with water and mercury this is not so.

McClelland Method.

It is interesting in the light of the new information about the growth of the ions and the possible existence of intermediate groups to discuss the previous results obtained with the McClelland method. Blackwood shows that the Zeleny method is much better than the McClelland as a detector and resolver of group ionization. The question then arises as to how the groups were obtained with the McClelland method and not with the Zeleny method as used by Blackwood. Comparison of the length of the time which the ions spent in the tube explains this to some extent; in previous work with the McClelland method that time-interval was generally of the same order as that used in the present work and was thus smaller than the time in Blackwood's experiments. But there is another important consideration which shows that the flattening out of the theoretical curve due to the growth of the ions is much more serious in the Zeleny method than in the McClelland method. The ions captured in the insulated terminal of the Zeleny tube are those which have moved along the greater part of the tube; in the McClelland method ions begin to be captured on the insulated terminal at the moment of their entry into the field. Hence the effect of change of the ions is smaller in the McClelland than in the Zeleny tube; the larger the ratio between the earthed and the insulated electrodes of the Zeleny tube, the greater will be the effect of growth.

If intermediate groups exist it is not surprising that they were not detected by the McClelland method. They are probably present in small quantities and the McClelland method was not able to resolve them from groups nearly equal in mobility present in large quantities. It was only when the Zeleny method was being used in a highly resolving manner that indications of these intermediate groups were noticed.

CONCLUSION.

The results obtained in the present work place beyond doubt the existence of homogeneous groups of large ions. The curves admit of no other explanation. The numerical agreement between the mobility values of the different groups deduced from the Zeleny curves and from the McClelland curves shows that both types of curves have been correctly interpreted. The present paper suggests the reasons for Blackwood's failure to find any evidence of groups. Furthermore, it shows how it was possible to detect and examine groups of ions in the previous work although it was carried out by a method of admittedly inferior resolving power.

The results of the present paper do not enable us to add to the hypothesis put forward by J. J. Nolan to account for the production of different classes of ions. It seems likely that further knowledge of the nature of this ionization will be obtained more readily by attacking the question from a different angle rather than by continuing work on the present lines.

The author wishes to express his indebtedness to his brother, Professor J. J. Nolan, for his help and advice during the course of this research.

UNIVERSITY COLLEGE, DUBLIN,
March, 1921.

THERMOELECTRIC ELECTRIC EFFECTS IN IRON AND MERCURY DUE TO ASYMMETRIC HEATING.

By J. M. Benade.

Synopsis.

Thermo-electric Effect Due to Temperature Gradient Along a Constriction.—(1) *Iron rod.* Using constrictions from 0.5 to 4 mm. in diameter and from 0.5 to 1.5 mm. in length, electromotive forces were obtained with gradients of a few degrees per mm. large enough to be measured easily with a sensitive galvanometer. For each constriction the effect was found to be proportional to the temperature gradient. The fact that it decreased as the mean temperature was increased indicates that it is not an inverse Thomson effect. Moreover it depends on the dimensions of the constriction, being less for the smaller diameters. (2) *Mercury.* By using two relatively large quantities of mercury connected through a pin hole in an intervening sheet of mica, a temperature gradient of at least 600° C. per m.m. could be maintained in the constriction, but a high sensitivity galvanometer failed to indicate any electromotive force although according to Benedick's conclusions[1] a deflection of over 50 divisions should have been observed. His results are explained as due to a true thermo-electric effect between the mercury and the slate tube containing it. (3) *In explanation* it is suggested that the effect is associated with the non-homogeneity produced by the mechanical stresses set up in the iron as a result of unequal thermal expansion and is not due merely to the flow of heat through the constriction.

It has been generally assumed that the e.m.f. produced by the asymmetric heating of conductors is an inverse Thomson thermoelectric effect. It was on this assumption that an attempt was made to measure the temperature coefficient of the Thomson effect by observing the rate of change of this thermo-e.m.f. with a change of mean temperature of a constriction in various metals, the temperature gradient in the constriction being maintained constant. With this object in view various forms of apparatus were used, the one shown in Fig. 1 being typical. The temperature gradient in the constriction as well as the absolute temperature was controlled by means of heating coils. The temperatures were measured by means of the usual copper constantan thermo couples, which it was found necessary to insulate carefully from the rod into which they were inserted. To the ends of the rod were soldered iron wires and to these in turn copper wires which were connected to a high sensitivity galvanometer. In order to prevent spurious thermoelectric effects both pairs of junctions were kept in constant temperature baths.

The thermo-junctions protected by thin-walled glass capillary tubes were inserted into holes 1.5 mm. in diameter in the iron rod on opposite

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[1] Comptes Rendus, 169, No. 13, Sept. 29, 1919.

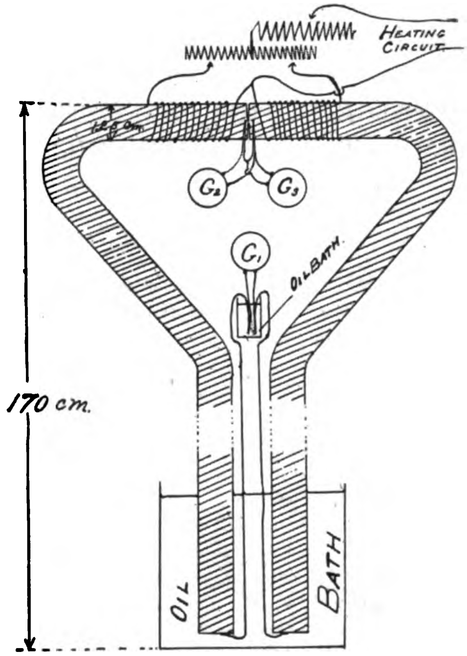
HEATING CIRCUIT
12.5 cm.
G2
G3
G1
OIL BATH.
170 cm.
OIL
BATH

Fig. 1.

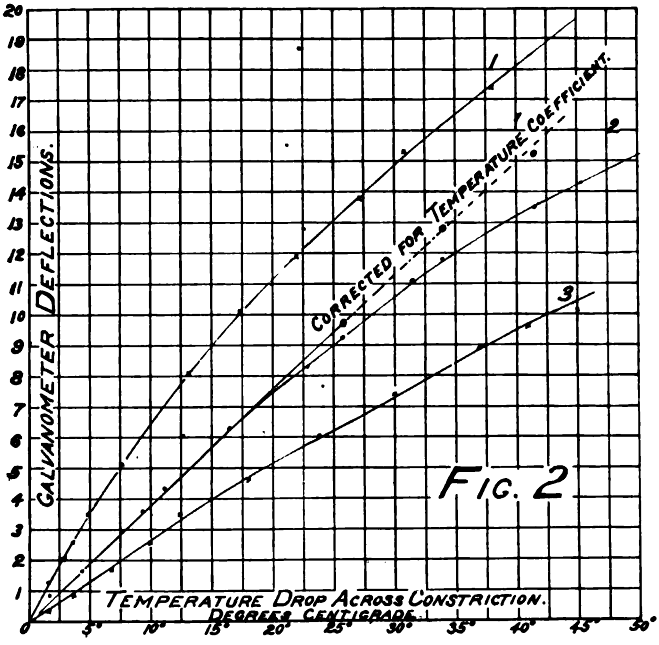
GALVANOMETER DEFLECTIONS.
TEMPERATURE DROP ACROSS CONSTRICTION. DEGREES CENTIGRADE.
CORRECTED FOR TEMPERATURE COEFFICIENT.
1
2
3
FIG. 2
0
1
2
3
4
5
6
7
8
9
10
11
12
13
14
15
16
17
18
19
20
5°
10°
15°
20°
25°
30°
35°
40°
45°
50°

Fig. 2.

sides of the constriction with a distance of about 4 mm. between their centers. The constrictions varied from 4 mm. to 0.5 mm. in diameter and from 1.5 mm. to 0.5 mm. in length. It was found that for very sharp constrictions with correspondingly abrupt temperature gradients, for a given difference of temperature between the ends of the constrictions relatively small e.m.f.'s were obtained. The greatest effects were obtained when the constrictions were not very deep, though this point has not been carefully investigated. The important fact for our present purpose is that the e.m.f. is found to depend not merely upon asymmetry in the temperature gradient but also upon the dimensions of the constriction. This is shown by the curves in Fig. 2, and in Fig. 3.

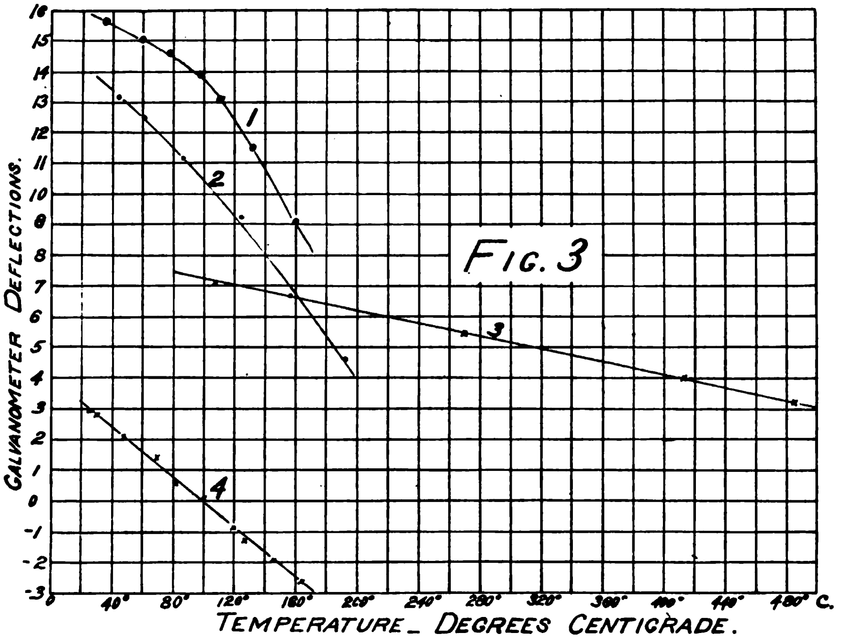

Fig. 3.

The curves in Fig. 2 show the relation between the e.m.f. and the temperature gradient in the constriction. Were it not for the fact that the mean temperature of the constriction during each run was constantly rising, the curves would have been rectilinear, for, as will be later shown the temperature coefficient of the effect is negative. In other words the e.m.f. for a particular apparatus is directly proportional to the temperature gradient in the constriction. It is assumed always that the only asymmetry in the temperature gradient is at the constriction. Small errors are actually introduced because this is not always strictly true, though in general these errors are negligible. Curves 1, 2 and 3 were obtained under similar conditions except that the constriction was reduced after getting the data for the first, and again after the readings for the second curve, the third curve corresponding to the sharpest constriction. These curves show clearly that for given differences in temperature between the ends of the constriction the e.m.f. actually decreases as the abruptness of the temperature gradient increases.

In order to measure the temperature coefficient of the effect for any particular form of apparatus, the e.m.f.'s corresponding to mean temperatures of the constriction in which the temperature gradient was maintained constant, were observed. For this purpose a temperature difference between the ends of the constriction of about 20° C. was sufficient to produce convenient galvanometer deflections. In Fig. 3 are several typical curves representing the variation of the e.m.f. with the mean temperature of the constriction. The curves show that in each case the temperature coefficient is negative,[1] *i.e.*, the e.m.f. produced by a constant temperature gradient in the constriction decreases as the temperature rises. The value of this coefficient is found to depend upon the size and form of the constriction as well as upon the material concerned. The variations in the degree of slope and especially the direction of slope show definitely that the effect is not simply an inverse Thomson effect, for in that case the slope should be the same in all cases and should show a positive temperature coefficient as found by Berg,[2] and others. It seems probable that the effect is due to mechanical stresses set up in the iron by unequal thermal expansion which in some way alter the structure of the metal in or near the constriction sufficiently to give rise to the observed thermoelectric effects, as has been repeatedly suggested.

The unexpected results above recorded indicated that the observed e.m.f.'s are due to a lack of homogeneity in the asymmetrically heated conductor. The crucial test for the existence of the effect is its detection in the case of mercury. Benedicks[3] claims to have observed and measured the effect in mercury though Haga and Zernike[4] using a very much better form of apparatus conclude that there is no true effect due merely to asymmetric heating of the conductor. Repeated attempts with various forms of apparatus were made but with negative results in each case though an effect of the same order of magnitude as that obtained in iron should have been very easily obtained. A diagram of the final form

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[1] Curve No. 4 which indicates a reversal of the effect is rather less reliable than the others though there is no reason to believe that it is not correct. The apparatus was broken before getting other readings.

[2] Annalen der Physik, Vol. 32, 1910, p. 477; Vol. 33, p. 1195.

[3] Comptes Rendus, 169, No. 13, Sept. 29, 1919.

[4] K. Akad. Amsterdam, Proc., XXI 10, p. 1262, 1919.

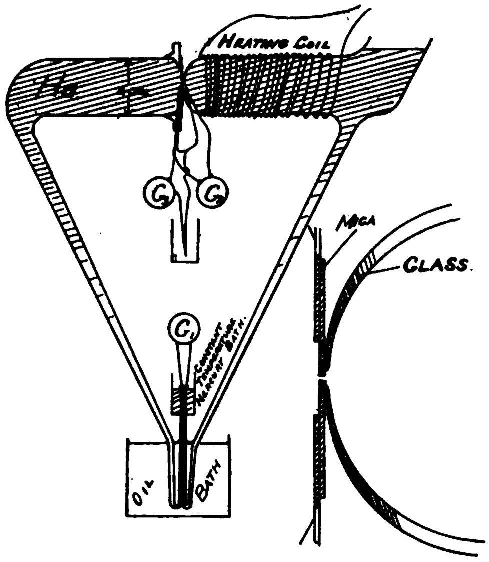

Fig. 4.

of apparatus used is shown in Fig. 4. The relatively large quantity of mercury used was contained in two tubes connected together by means of a pin-hole 0.35 mm. in diameter through an intervening sheet of mica 0.3 mm. in thickness. For the galvanometer connections smaller glass tubes were sealed on to the larger ones and suitably drawn out so that the necessary junctions with the galvanometer lead wires could be kept at exactly equal temperatures by means of a cup of mercury, in order to avoid all spurious thermoelectric effects. Differences in temperature on opposite sides of the constriction were measured by means of thermocouples encased in very fine capillary tubes. The galvanometer used gave deflections of one scale division per 10^{-8} volt, which though not so sensitive as the one used by Benedicks was sufficiently sensitive for the purpose. With this arrangement it was quite possible to keep the mercury on one side of the constriction at or below room temperature indefinitely while the mercury on the other side was slowly boiling. Though it was of course not possible to determine accurately the temperature gradient in the constriction it must have been not less than 600° C. per mm. and was probably nearly if not quite 1000° C. per mm. when the temperature difference between the two sides of the mica was 330° C. or more. Special care was taken in constructing the apparatus to reduce to the minimum conduction of heat from one side to the other through the glass and mica and at the same time to make full use of convection currents in the mercury. The only deflections obtained amounted to four or five scale divisions and these were found to be due entirely to leakage from the heating circuit. Had there been an effect of the magnitude affirmed by Benedicks a deflection of at least 50 scale divisions should have been obtained, for the temperature gradient in the present experiment must have been very much greater than it is possible to obtain in small glass or slate tubes.

If there were in mercury an effect of the same type as that obtained by the asymmetric heating of solid conductors the order of magnitude of the effect would probably be the same as in iron, in which case a drop of 0.005° C. in the constriction should have produced an appreciable deflection, though an actual drop of more than 330° C. produced no observable effect. Since it has been suggested that an e.m.f. may be produced by the rapid conduction of heat across a constriction a modification of the above described apparatus was made in order to test this hypothesis. As described the apparatus was designed to reduce heat conduction to a minimum so that if this were the true explanation of the effect at most only a small effect would have been obtained. The pin-hole in the mica separating the two sections was enlarged to a diameter of about 2 mm. With this arrangement heat was rapidly conducted through the constriction though as before there was absolutely no indication of the existence of any e.m.f. in the circuit.

In explanation of the results obtained by Benedicks and of his cubic relationship, *i.e.*, that the e.m.f. produced in mercury when heated on one side of a constriction in a tube is proportional to the cube of the temperature drop in the constriction, it is suggested that the effect measured was due entirely to a true thermo-e.m.f. produced by the mercury and slate, the abnormal rate of increase being caused by the change in the conductivity of the slate containing-tubes. The thermo-e.m.f. produced by the slate mercury couple with 300° C. difference in temperature between the junctions is very large indeed in comparison with the quantities measured and the conductivity of slate is not extremely low, so that even though the mercury column was continuous the relatively small e.m.f.'s observed may thus be fully accounted for If this be the correct explanation it is to be expected that the effects in glass will be smaller. The apparatus described above should be nearly if not entirely free from the possibility of disturbances of this nature.

Results.—It seems to be definitely shown that if there be any true thermoelectric effect in mercury due to asymmetric heating it must amount to less than 10^{-8} volt for a gradient of 600° C. per mm. Also that if there be any effect due to a flow of heat it must be extremely small.

The effect in the case of iron and presumably of other metals is not an inverse Thomson effect. Since there is no effect in mercury to compare with that observed in solids it may be assumed that non-uniform temperature gradients in solids sufficiently disturb the homogeneity of the material to give rise to the observed e.m.f.'s and that these effects are not due simply to a flow of heat across a constriction.

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NOTE ON THE DYNAMICS OF CAPILLARY FLOW.

BY WILLARD GARDNER.

SYNOPSIS.

Theory of the movement of moisture in soil.—Attention is called to experimental data which show that there usually exist in the soil appreciable moisture gradients which indicate that it cannot be regarded as a bundle of capillary tubes and that therefore *Washburn's theory of capillary flow* does not rigorously apply in this case. However by using Poiseuille's equation modified by the assumption that the variation of moisture content does not appreciably change the kinematical resisting force, together with empirical equations derived from soil moisture studies connecting the capillary pressure and vapor pressure with the moisture content, an *equation for horizontal flow* is derived, $V = k(1 - e^{-\beta t})$, which is in qualitative agreement with experimental results. A more extended treatment of the subject is given in an article by the author in Soil Science.

In the March number of the PHYSICAL REVIEW there appears an article by Washburn on "The Dynamics of Capillary Flow." In part 7 of this article he discusses the rate of penetration of a porous body by a liquid, arriving at a tentative relation:

$$V = k' \left(\frac{\gamma}{\eta}\right)^{1/2} t^{1/2},$$

where V expresses the volume of liquid absorbed, t the time, γ the surface tension, η the coefficient of viscosity, and k' a constant. He finds, however, that for large values of V this relation does not hold in the case of the absorption of water by charcoal.

In this connection, it may be of interest to note that a considerable amount of data are available from soils literature[1] showing that the volume of water absorbed by a soil is not always proportional to the pore space, as his equation (15), viz.,

$$V = \pi r^2 l$$

would indicate. On the other hand, the liquid "density" ρ (*i.e.*, the mass of liquid per aggregate unit volume of soil) is a function of the distance x from the source. Uniformity of soil moisture content is exceptional, whereas moisture gradients are frequently observed in moisture studies, although their magnitudes are usually small except at the immediate water front where an abrupt change is often observed. This fact would seem to invalidate also the application of Washburn's equation (8) wherein he has made use of the reciprocal of r (*i.e.*, the radius of the equivalent cylindrical tube) as a measure of the curvature of the liquid surface. It is clear of course that his equations have been derived on the assumption that the porous body should approximate in character a bundle of capillary tubes and an attempt to apply the equations to a mass of soil is perhaps somewhat beyond his expectations. If, however, we regard the curvature factor of his equations (8) and (9) as a function of the liquid "density," which in turn depends upon the distance x from the source, these equations will apply provided Poiseuille's equation holds for this case.

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[1] See, for example, "Soils," by Hilgard, p. 209; "Physics of Agriculture," by King, p. 134; "Studies of the Movement of Soil Moisture," by Buckingham, *in* U. S. Dept. of Agr. Bur. of Soils Bul. 38; also "The Movement of Soil Moisture" by Gardner and Widtsoe, *in* Soil Science, Vol. XI., No. 3, March, 1921, Fig. 3, p. 224.

In the March number of *Soil Science* the writer has made use of Poiseuille's equation modified for variable "density" by introducing the assumption that the kinematical resisting force is independent of the "density." Or, what amounts to the same thing, that the average velocity v of the moisture is proportional to the pressure gradient and independent of the moisture content,

$$v = K_1 \nabla p. \tag{1}$$

The capillary pressure p is determined by the curvature of the air-water surface and this in turn is determined by the moisture content. Due to the complexity of this surface configuration, however, we are forced for the present to rely upon experiment for this functional relation. The experimental data of Briggs[1] on the moisture content as a function of the reciprocal of the "centrifugal" force to which the moist soil is subjected leads to an equation which holds over a wide range of moisture contents of the form,

$$p = \frac{c}{\rho} + b. \tag{2}$$

This involves the assumption that the capillary pressure is proportional to the equilibrium "centrifugal" force. Thomas's[2] vapor pressure measurements for varying moisture content lead also to an equation of the form,

$$\pi = \pi_0 - \frac{c_1}{\rho}, \tag{3}$$

where π is the vapor pressure and c_1 is a small constant, this equation representing the experimental facts over a considerable range of moisture contents.

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[1] U. S. Dept. of Agr. Bur. of Soils, Bul. 45.

[2] Soil Science, Vol. XI., No. 6, June, 1921.

By a familiar method it may be shown that the vapor pressure and the capillary pressure are related as follows:

$$p = \pi - nRTln\frac{\pi}{\pi_0} \tag{4}$$

and a simple algebraic substitution leads to a converging series which, for finite values of π differing only slightly from the average vapor pressure π_0 of water, gives a relation also of the form,

$$p = \frac{c_2}{\rho} + b, \tag{5}$$

from which,

$$\nabla p = \frac{c_2}{\rho}\nabla\rho. \tag{5'}$$

Combining (1) and (5′), we obtain,

$$v = K_2\frac{\nabla\rho}{\rho}. \tag{6}$$

And combining equation (6) with the equation of continuity,

$$\frac{\partial\rho}{\partial t} = -K_3\nabla(\rho v) \tag{7}$$

we obtain,

$$\frac{\partial\rho}{\partial t} = -K_4\nabla^2\rho. \tag{8}$$

From (8), we may obtain a solution of the form,

$$\nabla\rho = K_5l^{-\alpha x}l^{-\beta t}. \tag{9}$$

From equation (9), equation (6) may be reduced, for one-dimensional flow, to the following form,

$$v_l = \frac{dl}{dt} = \frac{K_2l^{-\alpha x}l^{-\beta t}}{\rho_l} \tag{10}$$

where l is the coördinate of the water front, α and β are constants. If α (which is a measure of the moisture gradient) is small, ρ_l and the factor $K_2l^{-\alpha x}$ remain practically constant, giving an equation of the form,

$$\frac{dl}{dt} = K_6l^{-\beta t}, \tag{11}$$

which becomes when integrated

$$l = K_6(1 - e^{-\beta t}) \tag{12}$$

and the approximate value of V

$$V = K_7(1 - e^{-\beta t}), \tag{13}$$

which is in qualitative conformity with the experimental data of Cude and Hulett as quoted by Washburn. Equation (8) and a somewhat more general equation taking account of gravitational and hydrostatic potentials have been solved for special cases with qualitative experimental confirmation, as reported more in detail in the article noted.

It is true, of course, that these equations rest upon empirical data, together with the assumption which has been made with reference to the application of Poseuille's equation, but a considerable amount of material has been presented in the article cited indicating that they are not far from correct.

Utah Experiment Station,
Logan, Utah.

THE CRYSTALLINE STRUCTURES OF SPUTTERED AND EVAPORATED METALLIC FILMS.[1]

By H. Kahler.

Synopsis.

Crystalline structures of evaporated films of bismuth and silver, condensed on a cool surface, were determined by the X-ray powder method. The films were found to contain no pure metal in the crystalline state, but to consist of amorphous metal and of metallic compounds. Optical evidence indicates that *selenium* also condenses on a cool surface in the amorphous form, although on a heated surface crystals are deposited whose size and color depend on the temperature.

Crystalline structures of sputtered films of bismuth, cobalt, gold, platinum and silver, as determined by X-ray analysis, were in each case the same as that of the unsputtered metal. The crystal grains had a random distribution of axial direction. Evidently in sputtering crystalline grains are knocked off the cathode and are transported and deposited grain by grain, whereas evaporation takes place atom by atom.

Relative sputtering of hexagonal and cubic forms of cobalt. The hexagonal form is easily sputtered whereas the other form is not.

Crystalline structure of bismuth, determined by X-ray analysis of the powdered metal and sputtered film, is rhombohedral trigonal, with elementary triangular prism of side 4.54 Å and height 5.91 Å.

Introduction.

The optical and electrical constants of thin metallic sputtered and evaporated films have been the subject of investigation for many workers since the pioneer experiments of Plücker. The crystallographic study of such films, however, has never been undertaken. As a result of the conductivity experiments[2] by Professor Richtmyer, Mr. J. A. Becker and Mr. L. F. Curtiss on sputtered films of bismuth it was questioned by the latter whether their films were crystalline. It was in part to answer this question that the writer's experiments were originally undertaken.

Method.

The x-ray powder method as developed in Germany by Debye and Scherrer[3] and in America by Hull[4] was used in this study. As a source of Rontgen radiation a small General Electric molybdenum-target x-ray tube which normally operates on two milliamperes current and 30 kilovolts potential was used. The photographic films consisted of two thicknesses of duplitized material which were clamped half way around two rigidly fastened metal rings turned on a lathe to an exact circle of 17 cm. diameter. One ring was supported about two centimeters vertically above the other by means of iron straps. The powdered material to be examined was in the form of a flat sheet and was put at the center of the circular films. A filter of zirconium oxide of thickness four tenths of a millimeter was inserted in the path of the x-rays between the tube and the lead slits used in defining the beam. The slits were each about a half millimeter wide. The slits, powder and circular photographic films were enclosed in a heavy lead box to screen off stray radiations.

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[1] An abstract of this paper was read before the Chicago section of the American Physical Society, Dec., 1920. Phys. Rev., 17, p. 230, 1921.

[2] J. Becker and L. Curtiss, Phys. Rev., 15, p. 457, 1920. F. Richtmyer and L. Curtiss, Phys. Rev., 15, p. 465, 1920.

[3] P. Debye and P. Scherrer, Phys. Zeits., 17, p. 277, 1916; 18, p. 291, 1917.

[4] A. Hull, Phys. Rev., 10, p. 661, 1917.

Intensifying screens were discarded after a few tests as they were found of little value for a tube of low power output.

Bijl[1] has pointed out that under some conditions a large correction must be applied to the calculation of the planar spacings. This correction was avoided in the present work by using a layer of crystalline powder of such small thickness that it was easily penetrated by the x-rays.

Structure of Bismuth Metal.

Filings from a rod of bismuth were sifted through a screen 100 mesh to the centimeter, mixed with a small amount of soft wax, pressed into a flat sheet and put at the center of the circular photographic films. Exposures of 150 milliampere hours were made.

Knowing the crystal system and axial ratio from the standard crystallographic works, tables of planar spacings could be computed. It was found that the lattice may be considered as being made up of three interpenetrating sets of unit equilateral-triangular prisms, the atoms of the second set being raised one third the prism height vertically above the first and placed so that their vertical projections center alternate triangles of the basal plane. The third set are raised two thirds of the prism height above the first set, and center the remaining alternate triangles. The experimental values were in agreement with the theoretical values for the rhombohedral trigonal structure with elementary triangular prism of side 4.54 A and height 5.91 A with an axial ratio of $C = 1.303$.

Beyond the first eight lines the intensities were too faint to place with certainty the weaker experimental lines.

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[1] A. Bijl and N. Kolkmeyer, Proc. Amst. Acad., 21, 1, p. 495, 1918–1919.

TABLE I.

Indices.	Theor. Spacing.	Exp. Spacing.	Intens.	No. of Planes.
$10\bar{1}1$	3.27	3.28	V. Strong	3
$10\bar{1}2$	2.36	2.38	V. Strong	3
$11\bar{2}0$	2.27	2.25	V. Strong	3
0001	1.97	1.96	V. Weak	1
$20\bar{2}1$	1.84	1.83	Weak	3
$10\bar{1}1$	1.64	1.64	Medium	3
$11\bar{2}3$	1.49	1.50	Strong	6
$21\bar{3}1$	1.44	1.44	Strong	6
$10\bar{1}4$	1.38	——	——	3
$12\bar{3}2$	1.33	1.33	Strong	6
$10\bar{1}0$	1.31	——	——	3
$10\bar{1}2$	1.18	——	——	3
$10\bar{1}5$	1.14	1.15	V. Weak	3

For a check the number of atoms for the three interpenetrating unit prisms, in this case 1.5 may be computed;[1]

$$n = \frac{52.8 \times 9.7}{1.663 \times 208} = 1.49.$$

SPUTTERED BISMUTH.

Pure cast bismuth was used as a cathode in the sputtering jar. The current used was about one milliampere, the jar being in series with a kenetron rectifying-tube. The time of sputtering was about four hours. The bismuth was sputtered on several thin cover glass slides. When these slides with their metallic films, were superposed and put at the center of the photographic films the result was striking. The positions and intensities of the lines were nearly identical with those obtained with the bismuth rod filings, the only difference being a greater fogging of the lines. This fogging and lack of definiteness is due to the presence of small amounts of oxides of bismuth and to amorphous material.

The conclusion to be drawn from this comparison is that sputtered bismuth is crystalline, the crystal axes being turned in all possible directions, and the structure being the same as the ordinary metal. Although compounds of bismuth with the gases in the sputtering jar were undoubtedly present in small quantities, the pure metal was so predominant as to mask the spectra of the impurities.

EVAPORATED BISMUTH.

A piece of bismuth was put in the bottom of a long pyrex tube and the tube evacuated to approximately the same pressure as was used in sputtering the bismuth. On heating the bottom of the tube till the bismuth came to a low red heat a heavy opaque film was deposited on the cooler parts of the tube. This film was examined by the method previously described. The photograph showed four widely spaced lines on a heavily fogged background indicating the absence of bismuth crystals but probably the presence of amorphous bismuth and a bismuth compound which however could not be identified.

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[1] A. Hull, PHYS. REV., 10, p. 661, 1917.

A second sample of evaporated bismuth was prepared. The film was scraped from the glass, mixed with wax as in previous experiments and pressed into a flat sheet. When this was exposed to the x-rays in the usual manner the result was the same as that described above.

This would indicate that sputtering and evaporation are essentially different processes from the crystallographic standpoint.

THE STRUCTURE OF SPUTTERED GOLD, SILVER AND PLATINUM.

In a similar manner the structures of sputtered films[1] of gold, silver and platinum were determined. In each case the sputtered film was crystalline and of the same fundamental crystal structure as the ordinary metal. The three metals have the face-centered cubic lattice with the length of the elementary cube 4.08, 4.06 and 4.02 Å respectively. The crystals are very minute and have a random distribution of axial directions.

EVAPORATED SILVER.

Several opaque layers of silver were prepared by evaporation from an electrically heated silver wire in an evacuated vessel. These films were deposited on strips of lime glass. On examination they were found to be without the typical silver structure. The photographs were taken within one day after making the silver film.

In this connection it is interesting to note that Hull has found that distilled sodium and potassium are amorphous.[2] It is obvious beforehand that rapid evaporation in vacuo to a cold surface offers a favorable opportunity for the formation of a metal in the amorphous state.

SPUTTERED COBALT.

This metal[3] occurs in two systems, the face centered cubic and the close-packed hexagonal, the former of which is made by annealing in a hydrogen furnace while the latter is made by rapid electrolysis.

The electrolytic cobalt sputtered readily and the structure of the film obtained from it was hexagonal.

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[1] The films of gold and silver were sputtered by Mr. L. Koller.

[2] A. Hull, PHYS. REV., 10, p. 688, 1917.

[3] A. Hull, Trans. Am. Inst. El. Eng., 38, p. 1189, 1919; PHYS. REV., 17, p. 571, 1921.

On the other hand, samples of cobalt consisting largely of the cubic variety did not sputter at all under the same conditions as obtained in the case of the hexagonal variety.

This indicates that rapidity of sputtering is determined in some cases by the crystal structure of the cathode. In the case of cobalt the less stable hexagonal arrangement seems to sputter better than the stable cubic variety. However as films of oxide exert very great influences upon the rate of sputtering and as the cubic variety may have been coated by more oxide than the hexagonal sample these results should be considered as preliminary only.

SPUTTERED SELENIUM.

Longden,[1] Pfund[2] and Nicholson[3] have found that metallic cathodes of gray selenium when sputtered give films of amorphous material. On continued sputtering these films become black and granular.

In the present experiments a cathode of the metallic variety when sputtered at a very low current density over a period of several hours gave a film of the same character as the cathode, namely, the gray metallic modification. When the current density was increased so that the surface of the cathode became molten the sputtered sample turned to the vitreous modification which appears red by transmitted light. The thinnest parts of the gray sputtered film did not transmit red light.

Other cathodes made of the red vitreous selenium were sputtered at several current densities and in all cases gave the red vitreous variety as the sputtered film.

Attempts made to obtain x-ray photographs of the various kinds of selenium proved futile. This may be ascribed to the fact that the characteristic fluorescent radiation predominates over the scattered beams. By suitably choosing a filter to stop the selenium characteristic radiation and prolonging the exposure by the order of ten times it may be possible to obtain suitable photographs.

OPTICAL COMPARISON OF EVAPORATED AND SPUTTERED SELENIUM.

Selenium crystals were formed by using the method of evaporation described by Brown.[4] If selenium is put in a long tube heated at one end and having a uniform temperature gradient, a great variety of crystals form along the tube. These crystals have characteristic colors and structures. In the hotter portions of the tube the crystals are largest in size and are of metallic luster, their colors being blue-black, gray and white. At the cold edge, the amorphous red variety collects. On continued heating the red kind is transformed into the gray modification.

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[1] Longden, Am. Jour. of Sci., 10, p. 55, 1900.

[2] Pfund, PHYS. REV., 28, p. 324, 1909.

[3] Nicholson, PHYS. REV., 3, p. 1, 1914.

[4] F. Brown, PHYS. REV., 4, p. 85, 1914; 5, p. 236, 1915.

It is obvious that when sputtering with a high voltage and current density the temperature will be sufficiently elevated to cause rapid evaporation simultaneously with the sputtering as selenium melts at 220° C. On the other hand at low current densities the gray metallic crystals were sputtered intact, indicating that sputtering took place without appreciable evaporation of the selenium cathode. If the selenium had evaporated, the red modification would have formed, on condensing at the cold glass slide. Evidently when selenium is sputtered, under suitable conditions, no alteration of crystal structure takes place, the crystals leaving the cathode in grains.

General Discussion of Data.

Reinders and Hamburger[1] examined with the ultramicroscope films of sputtered silver and tungsten. The grains making up the silver films were found to range from 6 μ to 8 μ in diameter while those making up the films of tungsten varied from 2 μ to 5 μ. These comparatively large grains were found to be imbedded in a thin layer of amorphous metal. When sputtered at high temperatures the particles were of larger size than when sputtered at low temperatures.

In the present work it has been shown that the sputtered films are crystalline and that the crystalline particles making up the film are arranged at random orientations with respect to each other.

The writer's data show that evaporated metallic films do not consist of the pure metal in the crystalline state but of metallic compounds and amorphous metal.

It has been shown by Langmuir[2] that metallic evaporation in a fairly high vacuum takes place atom by atom. (However it appears from the work of Reinders, Hamburger, Kaye and Ewen[3] that the low melting-point elements occasionally eject large particles during the process of sublimation.) The vapor of tungsten was found to be monatomic as long as the gas pressure in the tube was kept below a definite value. At every collision between a nitrogen molecule and a tungsten atom the compound WN_2 was formed. Other gases were found to behave in a similar manner with respect to tungsten.

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[1] W. Reinders and L. Hamburger, K. Akad. Amsterdam Proc., 19, p. 958, 1917.

[2] I. Langmuir, Jour. Am. Ch. Soc., 35, p. 944, 1913; 37, p. 1139, 1915.

[3] Kaye and Ewen, Proc. Roy. Soc., 89, p. 58, 1913.

Using the ultra-microscope, Reinders and Hamburger[1] found that the high melting-point elements W, C, Mo, Pt, Ni and Fe gave evaporated films that were mainly "amorphous." The low-melting metals Au, Ag, Cu and magnesium show a tendency toward a coarser condensation and seemed to form a connected network of ultra-microscopic particles.

The writer's spectrograms of the evaporated films show that the vapor in transit to the film has combined with the gases present in the tube and that if pure metal is present it exists only in the amorphous state. Indeed the conditions are favorable during evaporation in a high vacuum for the formation of an amorphous metal.

Maurain[2] has shown from magnetic deflection tests that the cathodic particles while charged electrically have the ratio e/m of such a value as to indicate that the particles in transit consist of large groups of atoms.

Lack of fluorescence when these particles impinge upon a suitable material is a further indication that the sputtered particles are large and slow moving.

In the present tests the relatively pure condition of the sputtered films indicates that the number of collisions per atom with gas molecules in transit has been small, showing that the particles in transit consist of a large number of atoms.

Concerning the origin of the sputtered particles there has been much difference of opinion in the past. Some have maintained that sputtering is an evaporation phenomenon and others have held that the particles were bodily knocked off from the cathode by the impinging gas ion. The evidence here presented seems to prove that the particles initially leave the cathode in granular crystals rather than by mon-atomic evaporation. In the latter case it would have to be assumed that the large particles in transit had been formed by condensation of the vapor near the surface of the cathode. This appears very improbable as the gas pressure used in sputtering is lower than that for which condensation of metallic vapor occurs.

Worthing and Baker[3] have recently shown that the rate of metallic evaporation is not affected by electrostatic fields of less than 800,000 volts per cm. Hence the comparatively low field strengths used in sputtering would exert no influence on the rate of evaporation or size of particles. This eliminates the hypothesis of large particles being thrown off by evaporation under the influence of an electric field such as that present in sputtering.

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[1] W. Reinders and L. Hamburger, K. Akad. Amsterdam, Proc., 19, p. 958, 1917; 20, p. 1135, 1918.

[2] C. Maurain, Compt. Rend., 161, p. 1223, 1905; 162, p. 870, 1906.

[3] A. Worthing and W. Baker, PHYS. REV., 17, p. 239, 1921. A. Worthing, PHYS. REV., 17, p. 418, 1921.

In conclusion the writer wishes to thank Mr. L. F. Curtiss for preparing many of the sputtered films and Professor Ernest Merritt for his kindly interest in the work.

Cornell University,
April, 1921.

Note Added August 16, 1921.

After this paper had gone to press articles appeared on the structure of bismuth by Ogg, Phil. Mag., 42, p. 163, 1921, and James, Phil. Mag., 42, p. 233, 1921. They report a more detailed study of bismuth using a large crystal and the original Bragg method, which of course would be impossible with thin films. The dimensions given by Ogg and James agree with those here given when expressed in the same terminology. Instead of the simple rhombohedral lattice as I have stated the structure of bismuth to be they have found it to be best represented by a slightly distorted rhombohedral lattice resolvable into two face centered interpenetrating rhombohedral lattices.

ON THE EMISSION AND ABSORPTION WAVE-LENGTHS OF THE CHARACTERISTIC RADIATION IN THE L-SERIES.

BY D. COSTER.

SYNOPSIS.

Wave-lengths of the lines L_{β_5}, L_{γ_2} and L_{γ_4} of the X-ray spectra of W, Pt, Au, Tl, Pb, Bi, Th and Ur have been measured by an improved photographic method developed by Siegbahn, for comparison with the corresponding absorption lines L_{a_1}, L_{a_2} and L_{a_3} measured by Duane and Patterson.

Test of Stokes' law as applied to X-radiation.—The conclusion of Duane and Patterson that contrary to Stokes' law the absorption wave-lengths are in some cases longer than the corresponding critical emission wave-lengths is found not to be true for any of the elements from Pt to Ur. In the case of W there is a discrepancy, but this may be attributed to the effect of the characteristic radiation from the W target used.

In two very interesting papers Duane and Patterson[1] give the results of their measurements of the three critical absorption wave-lengths of the elements tungsten-uranium. From their measurements they conclude that these critical wave-lengths are not always shorter than the emission wave-lengths, which belong to the same part of the series, and that consequently Stokes' law breaks down in the x-ray series. Now this conclusion seems to be contradictory to many views in modern physics and especially to Bohr's theory. It therefore seems to me very desirable to examine thoroughly the experimental evidence, before drawing conclusions from these experiments.

In Prof. M. Siegbahn's laboratory, I have determined[2] the wave-lengths of most of the L-lines of the elements tungsten-uranium (the wave-lengths for tungsten have been determined by Prof. Siegbahn himself) by the photographic method, which has recently been improved in its essentials by Prof. Siegbahn.[3] As is known, the lines of the L-series are divided into three groups, which are related respectively to the three absorption bands, denoted by Duane as La_1, La_2 and La_3. The shortest lines belonging respectively to these three groups are β_5, γ_2 and γ_4. Unfortunately these lines are rather faint and they could not be measured with the same accuracy as the stronger lines α_1, α_2, β_1, β_2 and γ_1. The accuracy is sufficient, however, to allow of some provisional conclusions. In the following table I have combined the values obtained by Duane

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[1] Proc. Nat. Ac. Sci. Washington, Sept., 1920.

[2] Zeitschrift für Physik, 4, p. 178, 1921.

[3] Phil. Mag., vol. 37, p. 601, 1919.

El.	Wave-lengths in 10^{-11} cm.					
	$L\alpha_1$	$L\beta_5$	$L\alpha_2$	$L\gamma_2$	$L\alpha_3$	$L\gamma_4$
W	1213.6±0.1	1203.1	1072.6±0.5	1065.84	1024 ±3	1026.47
Pt	1070.5±0.3	1070.1±0.4	932.1±0.3	931.7±0.4	888.5±0.9	895.0±0.4
Au	1038.3±0.3	1038.2±0.4	899.3±0.3	901.2±0.4	860.6±0.8	866.3±0.4
Tl	977.6±0.3	978.3±0.4	841.5±0.3		805.5±1.4	810.0±0.4
Pb	949.7±0.3	949.5±0.2	813.3±0.3	813.7±0.2	780.3±0.9	783.6±0.4
Bi	921.6±0.3	922.3±0.4	787.2±0.3	787.4±0.4	753.2±0.9	761. ±2
Th	759.6±0.3	762.6±0.2	628.6±0.3	630.1±0.4	604.4±0.7	
Ur	721.4±0.3	724.1±0.4	591.8±0.3	592.6±0.4	568.5±0.7	

and Patterson for $L\alpha_1$, $L\alpha_2$ and $L\alpha_3$ and my values for β_5, γ_2 and γ_4, in so far as the same elements were measured.

From this table it appears that, for the elements Pt, Au, Tl, Pb and Bi: $L\alpha_1$ and $L\beta_5$ and also $L\alpha_2$ and $L\gamma_2$ coincide within the limit of possible error. For the elements Th and Ur the $L\alpha_1$ and $L\alpha_2$ wave-lengths seem to be slightly shorter then $L\beta_5$ and $L\gamma_2$. In the case of $L\alpha_3$, the wave-length is considerably shorter than $L\gamma_4$ for all the elements. There is consequently perfect agreement with Stokes' law.

This close agreement strongly supports our confidence, both in the ionization-method, which has been greatly improved by Prof. Duane and in the photographic method used in the laboratory of Prof. Siegbahn.

For the element tungsten however there is a remarkable discrepancy, which lies wholly outside the limit of possible error. Now it seems to me, that in the measurements of the tungsten absorption-bands there is a complicating feature which is not found in the measurements of the other elements. The bands were measured with a Coolidge-tube with a tungsten anti-cathode and a tungsten hot-wire, which gives not only a heterogeneous radiation, but also a very strong characteristic L-radiation of tungsten. Now it is known, that the heterogeneous radiation of an anticathode in the region of its own characteristic radiation is of considerably less intensity for both the long-wave and the short-wave end of the spectrum than it would be, if under the same conditions another anticathode-material were used. To determine the L-bands of tungsten in a manner quite free from criticism, it would perhaps be better to make use of an anticathode-material other than tungsten. Moreover the tungsten hot-wire should be avoided, as by the tungsten sputtering, the anticathode would be coated with a thin sheet of tungsten. It therefore seems desirable to employ a tube of the old type for the measurements in question.

LUND, April, 1921.
PHYSICAL LABORATORY OF THE UNIVERSITY.

PHYSICAL PROPERTIES OF THIN METALLIC FILMS.[1]

III. Some Factors Affecting the Resistance of Sputtered Platinum Films.

By Lewis R. Koller.

Synopsis.

Resistance of sputtered platinum films; *aging in vacuo and in various gases*.—The rapid decrease of resistance which occurs if a film is kept in vacuo after sputtering, is doubtless due to coalescence or agglomeration of the particles making up the film. In air, oxygen and hydrogen, however, this aging practically stops, probably because the adsorption of gas around the particles prevents further coalescence until the gas is removed by reëvacuation or by chemical means. This adsorption theory was tested in various ways and was found in accord with all the facts. Nitrogen and carbon dioxide are only slightly adsorbed. The *effect of heating* is to accelerate and further the aging process but with prolonged heating the resistance reaches a minimum and then increases indefinitely. *Temperature coefficients of resistance* to 200° C. were measured for films previously aged at a higher temperature and were found to depend, like the resistance itself, more on the physical condition, as determined by heat treatment, than on the thickness of the film. The highest coefficient measured was 0.002 which is about 4/7 of that of platinum in bulk. Very thin films have negative coefficients.

Activation of platinum by sputtering.—Both the sputtered film and the cathode used in sputtering cause hydrogen and oxygen to combine at room temperature so rapidly that the film and the cathode may become very hot.

Historical Summary.

ONE of the most noticeable properties of thin metal films, deposited by cathode sputtering, is that they undergo a change in resistance with time. For some metals this change is an increase, for others it is a decrease, while some pass through a minimum. In general the change is accelerated by heat. This process of change in resistance with time has been called aging.

These facts have been observed by several investigators. Kohlschutter and Noll[2] found that silver films at first decrease in resistance with time after sputtering and then increase, the manner of this change depending on the nature of the film as determined by the gas in which it was sputtered and on the heat treatment which it received after sputtering. Patterson[3] found that bismuth films increase in resistance after sputtering, and that platinum films decrease in resistance on heating. Longden's[1] results were similar to those of Patterson. Miss Hobbs,[2] also working with platinum, found that the films decreased in resistance when allowed to age in vacuo, but rose in resistance when air was admitted to the sputtering jar.

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[1] The writer is pleased to acknowledge assistance from a grant from the Rumford Fund to Prof. F. K. Richtmyer.

[2] Kohlschutter and Noll, Zeit. fur Elektrochemie, 18, 419, 1912.

[3] Patterson, Phil. Mag., Series 6, *4*, 652, 1902.

It was the purpose of this investigation to study the aging of sputtered films, particularly with reference to the effect on aging of adsorbed gases. Unless otherwise specified the films studied were of platinum.

DESCRIPTION OF APPARATUS AND PROCEDURE.

The sputtering apparatus was of the type usually used for this purpose and needs no special description. The high vacua when required were obtained with a Langmuir pump. Pressures were measured by means of a McLeod gauge placed on the high vacuum side of the Langmuir pump. No attempt was made to keep mercury vapor out of the sputtering jars. These were small bell jars provided with leads sealed through the glass, so that the resistance of the films could be measured in situ. Resistances were measured on a Wheatstone Bridge.

The films were sputtered on small strips of glass prepared in a manner similar to that described by Richtmyer and Curtiss,[3] and were about 11 mm. long by 13 mm. wide. The films were deposited at a pressure in the neighborhood of 0.03 mm. of mercury. In order to have fairly constant pressure conditions during deposition a discharge was first passed through the jar for about an hour. During this time the blank was protected by an aluminum shield of the shape shown in Fig. 1, which was mounted on a pivot in a horizontal position. This shield carried several small pieces of iron on its upper surface so that, when the jar was sufficiently freed of occluded gases, it could be swung around from without, by means of a magnet, and the opening placed over the blank. The distance between the blank and the cathode was from 1.5 to 3 cm. The length of the cathode dark space under the conditions of operation was about 1 cm. The necessary voltage was obtained from a transformer operating on the 110 volt circuit and was in the neighborhood of 10,000 volts. In order to avoid excessive heating of the jar and of the film during deposition, an interrupter was placed in the primary of the transformer circuit which opened the circuit for about 5 seconds out of every 6. The stopcocks in the system were

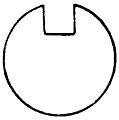
Fig. 1.

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[1] Longden, PHYS. REV., 11, 40, 1900.

[2] Hobbs, Phil. Mag., 32, 141, 1916.

[3] PHYS. REV., 15, 465, 1920.

greased with a special grease having a low vapor pressure, prepared as described by Booth.[1]

Normal Aging in Vacuo.

If a sputtered film is allowed to age in the air of the laboratory its resistance will decrease rather slowly, and perhaps somewhat irregularly, for a period of many weeks or even months before it settles to a nearly constant value. If the film is allowed to age in vacuo (that is if the pressure is kept below 0.005 mm. by running the pumps) the time resistance curve will be of the form shown by P. 6 Fig. (2). The initial reading here was taken immediately after stopping sputtering, and the pumps were kept running during the entire course of the observations. The initial value of the resistance was about 1,400 ohms and it dropped to a final value of approximately one third of this. This curve is typical in general form for both thicker and thinner films. In all cases the resistance decreases very rapidly at first and then gradually more and more slowly until finally after about 12 hours the rate of decrease is almost negligible. In the case of thinner films the total drop is relatively greater and of thicker ones relatively less for reasons which will be discussed later. The general form of the curve, however, is the same for all films.

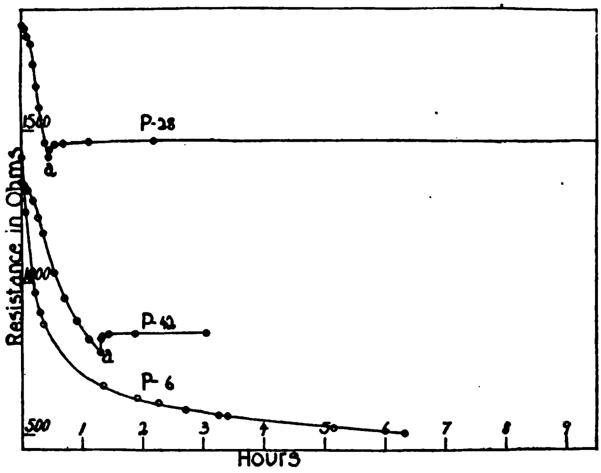

Fig. 2.

Evidently what is taking place here is a process of settling down and of agglomeration, or coalescence or sintering, the last three terms being more or less synonymous. In view of the lack of exact knowledge of the process coalescence is probably the best term to apply. This process has been studied for many substances both crystalline and amorphous. Bancroft[1] cites the cases of the sintering of tungsten and tantalum powders at temperatures far below the melting point and comes to the general conclusion that "liquid drops or solid grains will coalesce or agglomerate if brought into actual contact." Since we know from the work of Mr. Kahler[2] done in this laboratory that sputtered films are made up of small crystals, it is easy to form a picture of the mechanism by which the resistance is changed. The resistance of a film having this granular structure must consist of two parts; first, a purely metallic resistance, and secondly, a part due to the contacts between the particles. The first part of the resistance is not changed by aging, but the second, namely that due to the contacts, must be diminished, since as the small particles coalesce to form larger ones some of the contacts must disappear. This process, of course, can only result in a lowering of resistance when the film contains sufficient metal so that the coalescence may proceed without actually causing breaks. For this coalescence to take place readily the particles must actually come *into contact*, which can only take place in vacuo.

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[1] H. S. Booth, Thesis, Cornell University, 1919.

EFFECT OF ADSORPTION OF AIR.

If, instead of allowing the process to run to an end in vacuo, air is admitted to the jar a different result is obtained, as is shown by curves P. 28 and P. 42 in Fig. 2, where air at atmospheric pressure was admitted at the points marked *a*. The result is the same whether air is admitted immediately after sputtering or several hours later. Upon the admission of the air two things take place; first, a slight rise in the resistance of the film, and second, a nearly complete stoppage of the aging. This is due to the fact that each particle becomes surrounded with an adsorbed air film which tends to prevent further contact and coalescence. This prevention of coalescence by an adsorbed air film is merely an illustration of what takes place in every colloidal suspension; namely, the suspended material is kept from precipitating out by an adsorbed film either of a peptizing agent added for this purpose, or by an adsorbed film of the dispersion medium. In this case the disperse phase (platinum) is peptized by an adsorbed film of the dispersion medium (air). The rise in resistance is evidently due to a rearrangement of the little particles, most likely with a widening of the gaps between them as the air film is adsorbed.

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[1] Bancroft, Applied Colloid Chemistry, p. 153.

[2] Kahler, PHYS. REV., 17, 230, 1921.

If this theory is correct, since true adsorption is a reversible phenomenon, it ought to be possible to remove the air film by reëvacuating the sputtering jar and then the aging ought to continue. This is shown by curve P. 22 of Fig. 3. Air admitted at the point marked *a* had brought the aging to a stop. The pumps were started operating at the point marked *p*. The resistance began to fall, at first gradually and then more rapidly, until it finally reached almost the same rate of fall as before the admission of the air. At *a'* the air was again admitted and the aging stopped, and at *p'* the pumps were started with the same results as before. Similarly at *a''* and *p''*. Curve P. 11 in the figure is plotted for comparison and shows the aging in vacuo of a film having about the same initial resistance. It will be noticed that the final values of the resistance reached are of the same order, and that the corresponding portions of the aging curves are of very much the same form. The only effect of the air has been to halt the aging temporarily without producing any permanent change in the film.

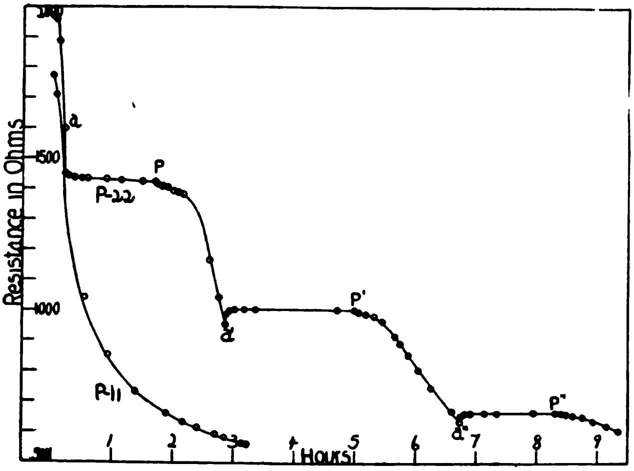

Fig. 3.

A closer study of some of the portions of the aging curves gives additional evidence of the correctness of this adsorption theory. First of all consider the rise in resistance when air is admitted to the sputtering jar. The greater part of this rise takes place in the first two or three minutes and after that it continues at a gradually decreasing rate for several hours. (This behavior is characteristic of all adsorption phenomena, where the equilibrium is nearly reached in the first few minutes, but is not complete until after a lapse of several hours.) Later a slight gradual fall in resistance may take place. Although the film may continue to age thus at a slow rate for a long time it shows no signs (over a period of several months) of dropping to the value which it would have reached if it had been aged in vacuo. The time required to reach the maximum depends on the resultant of the rate of aging and the rate of rise due to the gas.

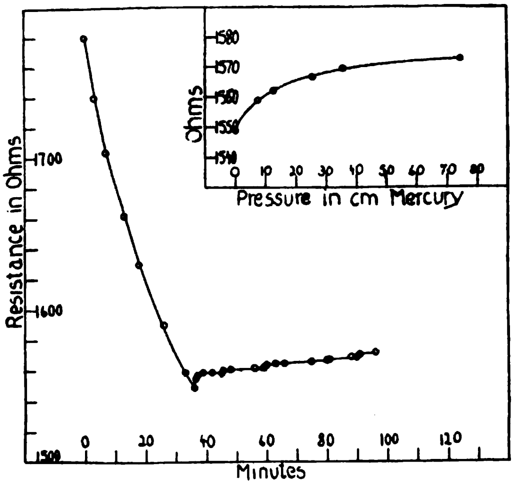

Fig. 4.

If this rise is due to adsorption of gas by the film it should be a function of pressure. This was actually found to be the case as shown in Fig. 4. In this case instead of admitting air at atmospheric pressure at once it was admitted in several instalments, allowing time in each case for equilibrium to be established. According to the theory of adsorption, at each pressure a definite amount of gas must be adsorbed, corresponding to which there is a definite value of the resistance of the film. A further increase in pressure will result in more gas being adsorbed and a further rise in resistance. When the final values of resistance are plotted against pressure a smooth curve is obtained, similar in general form to the adsorption isotherm.

On reversing the process and pumping out the air the exact same curve is not reproduced. The resistance drops slowly with decreasing pressure until in the neighborhood of 0.020 mm., below which it falls off very rapidly. Apparently once the aging has been stopped only a very small amount of adsorbed gas is required to prevent it from continuing, whereas it takes a much larger amount of gas to bring it actually to a stop. This is shown by a closer study of the portions of the curve marked p in Fig. 3. This has been plotted to a larger scale as curve 1 in

Fig. 5. If the values of resistance are plotted against pressure instead of time the result is shown by curve 2 in the same figure. This shows the gradual decrease in resistance till in the neighborhood of 0.020 mm. and the more rapid rate of fall as the pressure falls below this. This also explains the apparent lag observed at the beginning of many of the aging curves. This is not really a time lag of the films, but merely represents the time required for the pumps to bring the pressure from the sputtering value down below the critical value.

The pressure at which the films are sputtered is well above the critical value. This was shown by closing the stopcock leading to the sputtering jar immediately after sputtering, thus keeping the film at the pressure at which it was sputtered. When this is done the aging is very slight and the resistance time curve is nearly flat. As soon, however, as the stopcock is opened and the pumps are started the adsorbed air is given off and the resistance drops very rapidly.

Effect of Adsorption of Gases other than Air.

Some rough qualitative experiments were undertaken to determine the behavior of the films towards gases other than air. Since adsorption is a specific phenomenon the behavior of the films ought to vary with the nature of the gas, which was found to be the case.

One factor which influences the resistance of the films greatly is that the process of sputtering activates the platinum so that both the sputtered film and the cathode from which it was sputtered cause hydrogen and oxygen to combine at room temperature. This reaction takes place very rapidly; both the cathode and the thick deposits on the walls of the jar becoming quite hot, while water vapor is condensed on the cooler parts of the jar. The platinum deposits on the edges of the mica backing of the cathode are heated to incandescence and cause a mixture of hydrogen and oxygen to explode. This property of platinum is well known but so far as the writer is aware this method of activating platinum by sputtering is new. In this connection it would be interesting to study the catalytic properties of other sputtered metals; and also to see whether the substances which are known to poison catalysts have any effect on sputtering, and whether substances which are known to prevent sputtering have any effect on catalysts.

Instead of admitting air to the sputtering jar, as already described, its chief constituents were tried separately. Nitrogen in small quantities (pressure 2–3 cm.) hardly affects the aging appreciably. At a pressure of one atmosphere it slows the aging down very much but does not bring it absolutely to a standstill. Even a small amount of oxygen is effective in stopping the aging and giving the characteristic rise which has already been described; while increasing the concentration (pressure) of the oxygen gives further rises in resistance. This shows that the active constituent of the air is the oxygen, which is strongly adsorbed, while the nitrogen is only taken up to a much lesser extent.

Carbon dioxide behaves in much the same manner as nitrogen.

If hydrogen is admitted, after oxygen has been admitted and pumped out again, the resistance of the film will drop several hundred ohms almost instantaneously, and will continue to fall at a gradually decreasing rate for some time. This is due to two causes. One is the heat developed by the reaction of the hydrogen with the oxygen previously adsorbed by the film, which accelerates the aging; and the other is due to the removal of the oxygen from the surface of the particles by combination with the hydrogen, which has the same effect as pumping. An increase in the hydrogen pressure will result in a further drop in resistance because the velocity of the reaction between the hydrogen and oxygen is a function of the pressure. Hydrogen has the same effect even if oxygen has not been previously admitted because the film contains oxygen which was adsorbed during the process of sputtering. When the hydrogen oxygen reaction has run to an end the aging is gradually brought nearly to a stop by the adsorption of hydrogen.

If the hydrogen is nearly completely removed by pumping and oxygen is admitted the characteristic rise is observed. This is because the oxygen is much more strongly adsorbed than the hydrogen, and the rise due to the adsorption of oxygen masks the heating effect of the reaction with the very slight amount of hydrogen in the film. If now the oxygen is removed by pumping and hydrogen is readmitted another sudden drop will take place due to the heating when the hydrogen combines with the additional oxygen which has been taken up by the film. Miss Hobbs[1] observed that when a film has been deposited in hydrogen the effect of admitting hydrogen is a rise in resistance, while in the case of a film which has been deposited in air the hydrogen may produce a fall in resistance which seems to substantiate the writer's theory.

Illuminating gas has the same effect as hydrogen (to a slightly lesser extent) probably because it contains a large percentage of hydrogen.

It was found that at times the admission of air caused a sudden drop in resistance instead of the usual rise. This effect is shown at *a* in Fig. 3. This is due to the presence of traces of illuminating gas in the air of the laboratory.

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[1] *Loc. cit.*

Temperature Coefficients.

Another very interesting property of sputtered films is that their temperature coefficients of resistance are smaller than the temperature coefficients of the same metal in bulk. Patterson[1] made some measurements of the temperature coefficients of platinum films and found an apparent relation between the temperature coefficient and the thickness. Swann[2] also made measurements of temperature coefficients on the basis of which he formulated a theory of the mechanism of conduction in thin films. According to Swann's theory the films owe their properties to their granular structure and not to their thickness. Longden[3] measured the temperature coefficients of very thin films and found them negative. Thicker films had positive coefficients, and some intermediate films had zero coefficients. Longden attempted to show from his work that there was a linear relation between thickness and temperature coefficient.

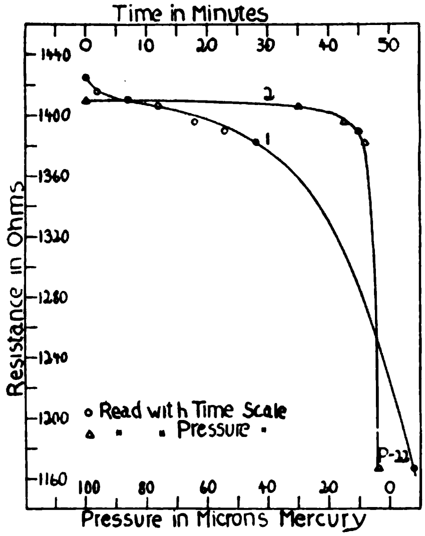

Fig. 5.

In this part of the work the resistances were measured as before on a Wheatstone Bridge. The films were in a test tube placed in an oil bath, which was electrically heated and constantly stirred. Temperatures were read on a thermometer immersed in the oil. The measurements covered the range from room temperature to 200° C.

The first time a film is heated its resistance may show an initial rise or it may begin to fall at once, but in all cases on further heating the resistance falls quite rapidly. This initial difference in behavior is probably due to the fact that the films were of different ages and had reached different stages in the aging process. The rapid drop in resistance when plotted against temperature gives a smooth curve which is rather misleading. This change in resistance is really a time effect and since the temperature of the bath is also a function of the time the resistance and temperature seem to be related. While the film is being heated two changes in resistance are taking place simultaneously. One is a small rise due to the temperature coefficient. The other is a large decrease due to the more rapid aging at the higher temperature. The higher the temperature is raised above room temperature the greater does the latter become, and until it has proceeded nearly to an end it completely masks the temperature coefficient effect. Thus with each heating the resistance of the film is permanently lowered. After several

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[1] *Loc. cit.*

[2] Swann, Phil. Mag., 28, 467, 1914.

[3] *Loc. cit.*

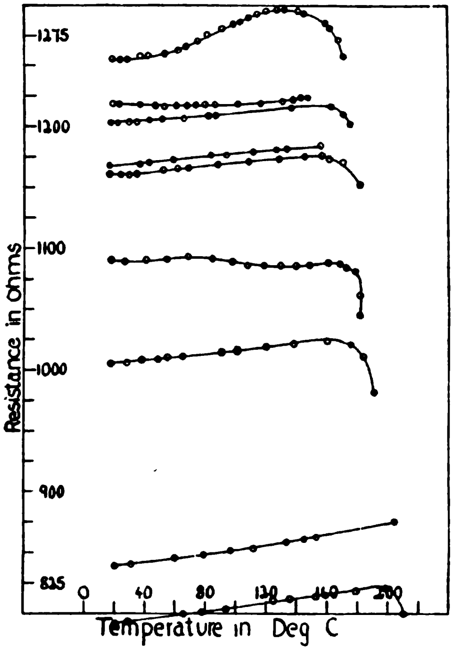

Fig. 6.

such heatings, when the resistance has nearly reached its minimum value the temperature coefficient becomes perceptible, and the heating and cooling curves become nearly straight lines and can be duplicated, provided the temperature is kept well below the maximum temperature of the previous run. Such a series of curves is shown in Fig. 6.

As the initial resistance of the film becomes lower with each run the temperature coefficient becomes greater but the slope of the lines changes very little. This agrees well with the suggestion made before as to the mechanism of aging; the effect of heating is to increase the resistance of the metallic portion of the film due to the true temperature coefficient effect, and to decrease the resistance of the contacts due to eliminating many of them as the particles coalesce. Until the film is well aged the latter effect is by far the greater. As long as the temperature of a film is kept below a higher temperature to which it has previously been raised the aging is small and so the chief change is in the true metallic resistance, and this rise must remain nearly the same for all stages of the aging.

The temperature coefficient then becomes a function of the degree of aging which in turn depends on the temperature to which a film has been heated. If, for example, a film is placed in the oil bath at a temperature of 100° the resistance will fall rapidly at first but after some hours will settle to a nearly constant value. The film will now be quite permanent for all temperatures below 100° and the temperature resistance curve will be nearly a straight line, as long as the temperature is kept below this value. On heating to say 200° aging proceeds further with a still further lowering in the resistance of the film until it finally reaches a constant value again. Again after this treatment the temperature resistance curve will be a straight line, while the temperature coefficient will have been nearly doubled. Patterson assumed that his films were completely aged by heating them for 60 hours at 100°, but the present work shows that heating to still higher temperature would have reduced the resistance still more and multiplied his temperature coefficients by as much as 4. After aging at the highest obtainable temperature of the oil bath (210°) the films were heated in an oven of which the temperature was estimated to be 500° C. This resulted in a still further lowering

Table I.

Film No..	Initial Resistance.	Temperature Coefficient.	Patterson's Temp. Coef.
85	1570	.000888	.00011 to .00033 approx.
77	1444	.000644	
72	1974	.00108	
58	1866	.00075	
73	506	.00108	
70	93	.00144	.0005 approx.
55	65	.00200	

of the resistance and raising of the temperature coefficient. In some cases the resistance became infinite due to the destruction of the gold platinum contact. In such cases a new pair of gold terminals was sputtered overlapping the old ones. This merely changed the effective length of the film. Table I. gives the initial resistances and the maximum values of the temperature coefficients obtained for some of the films, and for comparison the values for films of the same thickness obtained by Patterson. Very thin films were found to have negative temperature coefficients.

The aging due to heating even if done in air is very much greater than that due to a vacuum. The effect of heat is to accelerate the rate of aging, and furthermore heat will reduce the resistance far below the minimum value reached in vacuo at room temperature. For example, a film which had dropped to 376 ohms when aged in vacuo was heated for several hours at 110° and finally reached a value of 316 ohms. On further heating at 200° the resistance dropped to 192 ohms.

On prolonged heating at the higher temperatures the resistance of all except the very thickest films passes through a minimum, becoming very large and finally infinite. This point of minimum resistance is reached at lower temperatures the thinner the films. The explanation given by Kohlschütter and Noll for a similar phenomenon in the case of silver films probably holds good here, namely that as the coalescence proceeds it finally reaches a point where there is not sufficient metal in the film for it to take place without rupturing the film. In this condition the films must be very much like the extremely thin films in which Swann describes the distribution of particles as being similar to the condition of affairs on a pavement when rain drops begin to fall, and before enough drops have fallen to cover the pavement. The difference probably is that in the case of films which have been brought to this state by excessive heat treatment the individual particles must be larger. Since the very thin films have negative temperature coefficients it would be expected that these high resistance thick films would also have negative coefficients. These films, however, are in such an unstable condition that it is extremely difficult to make measurements with any degree of certainty. One such film was found to have a negative coefficient, but although several other films were studied this could not be confirmed.

A study of Table I. will show that the thickest films have the largest temperature coefficients and that the values decrease as the films become thinner. The maximum value is about four sevenths of the value of that for platinum in bulk. These values are higher than the ones obtained by Patterson because the aging was carried on at higher temperatures and therefore proceeded further. The fact that the temperature coefficients increase with increasing thickness is probably not due to the thickness or thinness of the film, but to the fact that the thinner films could not be aged to the same point as the thicker ones without destroying them. In other words the temperature coefficient of a film depends not on its thickness but upon its degree of agglomeration. Probably all films have the same temperature coefficients when aged to the same point.

Films deposited on quartz were found to have the same temperature coefficients as similarly treated films of the same thickness on glass.

In the table the initial resistances of the films are given, but as a matter of fact a serious error would be introduced if the thickness of films was to be determined on the basis of a comparison of initial resistances. A film having a resistance of the order of 10^6 ohms will fall on heating to 10^3 ohms, while one having an initial resistance of 1,000 ohms will fall under the same conditions to 100 ohms. The ratio of the final conductances of these two films, which is about the same as the ratio of the times of sputtering, gives a true idea of the amount of metal in them; while the ratio of the initial conductances would be entirely misleading. The reason for this is that the films age during the process of sputtering, probably partly due to the impact of the particles and mainly due to the heating. Thus a thicker film, which during the course of sputtering has been exposed to more heating has therefore aged more and so for the same amount of metal has a lower resistance.

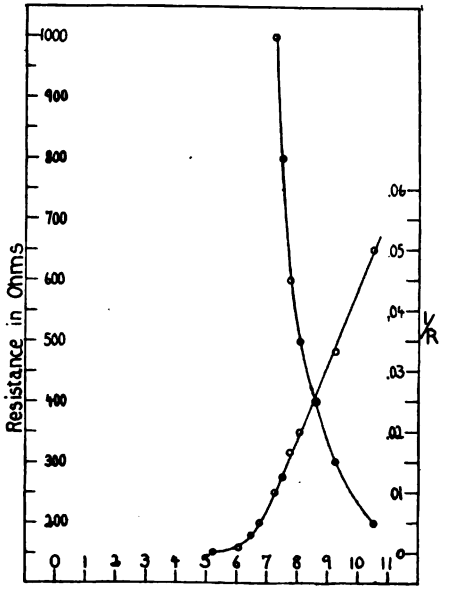

Fig. 7.

The aging due to the heat of sputtering was tested as follows. A film was sputtered in the usual way and the aging stopped by admitting air. A thin microscope cover glass was then placed directly on top of the film and the jar reëvacuated. At a pressure well above that at which aging takes place a film was sputtered on this cover glass. This resulted in a rapid drop in resistance which can only be explained by the heating effect of the sputtering.

In order to obtain some more information in regard to what takes place during sputtering measurements of resistance were made while sputtering and plotted against time as shown in Fig. 7. The reciprocals of these resistances when plotted against time give a straight line excepting during the very early stages of the deposition. If the conductivity increased by equal increments in equal times this curve should pass through the origin. The fact that it does not do so indicates that during sputtering in addition to depositing more metal the resistance of the metal already deposited is also growing less, thus giving the line a steeper slope than would be due to deposition of metal alone.

According to the theory of J. J. Thomson there should be a rapid increase in resistance below a certain critical thickness; this critical point being that at which the dimensions of the film become comparable with the mean free path of an electron. Patterson's results seem to show that the specific resistance of the films increases very rapidly with decreasing thickness below a thickness of 7×10^{-7} cm. which is in accordance with the theory. From this bend in the specific resistance thickness curve Patterson computes the mean free path of an electron. It is extremely doubtful whether the Thomson theory can be applied directly to sputtered films, the thickness of which is an indefinite quantity. In some places where the groups of crystals are piled up the thickness may be comparatively great while in others it may be very much less. While the specific resistance undoubtedly depends on the thickness the conditions in the film are such that the thickness is not uniform, and the Thomson theory which has been developed for uniform films does not necessarily apply here. The *physical condition* of the film is probably a much more important factor than the thickness.

Patterson's films were all aged at a temperature of 110°. Heating to a higher temperature would have lowered the resistance of all of them, but very likely not in the same ratio, and the thicker ones would probably have been lowered more than the thinner ones, so that possibly the shape of Patterson's specific resistance thickness curve would be altered.

Swann treats the films as "groups of molecules not pressed into intimate contact so that it is only those electrons which have more than a certain minimum velocity, depending on the closeness of packing, which can travel from one group to the next. This necessary velocity gets less as the time of deposit increases and the groups are pressed more intimately into contact by the greater molecular forces brought into play." This theory seems in accordance with the facts observed in the present work. The numerical values of Swann, however, are open to the same criticism as those of Patterson, namely that the films were insufficiently aged.

In conclusion the writer wishes to express his thanks to Professor Richtmyer who suggested the problem and whose enthusiasm and encouragement made the work possible.

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THE DISTINCTION BETWEEN INTRINSIC AND SPURIOUS CONTACT E.M.F.S AND THE QUESTION OF THE ABSORPTION OF RADIATION BY METALS IN QUANTA.

BY R. A. MILLIKAN.

SYNOPSIS.

Stopping potentials for photo-electrons from Na, K and Li surfaces, as determined with a Faraday cylinder in vacuo for monochromatic ultraviolet light, were found to be accurately the same for all. This result makes general the conclusion that the stopping potential for a given frequency is the same for all clean metals. It is not the same for an *oxidized copper surface*, however, or for any surface showing a spurious contact e.m.f.

Contact e.m.f.'s between Na, K and Li and the oxidized Cu surface of a Faraday cylinder were not found to be constant, but their changes were found to keep pace exactly with changes in the common stopping potential. These changes are attributed to the entanglement of electrons in the copper oxide surface, thus giving rise to a spurious e.m.f.

Intrinsic contact e.m.f. between any two metals is shown to be a definite quantity equal to $(h\nu_0 - h\nu_0')/e$, when ν_0 and ν_0' are the threshold frequencies for photoelectric emission from the two metals. This equation enables intrinsic to be *distinguished from spurious contact e.m.f.* For if the observed contact e.m.f. differs from this value, (which it does not do for clean metals), there is a spurious contact e.m.f. equal to this difference and also equal to the difference in stopping potentials.

The Photo-electric quantity $h\nu_0$ is shown to be equal to the thermionic work function ϕe measured at the same temperature.

Theory of photo-electric emission.—Proof is adduced for the conclusion that either the free, conduction electrons of a metal have the power of taking up the energy $h\nu$ from the incident light, or that, as Barkla has claimed, absorption does not take place in whole quanta. The difficulties associated with each conclusion are pointed out.

I. INTRODUCTION.

THE present paper is a continuation of, and a supplement to, one published in 1916, under the title "Einstein's Photoelectric Equation and Contact Electromotive-Force,"[1] in which there were pointed out certain necessary conclusions of great importance for the theory of quanta which might be drawn from a given type of photoelectric observations. At that time, however, it had not been possible to make these observations in full and report upon them was therefore deferred, the immediate object there being to obtain evidence for the then unsettled question as to the validity or invalidity of an equation of the type first suggested by Einstein.

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[1] PHYS. REV., VII., 18, 1916.

In view of the extraordinary sequence of exact determinations of h made with the aid of the Einstein relation, or its inverse suggested earlier by Bragg, a sequence which began with the author's studies in the region of ordinary wave-lengths,[1] and was continued by those of Duane, Hunt, Hull, and others, in the field of general x-radiation, then by those of Webster and others in the field of characteristic x-rays, and finally by those of McLennan, Foote, and a large group of observers in the field of ionizing potentials, we may now set down without fear of error the following equation as representing accurately the facts of observation, whatever may have been, or may now be, its theoretical basis:

$$\tfrac{1}{2}Mv^2 = (PD + K)e = h\nu - w = h\nu - h\nu_0 \tag{1}$$

in which v is the velocity with which a negative electron escapes from a metallic surface under the influence of a given frequency ν; w the work which it does in getting out of the metal; ν_0 the threshold value of the frequency, or the smallest value of ν which is capable, even with a strong accelerating field (say 20 volts), of detaching a negative electron from the metal at all; PD the stopping potential, or the positive potential which must be applied to the metal to just prevent the escape from it of a photo current, under the influence of the impressed frequency ν, and K the contact e.m.f. between the illuminated metal and the Faraday cylinder into which the liberated electron must escape if it gets away at all.

The introduction of the term K represents an improvement upon Einstein's equation, the necessity for which was first seen, clearly stated, and in large part experimentally justified by Richardson and Compton,[2] and the correctness of which was very accurately demonstrated by the author, by simultaneous observations *in vacuo* under identical conditions of contact e.m.f.s, stopping potentials and threshold frequencies (*l.c.*).

Now if we *assume* as was done in the 1916 paper, and as has in general been done by all workers in the field of the emission of electrons under the influence of ether waves, *that the photo-electrons come from the atoms of the metal*, an assumption which is apparently justified by the independence of both photo-currents and stopping potentials upon temperature, as well as by the fact that insulators have been shown by the oil-drop methods to exhibit a photo-electric behavior precisely like that of conductors,[3] then the w term in (1) must be made up of two parts w_1 and w_2, the former w_1 representing the work necessary to detach the negative electron from its parent atom, and the latter w_2 representing the work necessary to push the free electron through the surface of the metal. This last term must be precisely the same as that which Richardson has denoted by the product φe in his development of the thermionic equations.

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[1] Phys. Rev., VII., 355, 1916.

[2] Phil. Mag., 24, 592, 1912.

[3] Cf. M. I. Kelly, Phys. Rev., 16, 260.

If, now, we push two metals A and B in succession before the Faraday cylinder F (see Fig. 1), set up equation (1) with respect to each of them, and subtract, as below, we may make ourselves independent of a measurement of a contact e.m.f. between either metal and the Faraday cylinder. Thus we have

$$(PD_A + K_A)e = h\nu - h\nu_{0_A} = h\nu - (w_{1_A} + w_{2_A})$$
$$(PD_B + K_B)e = h\nu - h\nu_{0_B} = h\nu - (w_{1_B} + w_{2_B})$$

which give, by subtraction

$$(PD_A - PD_B)e + (K_A - K_B)e = (w_{1_B} - w_{1_A}) + (w_{2_B} - w_{2_A}) = h\nu_{0_B} - h\nu_{0_A}. \quad (2)$$

Now, $K_A - K_B$ is the contact e.m.f._{AB} (*i.e.*, the contact e.m.f. between the metals A and B), and *by definition* $(w_{2_B} - w_{2_A})/e$ is also the contact e.m.f._{AB}. It follows, then, at once from (2), as I pointed out in 1916, that

$$(PD_A - PD_B)e = w_{1_B} - w_{1_A} \quad (3)$$

and also that

$$\text{Contact e.m.f.}_{AB} = \left(\frac{h\nu_{0_B} - h\nu_{0_A}}{e}\right) - (PD_A - PD_B). \quad (4)$$

Equation (4) then, each term of which is capable of direct and independent measurement, must be found satisfied in all cases if Einstein's equation, as modified by Richardson, is correct, and it was subjected to searching experimental test in my former work and found to be always in agreement with observation. The term $h\nu_{0_A}$ for freshly cut lithium, for example, was measured by applying a potential of about 20 volts between the lithium and the Faraday cylinder (see Fig. 1), the inner surface of which was heavily oxidized so as to render it as light-absorbing as possible, and the wave-length of the incident light was then varied until the threshold frequency was found at $\nu_0 = 57.0 \times 10^{13}$ ($\lambda = 5263$ Å). In this experiment, the negative terminal of the 20-volt battery was of course connected to the lithium. The $h\nu_{0_B}$ for the copper oxide was then found by reversing the terminal of the battery and varying the wave-length of the light incident upon the lithium until this light, strongly reflected from the lithium surface to the inside of the cylinder, began to cause negative electrons to flow from the cylinder back to the lithium. ν_{0_B} was thus located at 118.2×10^{13}. ($\lambda = 2535$ Å.) This gave $h/e\,(\nu_{0_B} - \nu_{0_A}) = 2.53$ volts. A contact e.m.f. measurement was then made in vacuo by the Kelvin method, by turning the lithium surface through 90° to the position A' (Fig. 1) in a manner which has been previously described[1] and finding directly the potential which had to be applied to just neutralize the contact e.m.f. between it and a copper disc D, oxidized in every way as had been the copper Faraday cylinder. This measured contact e.m.f. was in the foregoing case 1.52 volts. Now, using line $\lambda = 2535$, the measured stopping potential for the lithium was found to be 1.00 volts, while for the CuO, whose threshold frequency lay at line 2535, it was of course 0. Equation (4) then gave, inserting numerical values, $1.52 = 2.53 - 1.00 = 1.53$ which is an identity within the limits of experimental error.

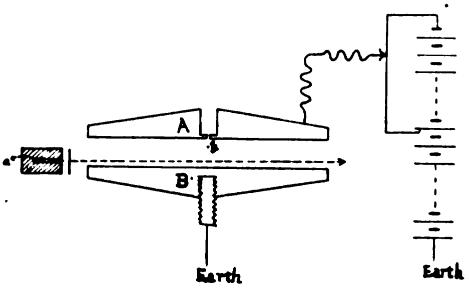

Fig. 1

I have thus given again one complete set of observations taken from the preceding work, to show how large the term $(PD_A - PD_B)$ often was, in this case a full volt, and to show how impossible it is that in these observations the contact e.m.f. can be given by the simple relation, contact e.m.f. $= h/e\,(\nu_{0_B} - \nu_{0_A})$. *I have now made many observations by the foregoing method using all three of the metals, sodium, potassium, and lithium, maintained all the time at a vacuum of* 10^{-6} *or lower and although the measured contact e.m.f.s do not in general remain at all constant over long periods of time, creeping in general towards smaller values, and showing variations in a month or two of time amounting to as much as a volt, the changes in the stopping potentials always keep pace with those in contact e.m.f. so that equation* (4) *has never yet been found to fail.* In a word, then, so far as the relations between photoelectric and contact effects are concerned, the demands of Einstein's equation are still found to be fully satisfied.

But, as pointed out in the preceding paper, there have been very strong experimental indications that a contact e.m.f. equation much more limited in scope than (4) actually holds in most cases, namely,

$$\text{Contact e.m.f.} = \frac{h}{e}(\nu_{0_B} - \nu_{0_A}) \qquad (5)$$

and the purpose of the present paper is to present the limits of validity of (5) and the reason for its failure in the above work, and also to discuss its significance both with relation to the theory of contact e.m.f.s and the theory of quanta.

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[1] Phys. Rev., 7, 362, 1916.

2. INTRINSIC AND SPURIOUS CONTACT E.M.F.S.

Equation (5) was thought by Richardson and Compton[1] to be consistent with their photoelectric work, but their contact e.m.f.s were either taken from tables or, in certain instances, measured in air, so that the point in question could not be considered as in any way settled by their measurements, since $(PD_A - PD_B)$ might well be expected to be small. The most refined and convincing way of testing it should consist in bringing different metals in succession before the same Faraday cylinder, and observing whether, with a given incident frequency, the stopping potential for the different metals is the same; equations (2) and (4) show that if $(PD_A - PD_B) = 0$ contact e.m.f. must be given by $h/e\ (\nu_{0_B} - \nu_{0_A})$. This was first done by Millikan and Winchester[2] in very early photoelectric work, designed primarily to test temperature effects. We did not use monochromatic light, however, and a number of metals were essentially in the same Faraday cylinder at one and the same time, with the consequent possibility of exerting a mutual influence upon one another's fields. The observed differences in stopping potentials, in general small, are not therefore convincing. Page[3] obtained *no* differences in stopping potentials when he used freshly scraped surfaces of copper, aluminum and zinc, and Drs. Hennings[4] and Kadesch in the Ryerson Laboratory, extended Page's conclusion to a considerable number of other metals.

In the 1916 paper, I left for further study the cause of the unquestionably different result which I had obtained in working with two surfaces one of which was an alkali metal and the other the oxide of copper.

I have now to report a considerable number of further observations, made with the same apparatus, in which all of the alkali metals, Na, K and Li, were brought in succession before the Faraday cylinder with the result that *the stopping potential was always exactly and accurately the same for them all, provided the observations were made at the same time.* It made no difference whether one of the metals had recently been shaved in vacuo and the other surface similarly treated weeks earlier, or whether they had all been shaved together. In other words, no surface effects due to residual gases modified the equality of the stopping potentials. The contact e.m.f.s taken at essentially the same time, between each of these three metals and the oxidized copper plate differed among themselves by very appreciable amounts. However, when the apparatus was allowed to stand for some days, or for weeks, the common stopping potential would be found to have changed, in one instance by more than 1 volt, but it had shifted by the same amount for all the metals, so that the equality between the stopping potentials for the different metals remained exact. Further, the measured contact e.m.f.s were all found to have shifted by precisely the same amount as the stopping potentials. *This shows clearly that the change which had taken place was due to the element which was common to all the measurements, namely the heavily oxidized surface of copper.* Further, the direction of the change was such as to be consistent with the view that it was due to the charging of the oxide negatively by the simple retention of a swarm of negative electrons by it. The oxide coating acts like a condenser of essentially infinite capacity, but the charge on this condenser varies with the time. This charging up of the copper oxide gives rise to a "spurious contact e.m.f." which was responsible for the finite value of $(PD_A - PD_B)$ in the work discussed above. It has no effect upon $(\nu_{0_B} - \nu_{0_A})$ because in the measurement of these threshold frequencies very strong accelerating potentials are externally applied. These completely neutralize any effect of a surface charge. But in the measurement of contact e.m.f. and stopping potential, the surface charge pushes each by the same amount in the direction of negative potentials. This is why the measured contact e.m.f. between sodium and the oxidized plate was at first observed to be about 2.5 volts, but had later fallen to less than .5 volt.

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[1] Phil. Mag., 24, 592, 1912.

[2] Phil. Mag., 14, 201, 1907.

[3] Am. Jr. Sci., 36, p. 501, 1913.

[4] PHYS. REV., 8, 217, 1916.

The accompanying experiments, then, taken in connection with those of Page and those of Kadesch and Hennings, show in the most beautiful, quantitative way that all clean metals possess intrinsic contact e.m.f.s and they also establish a criterion by which intrinsic and spurious contact e.m.f.s may be distinguished, namely this:

If equation (5) *is found by experiment to be satisfied, only intrinsic contact e.m.f.s are present. If equation* (5) *is not found to be satisfied, then the difference between the observed contact e.m.f. and* $h/e \cdot (\nu_{0_B} - \nu_{0_A})$ *is a measure of the presence and amount of a spurious contact e.m.f.*

Thus the century-old dispute as to the reality of intrinsic contact e.m.f.s appears to be quite definitely settled by experiments of this type. The experiment of bringing sodium and platinum, for example, in succession before the same Faraday cylinder and finding the stopping photo-potentials exactly the same for the two, despite the fact that the electron leaves the sodium under the influence of a given frequency, (*e.g.*, that corresponding to $\lambda = 2535$ Å.), with a much higher velocity than that with which it leaves the platinum, the threshold frequency for sodium being less than one half its value for platinum, is the most striking proof that each metal creates about itself a contact field which is inversely proportional to the work required to detach the electron from the metal. It is to be emphasized, too, that the result is a purely experimental one which is quite independent of any theory, although it is beautifully in accord with the theory of electron atmospheres.

3. SIGNIFICANCE OF THE FOREGOING RESULTS FOR THE THEORY OF QUANTA.

A finite value, then, of $(PD_A - PD_B)$ is merely a measure of the surface charge on the oxide and in no case a measure of the difference in the amounts of work necessary to detach an electron from two different metallic atoms, as is implied in equation (3), since in the case of all *clean* metals or indeed in the case of metals contaminated with invisible surface films this difference is zero; *i.e.*, $(PD_A - PD_B)e = 0$ and therefore (cf. (3)) $w_{1_B} - w_{1_A} = 0$. Now, the fact that $w_{1_B} - w_{1_A} = 0$ necessitates one or the other of two alternatives:

1. Either the works necessary to detach an electron from the atom of platinum and the atom of sodium, for example, are the same, or else,

2. The photo-electron which escapes with the highest velocity from a metal under the influence of the incident frequency ν has exactly the energy $h\nu$ *when it arrives at the inner surface of the metal on its way out, no loss of energy* w_1 *in getting out of the atom having to be taken into account.*

The first of these alternatives is so violently at variance with our definite knowledge of the relative strengths of the bonds which hold the outer electrons to different atoms (cf. the values of ionizing potentials, etc.) that it may at once be dismissed as impossible. If this be admitted *we have, then, no alternative but to adopt* (2) *as the inevitable consequence of the experiments discussed in this report.*

These experiments then prove conclusively in any case that *at the same temperature there is complete identity between the photoelectric work function* $h\nu_0$ *and the thermionic work function* φe. This identity has always been assumed by Richardson, but the experimental evidence has not heretofore been exact enough to demonstrate the correctness of the assumption. No comparison of experimental values of $h\nu_0$ and φe is likely to be as trustworthy as the foregoing evidence.

The significance of (2) for the quantum theory is found in the fact that it, in turn, requires one or the other of the two following alternatives:

(*a*) Either the incident light is able to take hold of a free electron of the metal and impart to it an energy $h\nu$, or (*b*) The incident light energy is not absorbed in whole quanta at all, but rather absorption continues within the atom until the electron is able to *escape* from the atom with the energy $h\nu$, the absorption being then $h\nu$ plus w_1, w_1 being the amount required to pull the electron out of the atom.

This last is precisely the conclusion which Barkla has drawn from his measurements upon the ionizing energy of the corpuscular radiations stimulated by characteristic x-rays.[1] I am not disposed to insist here upon the inevitableness of Barkla's conclusion, but I do wish to insist that the foregoing considerations demand either Barkla's conclusion or else the conclusion that a *free electron of a metal is able to absorb energy from an incident light wave and to be endowed by that wave with the energy* $h\nu$*; for they demand that within the metal there be free electrons which possess the full value of the energy* $h\nu$.

Either of the conclusions (*a*) or (*b*) required by these experiments is confronted by difficulties. It is very difficult to accept Barkla's conclusion (*b*) because it means either that there are an infinite number of natural frequencies within a metal, or else that an electron within an atom has the power of absorbing a quantity of energy which is characteristic of itself plus a quantity $h\nu$ which is characteristic only of the incident light. Neither of these assumptions is rationally satisfactory.

On the other hand, the conclusion that the free electrons of the metal take part in the photo-effect would seem to require that stopping potentials diminish with increasing temperature, and also that there be discontinuities in the photo-current potential curve corresponding to the potentials at which the photo-currents due to the bound electrons begin to add their effect to that of the free electrons. Neither of these predictions is as yet in harmony with the facts of observation. It is true that in the experiments of Winchester and the author[2] upon the effect of temperature upon photo-effects, the temperature was carried through but about 300° C. in the case of aluminum and about 100° C. in the case of other metals, and it is possible that a small temperature effect escaped detection. It is also conceivable that a further search for discontinuities in the photo-current potential curve might reveal them. Such a search for discontinuities and for a temperature variation in stopping potentials is now being made.

At any rate, I am at present disposed to adopt alternative (*a*) and to regard the present experimental work as justifying the conclusion that a light wave has the property of being absorbed by a *free* electron of a metal and of imparting to that electron the energy $h\nu$. If this conclusion is correct it is of much importance for the theory of radiation, for it means that the burden of accounting for the emission of electrons with the energy $h\nu$ can no longer be thrown back upon some unknown mechanism in the structure of the atom where many physicists have in the past sought to place it.

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[1] Phil. Trans., A, Vol. 27, 315–360, 1917.

[2] Phil. Mag., 14, 201, 1907.

Nor, on the other hand, can this burden be placed upon the elastic-solid properties of the complex of atoms and free (*i.e.* conduction) electrons constituting a metal. For, although, in view of the Debye and Born types of analysis of the phenomena of specific heats, etc., this complex should possess a practically infinite number of natural frequencies, each of which might get into resonance with the impressed wave of frequency ν, yet the phenomenon of discharging electrons with this energy would then be limited to systems which possess these elastic solid properties, *i.e.* to solids or at most to liquids. Gases, on the other hand, would be excluded from the possibility of exhibiting it. Yet, though exact quantitative proofs for the case of gases are perhaps not yet available, the evidence yielded by many experiments, including those of C. T. R. Wilson, upon the emission of secondary electrons from the atoms of gases by gamma and x rays is so convincing that it is at present extremely unlikely that gases do not emit electrons with energies which are at least approximately equal to $h\nu$.

If then conclusion "a" above is valid *the only remaining alternative is to consider the property of taking hold of an electron, whether free or bound and imparting to it an energy $h\nu$, as an intrinsic property of light itself.*

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X-RAY ANALYSIS OF THREE SERIES OF ALLOYS.

By Mary R. Andrews.

Synopsis.

Crystal structure of three series of alloys has been determined by Hull's method of x-ray analysis, using sheets of collodion or zapon in which the filings or powders to be investigated were embedded.

The *Fe-Ni and Fe-Co series* of pure binary alloys were each found to be solid solutions of one metal in the other with a single region of transition at 22–30 per cent. Ni and at 80–90 per cent. Co respectively. This means that Fe atoms can be substituted for up to 70 per cent. of the Ni atoms in the face-centered Ni lattice and Co atoms can be substituted for up to 80 per cent. of the Fe atoms in the cube-centered Fe lattice without changing the crystal structure. No evidence of a compound Fe_2Ni was obtained.

Fe-Ni alloys develop the cube-centered form when made magnetic by chilling in liquid air.

The *Cu-Zn series* of alloys form solid solutions of Zn in Cu up to 47 per cent. Zn, then change first to the cubic, then to the rhombohedral, and finally to the hexagonal form typical of Zn. These changes correspond roughly with changes in the hardness curve. The *effect of annealing* was not appreciable.

IN the past few years much work has been done by x-ray analysis on the crystal structure of those elements which are obtainable in pure solid form at room temperature, and of many of the simpler salts and oxides.

The question arose whether this same method might not be useful and enlightening when applied to alloys.

When a compound of elements of very different atomic weights is examined by x-rays, the diffraction pattern obtained is that of the crystal structure of the heavy element alone, the intensities of the lines of this pattern being modified by the position of the atoms of the light element. This often makes the interpretation of the diffraction pattern, in terms of the complete crystal structure of the compound, extremely difficult. And the complete interpretation of the structure of a compound containing three or more elements is, in the present state of our knowledge and experience, often almost impossible, even though the x-ray diffraction pattern may be perfectly definite.

For this reason we started with very simple and well-known series of binary alloys of metals whose atomic weights differed from one another only a little, so that we could be sure that they would both reflect x-rays in nearly equal amounts.

We used the Hull method[1] of x-ray analysis. Instead of powders we usually took fine filings, as many of our alloys were not brittle enough for powdering. The samples were prepared by mixing the filings with collodion or zapon and drying the paste to a thin sheet on glass. Such a sheet or film could be readily peeled off when dry and had the advantage of containing very little foreign material, and that little amorphous. Being composed of the light elements C, H, O, etc., quite transparent to x-rays, such a film, when placed in a narrow beam of x-rays, produces very little fogging.

The following data cover work done on nickel-iron, cobalt-iron, and copper-zinc alloys.

NICKEL-IRON ALLOYS.

Iron and nickel are known to form an unbroken series of solid solutions of all compositions from 100 per cent. iron to 100 per cent. nickel, though it has been suggested[2] that a compound may exist at 34.45 per cent. nickel (Fe_2Ni). As can be seen below, our work gave no indication of such a compound. The samples used in our work were prepared by the fusion of pure Swedish iron and pure nickel in a hydrogen resistance furnace. They therefore contained no carbon, and the percentage of other impurities was very low.

The crystallographic structure of alloys of different composition was determined, as shown in Table I.

TABLE I.

Nickel-Iron Alloys.

Per Cent. Ni.	Structure as Shown by X-ray Diffraction Patterns.
0	Centered cubic alone.
10	" " "
15	" " "
20	" " "
22	" " "
25	Centered cubic plus face-centered cubic.
27	" " " " " "
30	Face-centered cubic alone.
50	" " " "
75	" " " "
100	" " " "

There is no indication of a change in structure at 34 to 35 per cent. Ni, the composition of invar.

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[1] A. W. Hull, PHYS. REV., 10, 661, 1917. J. A. C. S., 41, 1168, 1919.

[2] Chevenard, Rev. de Metallurgie, 11, 841. Weiss, Tran. Faraday Soc., 8, 149. Weiss & Foex, Arch. des Sc. Phys. et Nat. (4), 31, pp. 4 and 89.

Fig. 1 shows the equilibrium diagram determined thermally as given in the circular issued by the Bureau of Standards: "Invar and Related Nickel Steels," with a small addition below it showing the structure as determined by x-ray analysis. The agreement is fair, and might be better below 22 per cent. Ni if the x-ray method were capable of detecting very small percentages of a crystal form. That is to say, though the x-ray pictures show a pure centered cubic form at 20 per cent. Ni, there might be one or two per cent. of the face-centered cubic form present. A ten to one artificial mixture of iron and nickel (*i.e.*, centered cubic with face centered cubic structure) showed both forms quite distinctly, but we have not tested mixtures containing smaller percentages of nickel.

Fig. 2 shows the diffraction patterns as photographed from some of the alloys of this series.

These patterns prove that nickel atoms can be substituted for iron atoms in the iron (centered cubic) crystal lattice until about 20 per cent. of the iron has been so replaced. On the other hand, iron can substitute for nickel in the nickel face-centered cubic lattice to a much greater extent; *i.e.*, until about seventy per cent. of the nickel atoms have been replaced by iron atoms. These replacements are possible because the atomic volumes of iron and nickel are nearly the same, iron being 7.2, nickel 6.7, and the shapes of the atoms are nearly enough alike to make substitution possible with little strain of the intra-atomic forces. Some of the photographs show, along the edge, a scale which we occasionally used. It consisted merely of a thin sheet of molybdenum in which slots were cut at spacings calculated so as to read Ångstroms direct on the photograph.

It will be noted that the transition from the centered cubic form to the face-centered cubic form takes place between 22 and 30 per cent. Ni. This is also the region in which some of the physical properties of the ferro-nickels have maxima or minima, such as hardness,[1] tensile strength,[1] magnetic saturation intensity[2] and hysteresis losses,[3] though most of the properties investigated[4] have maxima or minima at about 35 per cent. Ni,[4] this being the reason for the assumption of the compound Fe_2Ni.

The density, electrical resistance and elasticity of ferro-nickels in this transition range (20 to 30 per cent. Ni) varies with the history of the

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[1] Circular of the Bureau of Standards, No. 58 (Invar and Related Ni steels).

[2] Yensen, Journ. A. I. E. E., April, 1920, p. 396.

[3] Honda, Sci. Reports. Tohoku Inys. University, 7, p. 59, 1918.

[4] Ingersoll & Others, Phys. Rev., 16 (2), 126.

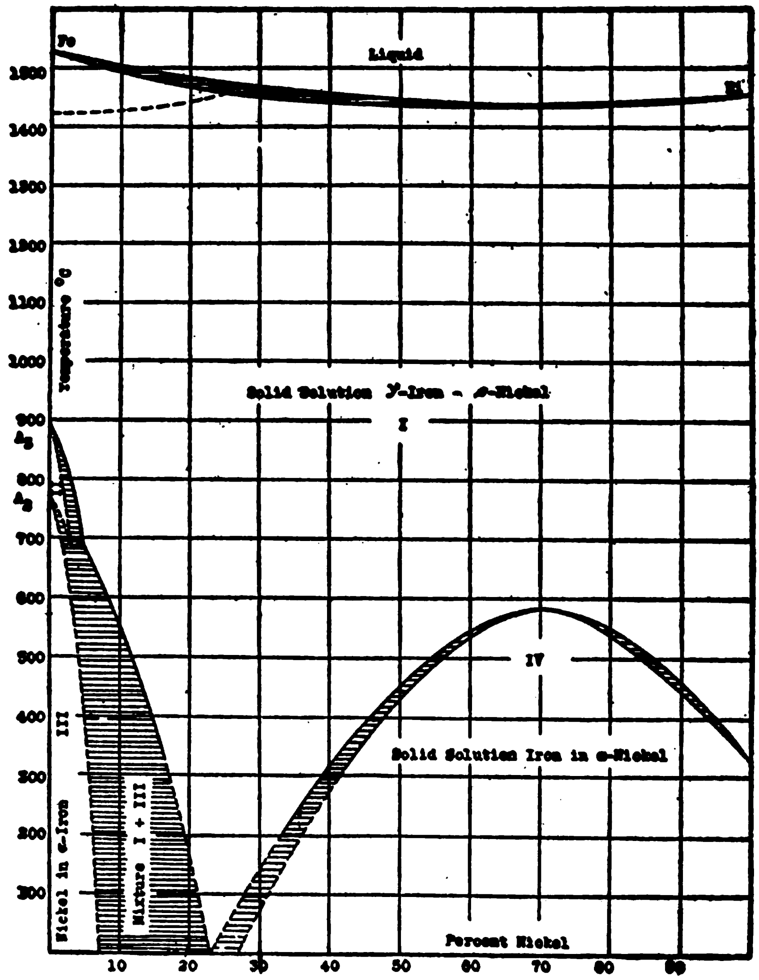

FIG. 1.—*The iron-nickel equilibrium diagram*

Structure by X-Ray Analysis

centered cubic	Mixture	Face-centered cubic

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PLATE I.
To face page 248.

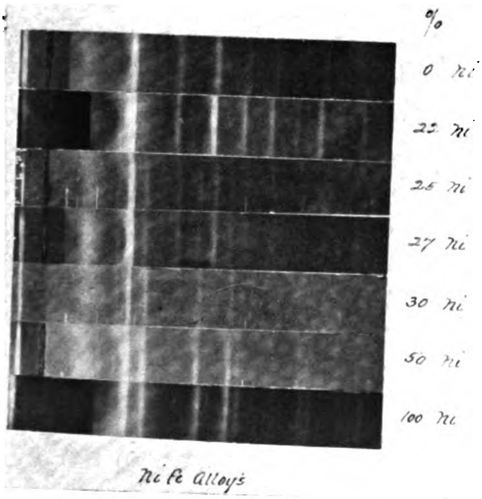

Fig. 2.

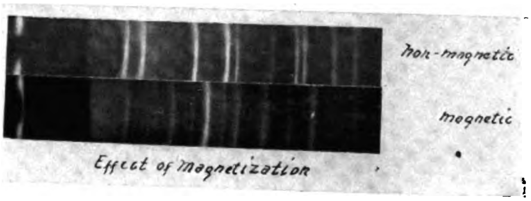

Fig. 3.

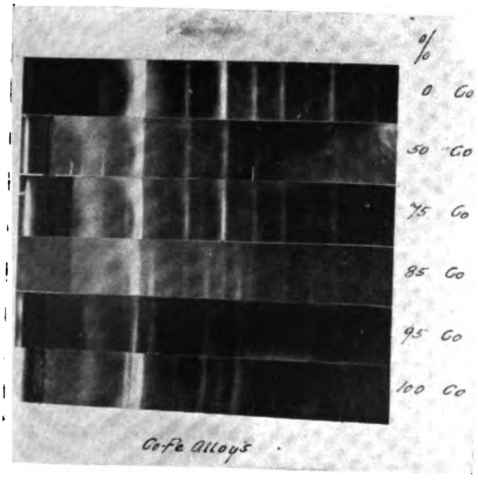

Fig. 4.

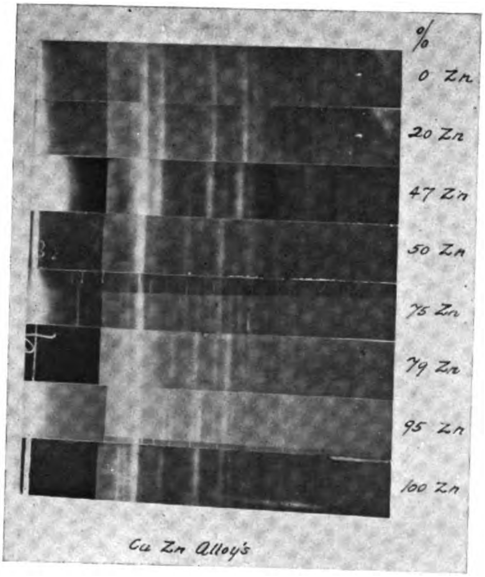

Fig. 6.

MARY R. ANDREWS.

specimen,[1] and this effect is also very marked with the magnetic properties.[2] The magnetic transformation curves as determined by Hegg[3] and others show a wide difference between the temperature of transformation on heating and on cooling for alloys between twenty and thirty per cent. nickel. If the alloys are free from carbon and manganese, both these curves lie above room temperature, and the alloys are, therefore, always magnetic at room temperatures. But if a few tenths of a per cent. of carbon be present, or a little manganese, these transformation curves are lowered so that it is possible to obtain an alloy that will be at room temperature, either magnetic or non-magnetic; *i.e.*, the irreversible alloys. If it has been cooled from a sufficiently high temperature it will be non-magnetic. If warmed from a low temperature, it will be strongly magnetic.

The alloys which were used for the photographs shown on Fig. 2 were all apparently so free from carbon, etc., that their transformation curves both lay above room temperature, as they were all strongly magnetic. But I obtained from the Bureau of Standards a sample of almost entirely non-magnetic nickel steel containing 20.1 per cent. Ni, which I photographed before and after chilling with liquid air. (See Fig. 3.) These photographs were made on the solid samples, since filings were always magnetic. The rod was filed to a wedge and the edge of this wedge was placed just in the beam of x-rays. I made photographs on samples which had been freshly filed down, and then on the same samples etched off slightly with hydrochloric acid, to determine whether there was a change at the surface due to the filing off, sufficient to show on the photographs, but could detect no difference in the pictures. However, on chilling for two or more hours in liquid air, there is, as the photographs show, a marked development of the centered-cubic form which was absent or very faint in the non-magnetic material. This test was made three times, and in every case the centered-cubic structure developed to a considerable extent. As nearly as possible the same surface of the alloy was exposed before and after chilling. The surface remained bright and free from rust or tarnish in two of the cases. In the third, the interval that elapsed between chilling and photographing was so long, due to difficulties with the x-ray apparatus, that a slight tarnish developed. This was wiped off as much as possible with a soft cloth. The penetration of the rays used is probably, for these alloys, sufficient to give the structure at a hundred or more microns depth, so that a very slight surface change would not affect the photograph to any extent.

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[1] Hopkinson, Proc. of the Royal Society, 48, 1; 50, 121.

[2] Honda, Idem.

[3] Archives des Sciences Physique et Naturelles, Vol. 30, p. 15.

This development of the cube centered form is interesting. It is an increase of the form characteristic of the stronger and more magnetic component of the alloy, iron, and therefore a decrease in the form (face-centered cubic) characteristic of the weaker and less magnetic component, nickel. Moreover, the centered cubic form is less closely packed than the face-centered, which would account for the change in density found by Hopkinson. Why the specific resistance should decrease, as he found, with an increase of the iron form, is less clear. These photographs would indicate that magnetic properties and crystal structure are closely connected, but it is not to be assumed that the centered cubic structure is necessarily magnetic. Chromium, molybdenum, and tungsten all occur in this system, and their magnetic properties are at best extremely feeble.[1]

It is to be noted that this non-magnetic alloy had a pure face-centered form, whereas the ferro-nickels of about the same composition, shown in Table I. and Fig. 2, which were magnetic, show a cube-centered form, the form that develops on chilling the non-magnetic alloy.

Small amounts of manganese, chromium, carbon, etc., doubtless influence the exact compositions at which the transition from cube-centered to face-centered form occurs. For instance, a sample of nickel steel prepared by Mr. A. W. Merrick, of the General Electric Company, which contained manganese and carbon, showed strong face-centered and weak centered cubic form, though it contained only about 18 per cent. nickel. It was weakly magnetic.

The whole question of magnetism of metals and alloys, including the Heusler alloys, offers a most fascinating field for investigation by x-rays, and I hope later to be able to do some work along these lines. A single Heusler alloy, sent to Dr. Hull by Dr. S. J. Barnett of Carnegie Institution, which was photographed, showed a centered cubic form.

COBALT-IRON ALLOYS.

Cobalt and iron, like nickel and iron, form an unbroken series of solid solutions. The atomic volume of cobalt is 6.85, of iron 7.2. The equilibrium diagram as given by Guertler,[2] shows a transformation curve that breaks at about 80 per cent. cobalt. Our photographs (see Fig. 4) show the centered cubic (iron) form for alloys from 0 to 80 per cent. cobalt, then the face-centered form begins to develop, so that at 85 per cent. Co we have both forms together, and above this percentage of cobalt the face-centered form only occurs. The alloy containing 98 per cent. of cobalt showed a trace of the hexagonal form, but chiefly the face-centered cubic. Pure cobalt crystallizes in both the face-centered and the hexagonal forms.[1]

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[1] A. W. Hull, The Positions of Atoms in Metals, Trans. A. I. E. E., 1920.

[2] Metallographie, 1 Band, Die Binäre Legierungen, p. 78.

Table II.

Cobalt-Iron Alloys.

Per Cent. Co.	Crystal Structure by X-ray Diffraction Patterns.
0	Centered cubic.
20	" "
30	" "
50	" "
75	" "
80	" "
85	Mixture.
90	Face centered cubic.
95	" " "
98	" " " plus trace of hexagonal.
100	" " " " hexagonal.

The physical properties of the iron-cobalt alloys do not seem to be connected with this crystal structure. Yensen[2] has done a vast amount of work on the magnetic properties, and finds that the magnetic permeability increases with increase in the per cent. of cobalt, up to 34.5 per cent. cobalt (Fe_2Co), being at this point about 13 per cent. higher than pure iron, but that further addition of cobalt reduces the permeability. Honda[3] finds that the thermal and electrical conductivity decrease from 0 to 10 per cent. cobalt, increase to 65 per cent., decrease to 90 per cent., and then increase to 100 per cent. cobalt.

Copper-Zinc Alloys.

The atomic volumes of copper and zinc differ rather widely, that of copper being 7.1 and of zinc 9.17. Therefore, only about half of the series forms solid solutions. Table III. shows the crystal structure of the alloys examined, and Fig. 5 the transformation curve for the copper-zinc alloys taken from Guertler,[4] below which has been sketched the divisions into crystal forms as shown by the x-rays. Type IV. of Guertler's diagram does not appear in the x-ray forms, at all. The pattern obtained from the alloys containing 79 and 82 per cent. zinc, agreed very well with the curves, to which we had access, prepared by Drs.

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[1] A. W. Hull, ibid.

[2] Trans. Am. Electrochem. Soc., Vol. 32, p. 165.

[3] Sci. Reports Tohoku Imp. Univ., Vol. 8, p. 51.

[4] Metallographie, 1. Band, p. 459.

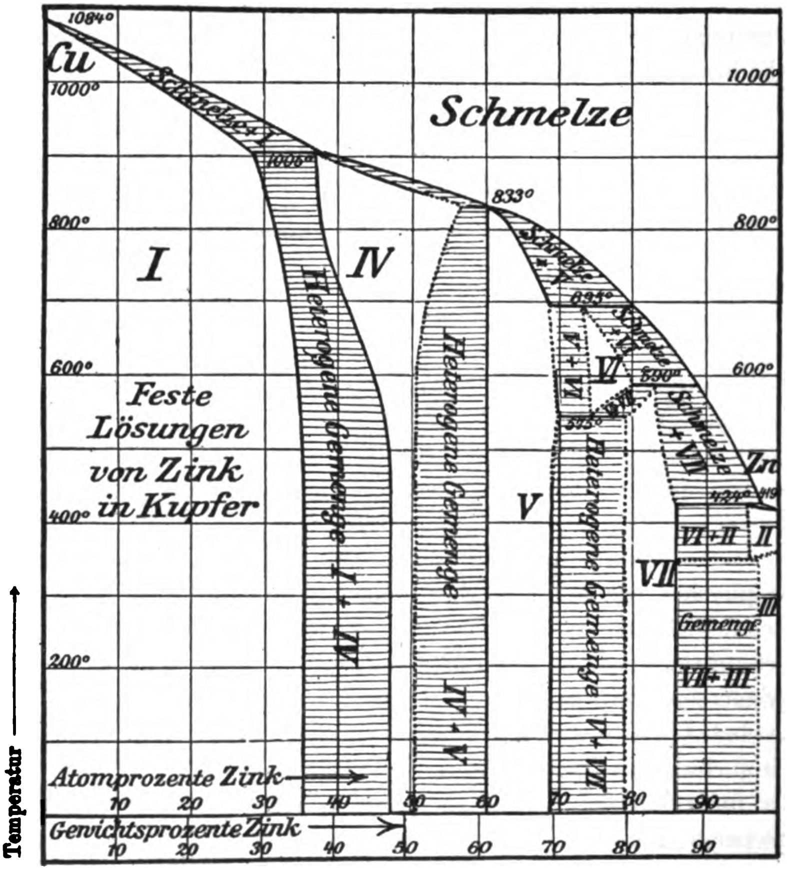

Fig. 189.
Zustandsdiagramm der Kupfer-Zink-Legierungen.

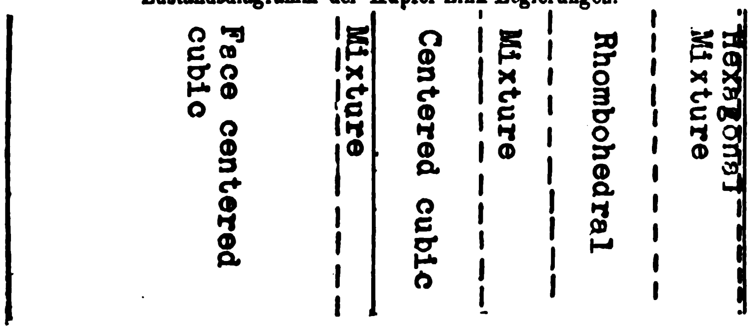

Fig. 5.

Hull and Davey,[1] for the rhombohedral form if taken at an axial ratio of .5. Some of the photographs are shown in Fig. 6.

TABLE III.

Per Cent. Zinc.	Crystal Structure as Found by X-ray Diffraction Patterns.
00 (pure Cu)	Face-centered cubic.
20	" " "
31	" " "
37	" " "
47	Mixture face-centered cubic and centered cubic.
50	Centered cubic.
63	" "
69	Mixture.
79	Rhombohedral.
82	"
90	Mixture rhombohedral and hexagonal.
94	" " " "
100	Hexagonal.

While there is voluminous literature on the copper-zinc alloys, most of it is confined to the commercial brasses and Muntz metals; *i.e.*, the alloys containing less than fifty per cent. zinc. These are all solid solutions of zinc in copper.

The Lunkenheimer Company has, however, been kind enough to allow me to print the hardness curve shown in Fig. 7 which was determined by Mr. G. K. Elliott, of that Company, with the Shore Scleroscope. This curve agrees roughly with the divisions of crystal forms.

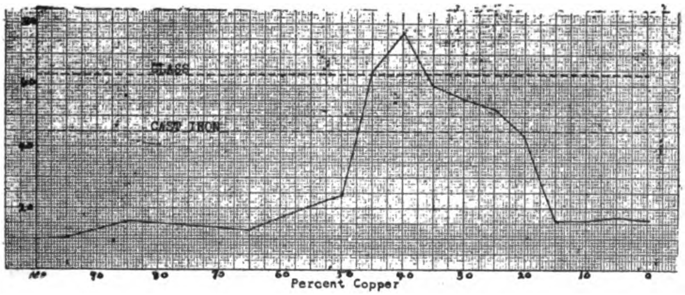

Fig. 7.

We tried the effect of annealing on a number of samples, but did not find that this treatment caused any change of structure.

Calculating the density of the alloy containing fifty per cent. of each component, which showed pure centered cubic structure, gave a value of 8.17. Maey[2] gives 8.14.

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[1] Phys. Rev., Feb., 1921, p. 266.

[2] Zeit. Phys. Chem., 38, 299.

We conclude that this method should be useful in determining the structure of other series of alloys.

Finally, I wish to express my hearty thanks to Dr. A. W. Hull, under whose direction this work was done, for the interest he has shown and for his many helpful suggestions.

RESEARCH LABORATORY OF THE GENERAL ELECTRIC COMPANY, SCHENECTADY, N. Y.
April 19, 1921.

Second Series *October, 1921* *Vol. XVIII, No. 4*

THE

PHYSICAL REVIEW.

PHYSICAL PROPERTIES OF THIN METALLIC FILMS.[1]

III. THE EFFECT OF TEMPERATURE ON THE CHANGE OF RESISTANCE OF BISMUTH FILMS IN A MAGNETIC FIELD.

BY L. F. CURTISS.

SYNOPSIS.

Ageing of sputtered bismuth films.—The films sputtered on glass between sputtered gold terminals, were sealed in glass tubes and kept in vacuo at 210–220° C. for a number of hours. As a result each film showed a gradual *decrease of resistance* amounting in all to about 40 per cent. Eventually the films became so thoroughly aged that temperature-resistance measurements in vacuo could be accurately reproduced over long periods.

Increase of resistance of bismuth films in a magnetic field for temperatures from – *190° to 230° C.*—The isothermal curves obtained for each film by varying the field up to 18,000 gauss, have the same form for all temperatures, differing only by constant factors, and closely resemble the corresponding curves for bismuth in bulk. The percentage increase of resistance is not proportional to the square of the field. It is greater the lower the temperature and the lower the resistance, that is, the thicker the film, reaching a value about 1/5 that of bismuth for the thickest film tested.

Variation of resistance of bismuth films with temperature, – 190° *to* 230° *C.*—The curves are parabolic in form, showing a minimum around 150–200° C.; the lower the resistance of the film, the lower the temperature of minimum resistance.

Electrically heated oil bath.—*It is suggested* that bakelite is a suitable material for protecting the nichrome heater.

THIS work continues the investigation of the behavior of the resistance of bismuth films in a magnetic field described in the second[2] paper of this series. A study of the change of resistance of these films in a magnetic field at various constant temperatures from near the melting point of bismuth down to liquid air temperatures has here been undertaken.

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[1] Third of a series of articles on this subject from the Physical Laboratory of Cornell University. This series of articles was made possible by a grant to Prof. F. K. Richtmyer from the Rumford Fund.

[2] F. K. Richtmyer and L. F. Curtiss, PHYS. REV., 15, p. 465, 1920.

For this purpose thoroughly aged films were desirable, since only with films which have been subjected to a thorough heat treatment can reproducible results be expected, as is evident from the results of the previous work referred to above. Since the oil bath had not been entirely satisfactory because of its destructive action on the films, it was thought advisable to try some other means of protecting the films from the action of the air during the heating. The procedure which suggested itself was to place the films in flat glass tubes and to evacuate the tubes to as high a degree as possible with a Langmuir mercury vapor pump, at the same time heating them and the enclosed films to near the melting point of bismuth. This heating served two purposes. It gave the films a preliminary heat treatment and it also helped to free the film and tube from adsorbed gases, thus affording a greater protection to the film during subsequent heatings.

The films were prepared in much the same way as for the previous work (*loc. cit.*). The glass slides on which the metal was deposited were made somewhat narrower for convenience in inserting them into the flat glass tubes. The actual dimensions of the bismuth film itself, however, were kept the same as those used formerly, *i.e.*, 10 by 3 mm. The rotating sector was dispensed with and a kenotron[1] for rectifying the secondary current of the transformer was placed in series with the discharge jar. This improvement cut out the reverse current and eliminated its heating effect, thus rendering the rotating sector unnecessary for the preparation of films for the present purpose. It also made possible the sputtering of much thicker films than had previously been possible. For, as stated in the preceding paper, after sputtering a certain length of time the surface of the bismuth film becomes gray and powdery and further sputtering has little effect upon the resistance of the film, since the particles of metal which come down seem no longer to adhere to the metal already deposited firmly enough to become active in conducting the current. It was found that this condition did not come about as soon when the kenotron was used.

The successive steps in the manufacture of a film are illustrated in the diagrams in Fig. 1, which are drawn to scale. First the glass slide was thoroughly cleaned chemically, rinsed with distilled water and dried. It was then covered across the middle squarely with a glass slide exactly 1 cm. wide with edges ground true and gold terminals were deposited on the exposed ends by cathodic sputtering, after which it had the appearance indicated in Fig. 1, *a*. Then a shield of mica with a slot 3 mm. wide and long enough to overlap the gold terminals was placed on this blank and a bismuth film deposited, likewise by cathodic sputtering, along the center of the slide connecting the two gold terminals, as shown in Fig. 1, *b*. A piece of fine bare copper wire was then wrapped tightly around each end as shown in Fig. 1, *c*. A thin layer of hot paraffin was spread across the face of the film, near the ends of the bismuth, which prevented the copper sulphate solution, used in plating the wires in position, from creeping up on to the bismuth. This was later dissolved off with xylol. First one end, then the other, of the film was dipped into the plating bath and a thick coating of copper deposited on the gold film and copper wire, the projecting wires serving as leads for the current in each case so that it did not pass through the bismuth film. This plated terminal insured positive contact with the film at all temperatures and proved to be far superior to the clamps which had been used before for this purpose. The film at this stage had the appearance shown in Fig. 1, *d*. Short pieces of Cunife wire were then welded onto the projecting ends of the copper wires for sealing through the glass and then longer pieces of copper wire were welded onto the Cunife to serve as external leads after the films had been sealed into the glass tubes. The tubes were prepared from flat glass tubing by sealing a short piece of glass tubing at right angles near one end for attaching to the pumps. The film with its lead wires was then sealed in and the tube sealed on to the pumps and evacuated for several hours at about 210° C. with a Langmuir pump, after which it was sealed off. The general arrangement of the film within its evacuated tube is also shown in Fig. 1. The film was now further aged by heating for four hour periods to about 220° in an oil bath. These heatings were repeated four or five times for each film after which it had become relatively stable and further heating had very little effect upon its resistance at room temperature.

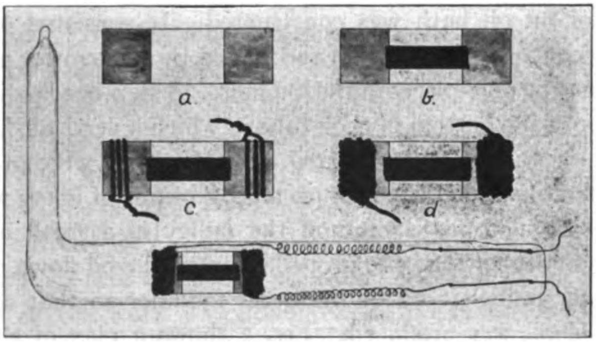

Fig. 1.

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[1] Kindly loaned by Prof. J. S. Shearer.

It is interesting to note that all films, without exception, *decreased* in resistance as a result of this heating. This is quite contrary to the effect produced by heating bismuth films in air, or even in an oil bath, since only in the case of two films out of a couple of dozen did such an effect occur in the previous work (*loc. cit.*). Thus Film V–5 had a resistance of 36.7 ohms at room temperature after it had been sealed into a glass tube and the tube exhausted. After it had been subsequently heated for four hours at about 220° its resistance at room temperature was only 21.7 ohms, thus decreasing about 41 per cent. in resistance during this treatment. Similarly Film V–13 had a resistance after being sealed into a tube of 30.8 ohms and after heating eight hours to about the same temperature it had a resistance of 17.8 ohms, or had decreased about 42 per cent. in resistance. Other films showed comparatively smaller changes during this aging process and in consequence were less stable, and showed further decreases in resistance during subsequent measurements at the higher temperatures. Thus the resistance of Film V–9 was 10.9 ohms after having been sealed into a tube and after heating for four hours its resistance was 7.4 ohms, decreasing about 33 per cent.; similarly Film V–14 had a resistance of 100.0 ohms after it was sealed into a glass tube and after heating eight hours its resistance was 83.6 ohms, decreasing only sixteen per cent. Before the final measurements on this film had been taken, however, it had decreased to about 55 ohms.

For measurements in the field at room temperature and above, a brass container for an oil bath was constructed. It consisted of two brass tubes, each cut away on one side and joined by a narrower neck so as to fit in between the pole-pieces of the magnet. A step-bearing was fastened in the center of the bottom of each tube in which a vertical shaft carrying a propeller for stirring was mounted. These shafts were provided with pulleys at the top which were connected by means of idlers with an endless belt which also passed around the pulley of a small motor. The shafts were so driven that one propeller forced the oil down one cylinder and the other pulled it up in the other cylinder so that a continuous circulation of oil was produced. Fig. 2 shows a plan of the container and illustrates its position with reference to the pole-pieces of the magnet. The bath was about 12 cm. deep and 22 cm. from the outside of one cylinder to the outside of the opposite cylinder. The distance between the pole-pieces was one centimeter. The bath was heated by a resistance element made by winding nichrome ribbon on a sheet of mica. It was found that some protection for the ribbon was necessary since the oil, although it was a good grade of gas-engine cylinder oil, carbonized badly at the higher temperatures and adhered to the ribbon, thus short-circuiting adjoining turns. Bakelite was found very satisfactory for this purpose. The current through the heating element could be controlled by rheostats so that the temperature could be brought to any desired value and held constant for considerable intervals of time. The temperatures were measured with a copper-advance thermo-couple made and calibrated in the usual way. Its e.m.f. was measured by a Leeds and Northrup potentiometer.

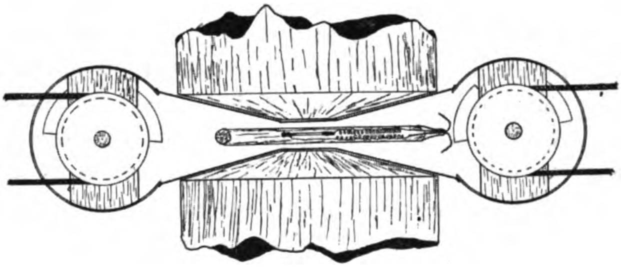

Fig. 2.

Since simultaneous measurements of field strength and resistance of the film were impracticable, a calibration of the magnet was made before the temperature bath was inserted between the pole-pieces. This was done by means of a bismuth spiral, calibrated for the purpose by the Bureau of Standards. Especial precautions were taken to keep the spiral during these measurements at the constant temperature at which it had been calibrated. By this means a curve between the current through the magnet coils and the strength of the resulting field in kilogauss was obtained which was used in determining the field throughout the experiments at room temperature and above. Control experiments showed that this method would give results which could be depended upon to within less than one per cent., which was deemed adequate for the present work. The current for the magnet was supplied by a set of twenty-six 15-ampere storage cells connected in series. The maximum current used in this part of the work was about seven amperes. A large Weston Laboratory Standard millivoltmeter and shunt were used in measuring the magnet current both during the calibration and throughout the subsequent experiments.

For convenience, all the measuring instruments and controlling rheostats were mounted on a table so that they were all under the control of one operator. Thus all factors could be varied by a single person without shift of position during an experiment. This also made possible a simultaneous determination of the various quantities, eliminating errors due to variation in the temperature or in the current through the magnet.

In making the measurements the procedure was as follows: The film was placed in its position in the oil bath and the rheostats controlling the current through the heating element were adjusted. About twenty minutes were necessary for the temperature to become steady. As soon as the potentiometer readings indicated that this state had been reached, measurements of the resistance of the film were made at various values of the current through the magnet coils, checking the zero-field resistance of the film and the potentiometer reading after each measurement. It was found possible in all cases to hold the temperature steady enough so that the zero-field resistance of the film did not vary more than 0.1 per cent. during a series of measurements at a given temperature. The results of the measurements can best be presented by a discussion of the curves plotted from the data taken with typical films. A set of data for one of the films, V-5, is given below to illustrate the magnitude of the quantities involved.

FILM V-5.

Potentiometer Reading, 0.001010; *Temperature,* 23.5°; *Resistance of leads,* 0.370 *ohms.*

Magnet Current.	**Bridge Reading.**	**Resistance of Film.**	dr	dr/r	H **(Gauss).**
1.0	21.603	21.233			
0	21.509	21.139	0.094	0.0044	2300
2.0	21.862	21.492			
0	21.516	21.146	0.346	0.0164	4725
3.0	22.219	21.849			
0	21.502	21.132	0.717	0.0339	7250
4.0	22.682	22.312			
0	21.498	21.128	1.184	0.0561	9700
5.0	23.170	22.800			
0	21.492	21.122	1.678	0.0795	11950
6.0	23.660	23.290			
0	21.492	21.122	2.168	0.1028	14125
6.84	24.014	23.644			
0	21.493	21.123	2.521	0.1196	15710

Another set of data for the same film at 230°, the maximum temperature at which measurements were made on this film, is given below also.

Temperature variations are more likely to occur and the zero-field resistance of the film is not as constant as in the set taken at room temperature. However the variations in it are still less than 0.1 per cent.

Potentiometer Reading, 0.011241; *Temperature,* 229.3°.

Magnet Current.	Bridge Reading.	Resistance of Film.	*dr*	*dr/r*	*H*
0	20.957	20.587			
1.0	20.943	20.573			
0	20.930	20.560	0.013	0.0005	2300
2.0	20.965	20.595			
0	20.938	20.568	0.027	0.0013	4725
3.0	21.001	20.631			
0	20.930	20.560	0.071	0.0034	7250
4.0	21.039	20.669			
0	20.930	20.560	0.109	0.0053	9700
5.0	21.086	20.716			
0	20.925	20.555	0.161	0.0078	11950
6.0	21.141	20.771			
0	20.923	20.553	0.218	0.0106	14125
6.8	21.186	20.816			
0	20.921	20.551	0.265	0.0129	15650

For measurements at low temperatures the oil bath was removed from between the pole-pieces of the magnet, and, to make possible better heat insulation, the pole-pieces were separated to a distance of 1.8 cm. The exposed parts of the magnet in between the coils were covered with a thick layer of heat insulation, a thinner layer also covering the faces of the pole-pieces. A cardboard container for the refrigerant was lowered into the rectangular compartment thus formed. A vertical section is shown in Fig. 3 which will give an idea of the arrangement.

Since the pole-pieces of the magnet had been separated to a greater distance it was necessary to increase the current through the coils, beyond that which was used during the experiments at the higher temperatures, in order to cover the same range of field strengths. The magnet was accordingly connected to the 110-volt direct current supply of the laboratory through suitable rheostats. This arrangement was not as satisfactory as the storage battery had been since the fluctuations in the line voltage caused small variations in the magnet current. However, by exercising extreme care, errors from this source could be reduced to a negligible amount. The maximum current used was about 18 amperes, giving a field of about 18,000 gauss. The magnet was recalibrated, a ballistic galvanometer, a standard of mutual inductance, and a snatch coil being used for this purpose, since an accident to the bismuth spiral had ruined it.

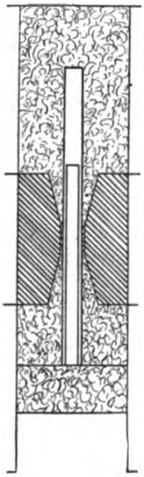

Fig. 3.

The films were removed from their protecting glass tubes for the measurements at low temperatures, as they have very slight tendency to oxidize at room temperature and below, and a considerable saving of space between the pole-pieces was thereby accomplished. They were mounted on a fiber support by means of which they were accurately centered between the pole-pieces. One junction of a thermo-couple for measuring temperatures was also mounted on the fiber strip so that it was close to the film.

Great difficulty, due to poor heat insulation and the conductivity of the large pole-pieces, was experienced in maintaining constant temperatures for any considerable period of time unless the films were actually immersed in the refrigerant. Consequently measurements in the field were only attempted when the films were immersed in either a mixture of carbon dioxide snow and ether or in liquid air.

The results of the investigation can be considered under four heads, viz., (1) effect of magnetic field on the resistance of the films at various temperatures; (2) the dependence of the values of the percentage increase of resistance upon the thickness of the film; (3) evidences of aging of the films; (4) relation between the temperature of the film and its resistance.

Observations were made on the behavior of a large number of films varying in resistance from 7 ohms to 70 ohms. No attempts were made to measure the thickness of the films. Since all of the films had the same length and breadth, the resistance can be taken as a rough indication of the thickness. Film V–10 has been selected as an example for a discussion of the effect of temperature upon the percentage increase of resistance of the films in a magnetic field. At 20.5° its resistance was 10.54 ohms, hence it was one of the thicker films studied. The results of the measurements on this film at various temperatures are shown graphically in Fig. 4. The percentage increase of resistance has been plotted against the field strength in kilogauss. The various curves have been labelled and were taken at the following temperatures:

I	20.5°	IV	147.5°	VII	− 80°
II	57.2°	V	195°	VIII	− 190°
III	109.0°	VI	227°	A	19.5°

As the temperature decreases the effect of the field becomes more strongly marked, especially at the low temperatures. For the weaker fields there is a decided curvature in the lines which gradually smooth out into practically straight lines. It is in this lower region that the "square law," proposed by Sir J. J. Thomson,[1] holds. As a check on the reproducibility of the results the data for Curve *I* were retaken immediately after taking the data for Curve *II*. The result is shown by the crosses (x) which practically coincide with the circles used to represent the initial data. Curve *A* was taken after Curve *VI* and also after a subsequent temperature-resistance run had been made with the film. The significance of the location of this curve will be discussed later in connection with evidences of aging in these films. The curves labelled (*I*), (*II*), and (*III*) were obtained by multiplying the ordinates

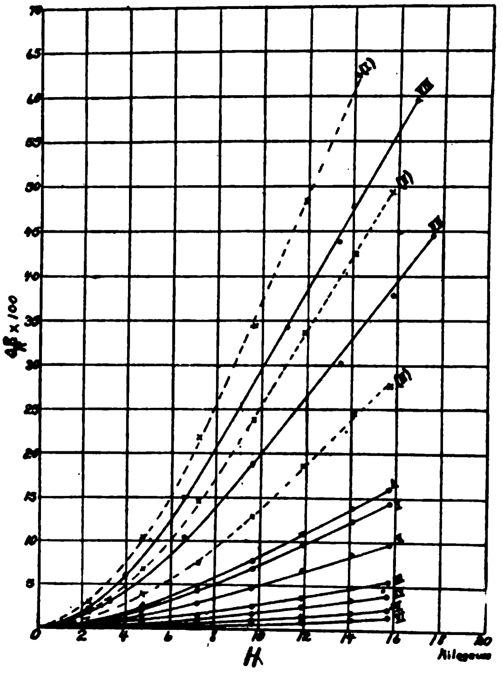

Fig. 4.

These curves represent the values of dr/r at various temperatures and values of the field for Film V–10.

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[1] J. J. Thomson, Rapports presentes au Congres International de Physique, 1900.

of the corresponding curves below by four. This has been done in order that it may be made plain that the form of the curves is the same throughout, so that the only effect of the change of temperature is to multiply the ordinates by a constant amount for any particular film. This is a rather significant and unexpected result. If we express the equation of the family of curves in terms of dr/r and some function of the field we may do so as follows:

$$dr/r = k\,f(H)$$

where k is a constant which depends on the temperature.

In Fig. 5 the effect of temperature is shown more directly for a similar film, V–9, where dr/r is plotted against the temperature for several

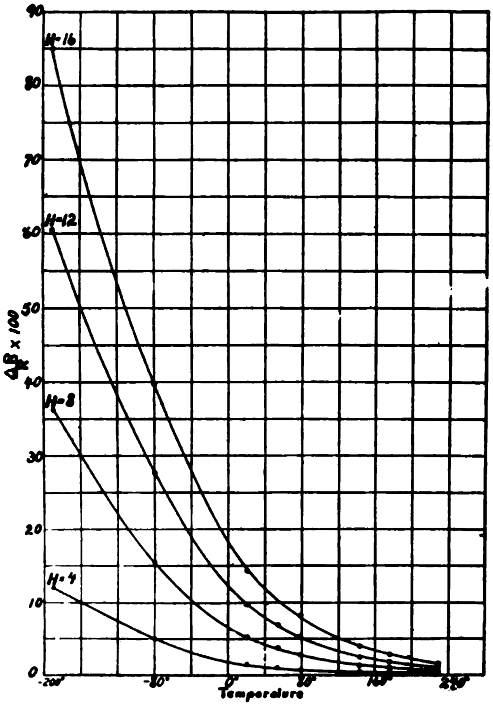

Fig. 5.

These curves, representing results obtained with Film V–9, shows directly the effect of temperature on the values of dr/r at a number of constant field strengths.

constant values of the field. Here again we see that as the temperature decreases the field produces correspondingly greater values of the increase of resistance. As in bismuth in bulk,[1] these curves indicate that the increase of resistance would vanish, or become very small, at least, at the melting point of the metal. In form the curves are very similar to the curves representing the same quantities for bismuth in bulk. In fact, all measurements made on these films in the magnetic field support the statement that they differ mainly from solid bismuth in the magnitude of the changes produced by the field.

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[1] Drude and Nernst, Gött. Nachrichten, No. 14, p. 471, 1890.

In order to give a basis for comparison of this series of films with those described in the previous paper (*loc. cit.*), in which the results were represented by plotting the percentage increase of resistance against the *square* of the field, a similar set of curves has been prepared for Film V–9. Since this film was of somewhat lower resistance than V–10, Fig. 4, it yielded larger values of dr/r under similar conditions. The result is shown in Fig. 6. It is quite evident that the values of dr/r are not directly proportional to the square of the field for these films as they seemed to be for those studied in the previous work. However, at the higher temperatures, *where the values of* dr/r *are small*, there seems to be an approach to such a proportionality for these films. This item is also quite in agreement with all previous work on this subject which has led to the conclusion that only where the effect of the field is small does the square law obtain,[1] and it seems to matter little as to what the cause of the smallness is, whether the nature of the substance or its condition. However, the result of this work tends to show that if the values of dr/r have been diminished by raising the temperature of a substance for which the square law did not hold at lower temperatures, it will also not hold at the higher temperatures. On the other hand there may be a limit above which the equation proposed in connection with the results shown in Fig. 4 does not hold, although at present there is no evidence of it within the range of temperatures here covered. It would be interesting

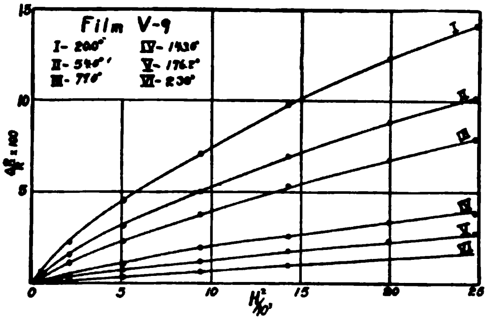

Fig. 6.

Data for Film V–9 showing that the values of dr/r are not proportional to the square of the field.

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[1] S. C. Laws, Phil. Mag., 19, p. 685, 1910.

to study the behavior of bismuth in bulk in weak fields, up to 3 kilogauss, to determine whether this square law holds when the values of dr/r are small as a result of the low values of the field strength used for a substance which does not obey the law in stronger fields. Although the tendency has been to assume that it did hold in this region, as far as the writer is aware, special attention has never been given to measurements in very weak fields.

For a discussion of the dependence of the values of dr/r upon the thickness of the film, which can only be considered here in a qualitative way, the following series of films has been chosen:

Film V-14, — Resistance 54.88 ohms at 24.5°.
Film V-13, — Resistance 17.48 ohms at 21.0°.
Film V-10, — Resistance 10.54 ohms at 20.5°.
Film V- 9, — Resistance 7.07 ohms at 20.5°.

The corresponding data for these various films have been plotted in Fig. 7. Each curve was taken at the temperature given above in connection with the zero-field resistance of the film. These curves show that the films of lower resistance have a larger value of dr/r under similar conditions. This is rather to be expected since the thicker the film the closer the approach to solid bismuth. To show further that no essential change in the form of the curve occurs as the thickness of the film in-

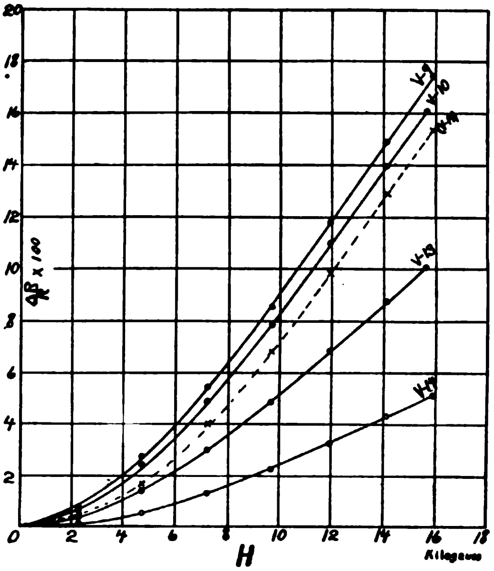

Fig. 7.

Curves showing the results for films of various thicknesses at room temperature.

creases, the ordinates for V–14 have been multiplied by three and replotted as indicated by the dotted curve which runs quite parallel to the neighboring curve for V–10. In Fig. 8 the similar results obtained for V–9, V–10, and V–14 at the temperature of liquid air are shown. Here the difference between V–9 and V–10 is much greater, indicating that the thicker films have a comparatively greater value of *dr/r* at the lower temperatures, a further evidence that in them the conditions are more nearly approaching those in solid bismuth.

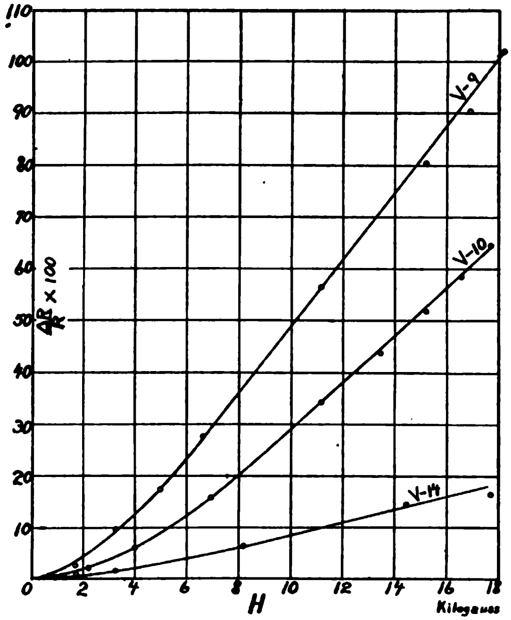

Fig. 8.

Showing the effect of thickness of the film upon the values of *dr/r* at the temperature of liquid air.

A discussion of the evidences of aging in these films and also of the form of the temperature-resistance curves may well be taken up together, since they are closely related. The temperature-resistance curves for Films V–14, V–13, V–5, and V–10 are shown in Figs. 9, 10, 11, and 12, respectively. In Fig. 9 we have a typical curve for a film of higher resistance. It is to be noted that the first cooling curve is exactly retraced by the second heating curve and the second cooling curve, whereas the first heating curve lies somewhat above these three. The explanation for this behavior is evident when attention is called to the fact that, as shown by the curves, care was taken in making the second heating not to exceed the previous maximum temperature for the first heating.

Hence no further aging occurred during the second heating and the resistance remained unchanged. This plainly shows that this film had not had its aging process carried to the limit before the temperature-resistance measurements here shown were undertaken, and that in making the first heating the previous maximum temperature to which the film had been heated was exceeded with a resultant decrease in resistance

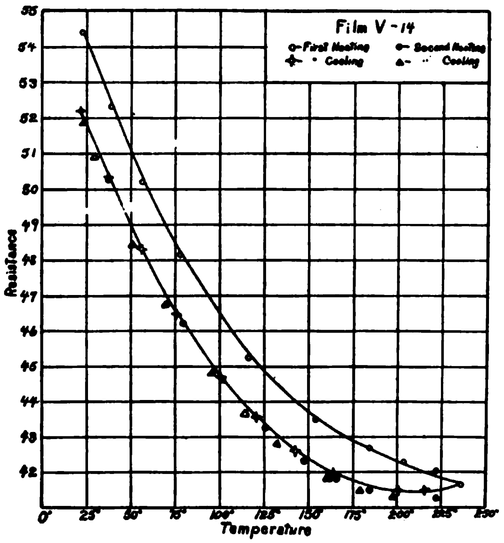

Fig. 9.

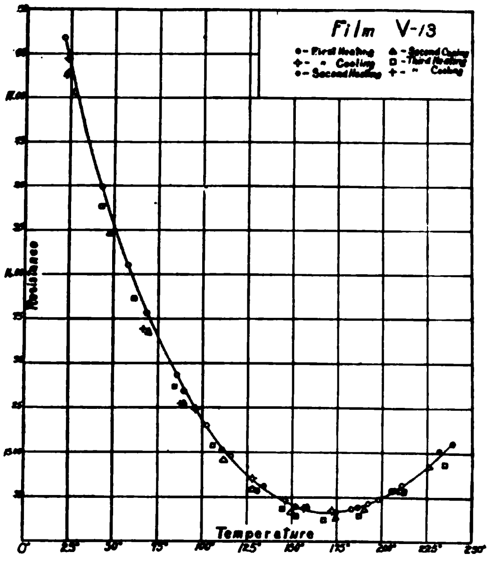

Fig. 10.

of the film. One would expect then that this would show itself in the behavior of the film in the magnetic field. Data taken in the magnetic field before and after these temperature-resistance measurements yielded the expected result. Since too many figures would be required to show all the results in detail, an idea of the relative magnitude of this change produced in the values of dr/r may be obtained by reference to Fig. 4,

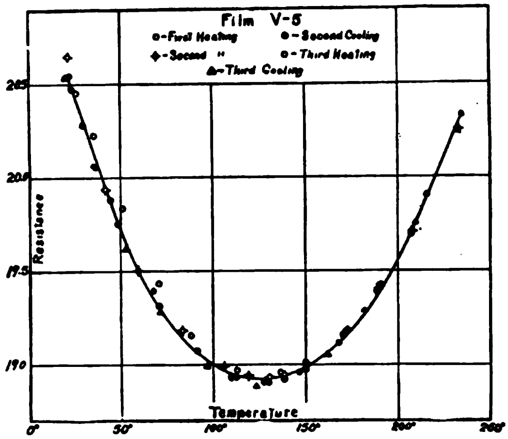

Fig. 11.

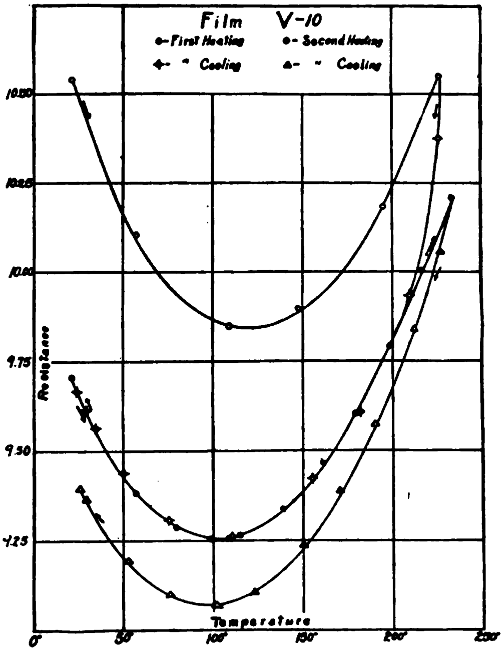

Fig. 12.

The above figures show typical temperature-resistance curves for films of different thickness.

where the similar results for Film V–10 are represented. The curves labelled *I* and *A* show the behavior of this film when treated in a like manner. Curve *I* was taken before the temperature-resistance run and Curve *A* afterwards at the same temperature. The result of the aging was, as shown, to increase the values of dr/r. A further point of interest in Fig. 9 is the suggestion of a minimum value of the resistance at about 210°. This minimum shows much more clearly in the curve for Film V–13 in Fig. 10. It occurs, moreover, at a lower temperature, in the neighborhood of 170°. In this connection it is to be noted that V–13 has a considerably lower resistance than the preceding film. Furthermore the preliminary heat treatment was carried out much more thoroughly, the results of which are strikingly shown by the reproducibility of the data. The observed points are here indicated by various symbols for three heatings and three coolings, taken over a period of several days, and yet a single curve represents all fairly well; only the very slightest traces of aging are to be detected. Likewise it was found possible to reproduce the measurements in the magnetic field with similar accuracy for this film. These results are quite contrary to any of the author's previous experiences with sputtered films. In fact, it had always been found impossible to so age a film that its resistance would remain constant for more than a few hours at room temperature, while exposed to the air, and reproducible measurements at higher temperatures were out of the question. To show more completely the form of the temperature-resistance curve the data for Film V–5 are shown in Fig. 11. Here both branches of the curve, which is perfectly symmetrical and in form approaches that of a parabola, are well defined. This film also has been thoroughly aged and the minimum resistance has shifted to a still lower temperature. An attempt to fit a parabolic equation to this curve has been made. Although the deviations are slight, they seem to be somewhat greater than possible experimental errors. All previous films of bismuth which have been studied by the author, or by others, so far as a search of the literature has revealed, have without exception had negative temperature coefficients of resistance. We have here, however, an example where the coefficient, initially negative at room temperature, actually reverses its sign as the film is heated and becomes positive. The maximum value of the positive coefficient which has been obtained is far less than that for bismuth in bulk, roughly of one quarter the magnitude. A further example of this form of curve is shown in Fig. 12, representing the results obtained with Film V–10. This is another example of a film that has been only partially aged, hence there was a big decrease in resistance as the film was heated for the first time. The effect of the second heating, as was to be expected, was considerably less, although it was carried out to a slightly higher temperature. The symmetry of the curves is considerably distorted by this aging effect which takes place at the higher temperatures. It is to be noted that the points representing the second heating lie closely on the curve for the first cooling, until the higher temperatures are reached, showing that the aging occurs chiefly at temperatures above 200°.

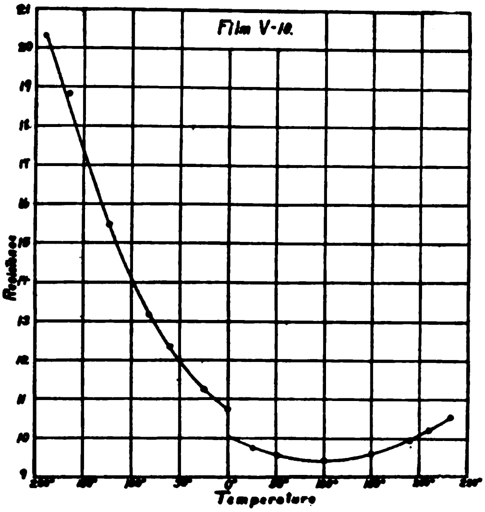

Fig. 13.

Showing the complete temperature resistance-curve for Film V-10.

In Fig. 13 the complete temperature-resistance curve for Film V-9 is represented. The part from room temperature to 230° was taken some months before the part at low temperatures. This fact, together with the effect of increased pressure to which the film was subjected when air was admitted to it, accounts for the jog in the curve at 0°. If the part taken at the higher temperatures were raised throughout by a constant amount, equal to the difference of ordinates at 0°, the two parts would then form a smooth curve which agrees in form with those taken for other films throughout this range of temperatures. Hence there seems to be no evidence of any peculiarities in the low temperature part of the temperature-resistance curves for these films.

To show what happens to one of these films when it is heated in contact with the air, a temperature-resistance run for Film V-8 is shown in Fig. 14. This film was prepared in the usual way and during the first heating and cooling behaved much the same as had all previous films which were thoroughly aged, the two sets of observations lying along the same curve. However, soon after the beginning of the second heating, the resistance began to increase rapidly. After cooling down it had a considerably higher resistance than before at room temperature. This anomalous behavior was hard to explain until the film was taken out of the bath and examined. It was then discovered that a yellowish coating of oxide had form on the surface, indicating that the glass tube had given way at some point, admitting air. On further examination this proved to be the case. There seems to be an additional effect of the air on the form of this curve beyond that produced by oxidation alone, however. The difference in ordinates at the maximum temperature is considerably greater than at room temperature, hence the increase of resistance at the higher temperature is not all permanent, as it would be if it were due solely to oxidation. This additional effect may well be caused by the pressure, although this point was not specially investigated.

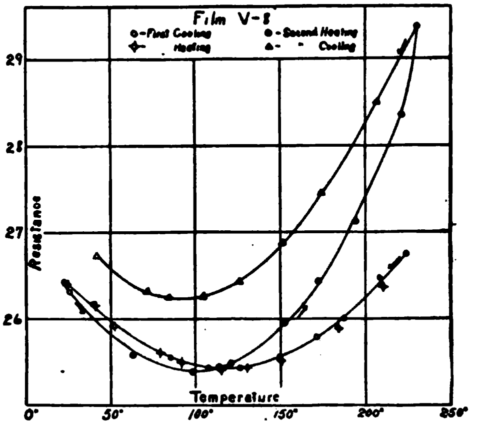

Fig. 14.

This figure shows the temperature-resistance curve obtained for Film V-8, which was attacked by the air as a result of a break in the protecting tube.

The foregoing covers the experimental results of this investigation. As far as the magnetic measurements are concerned little more can be said beyond the mere statement that these films approach the behavior of bismuth in bulk, the approach being nearer the thicker the film. Any theoretical discussion of these results and interpretation in terms of the electron theory would necessarily follow the same general lines as those for bismuth in bulk. Too little is known at present with any degree of certainty about the nature of electrical conduction in metals to enable very much progress to be made in this direction beyond what has already been accomplished by Thomson and others. This is very clear when attention is called to the fact that the most successful theories so far advanced,[1] either agree with the experimental data only for very limited ranges of the field strength, or require properties which bismuth does not possess, *e.g.*, magnetostriction.

It is also difficult to explain successfully the form of the temperature-resistance curves of these films. One explanation which might be offered for the reversal of the sign of the temperature coefficient and the consequent minimum resistance, the location of which varies from film to film, and is, in general, at a lower temperature the lower the resistance of the film, is as follows: It is based on the fact that the metal particles have a larger coefficient of thermal expansion than the glass. Hence as the film rises in temperature they expand more rapidly and bring more particles into contact, reducing the negative temperature coefficient of resistance, and finally, if the film is thick enough, the particles are crowded close enough together and at a rate fast enough to reduce this coefficient to zero, then finally to reverse its sign. Then the metallic film begins to behave somewhat like the metal in bulk, but is prevented from attaining a positive coefficient as large as that for solid bismuth by the fact that the glass also continues to expand, thus preventing the compacting process from going far enough to establish contact between all the possible particles. The study of this phase of the subject is limited by the comparatively low melting point of bismuth (about 260°). This explanation is far from satisfactory, however, since on the basis of it a platinum film sputtered on glass should have practically a zero temperature coefficient, which is contrary to the facts as reported by other observers. But then there must be taken into account also the effect of temperature on the conductivity of the metallic particles themselves. Hence we have at least two influences which must be considered simultaneously. The first is that due to the difference in the thermal coefficients of expansion which should cause an increase or decrease in the number of possible paths for electrons from particle to particle, according as the film is heated or cooled. The second is that which results from the effect of temperature upon the motions of the electrons within the particles, and also its effect upon the ease with which an electron may detach itself from one particle and move on to the next. This view of the structure of the films is strengthened by the fact that x-ray studies[2] of the films indicated that they were composed of small crystals of metal, with random orientation, which have been detached from the cathode and deposited on the glass during sputtering. This view is in contrast with the evaporation theory which holds that the metal of the cathode is vaporized and then condenses on the glass, in which case a film of entirely different structure from that of the cathode metal might result. However, this point is not settled.

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[1] E. P. Adams, Phys. Rev., 24, p. 248, 1907.

[2] H. Kahler, Phys. Rev., 17, p. 230, 1921.

For further comparison of the results here given with those obtained for bismuth in bulk, a reference to an investigation by Dr. F. C. Blake[1] is included. His work, published in 1909, covers practically the same range for the solid metal.

SUMMARY.

The preceding results may be summarized as follows:

1. By protecting sputtered bismuth films in a vacuum and heating them to temperatures near the melting point of bismuth it is possible to develop the property of an increase of resistance in a magnetic field to about one fifth that possessed by bismuth in bulk.

2. The higher the temperature of the film the smaller the value of the percentage increase of resistance for a given field.

3. The form of the curve between dr/r and field strength is independent of the temperature of these films, *i.e.*, $dr/r = kf(H)$.

4. The temperature-resistance curves for these films are parabolic in form, and the minimum resistance is at a lower temperature the thicker the film.

5. If these films have been subjected to a very thorough heat treatment the temperature-resistance curve is fixed and definite. Under these conditions reproducible data are easily obtained in the magnetic field.

6. If the film is only partially aged it *decreases* in resistance with continued heating and exhibits correspondingly greater values of dr/r in the magnetic field as a result of the heating.

In conclusion I wish to thank Professors Ernest Merritt and F. K. Richtmyer, under whose direction this work was done, and all others, who, by advice or suggestions, have helped the progress of this investigation.

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[1] F. C. Blake, Ann. d. Phys., 28, p. 449, 1909.

CONDUCTIVITY OF FLAMES CONTAINING SALT VAPORS.

BY A. B. BRYAN.

SYNOPSIS.

Conductivity of a Flame Sprayed with Salt Solutions, as a Function of the Concentration.—The method used is due to H. A. Wilson. The ratio of the potential gradients in the central parts of two flames, each about 7 cm. wide, through which the same current was passed, was determined by means of a quadrant electrometer connected alternately to each of two pairs of probe wires in the flames, by means of a rotating commutator. Both flames were sprayed with identical sprayers. When desired for greater sensitiveness the current through the flames could be increased by applying K_2CO_3 to the cathodes. Using the conductivity produced by distilled water as a standard, the relative conductivities of various concentrations of *potassium carbonate, sodium carbonate, cæsium chloride, calcium chloride, barium chloride, aluminium chloride and boric acid* were determined. For the last four the conductivity was found to be approximately proportional to the cube root of the concentration, whereas in the case of the others the relation is more complicated. A *theory* to account for the cube root relation is suggested. The conductivities due to *mixtures of two solutions* were found to agree with the values computed from the conductivities due to the components alone. *Uranium nitrate* gave little or no conductivity.

I. INTRODUCTION.

THE first extensive investigation of the electrical conductivity of flames containing salt vapors was made by Arrhenuis[1] in 1890. He used a vertical flame with carefully regulated air and gas supplies and measured the current between two platinum electrodes in the flame with a constant potential difference, taking this current as a measure of the conductivity. A sprayer was arranged to spray a salt solution into the flame with the air supply and the effect of the salt on the conductivity could thus be measured.

Arrhenius made experiments with salts of the alkali metals, alkaline earth metals and some of the heavier metals, such as copper, zinc, iron and aluminium. For the alkali metals, all salts of each metal were found to give the same conductivity, and this conductivity was proportional to the square root of the concentration of the solution used. The conductivities for the different alkali metals were found to increase with the atomic weight. For the alkaline earth metals, no measurements were made on the variation of conductivity with concentration. The heavier metals were found to give little if any conductivity. The values obtained with them were all so small as to fall within the limits of experimental error. For a mixture of two alkali metal salts (sodium and potassium sulphate or chloride), the conductivity produced was found to be greater than that produced by either of the two salts separately but less than the sum of the two.

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[1] Wied. Ann., Vol. xliii., p. 18, 1891.

Other measurements of the conductivity produced by salt vapors in flames have been made by A. Smithells, H. M. Dawson and H. A. Wilson.[1] The apparatus used by them was similar to that used by Arrhenius and they also found that for the halogen salts of the alkali metals, the conductivity was proportional to the square root of the concentration. For the oxysalts this was not true, except at very small concentrations.

The electrical conductivity of flames has also been measured by H. A. Wilson.[2] He found that for large concentrations, the conductivity produced by salts of alkali metals was proportional to the square root of the concentration used and independent of the nature of the acid radical, so long as solutions having equal numbers of metal atoms per cubic centimeter were used. Wilson's experimental method, which has several advantages over that used by Arrhenius, has been used in the present work and will be fully described later.

In this paper additional data will be presented on the conductivity produced by salts of the alkali metals sodium, potassium and cæsium and various mixtures of the same, and by calcium, barium and aluminum chlorides, boric acid and uranium nitrate.

II. Experimental Method.

The apparatus used in this experiment is in all essentials similar to that used by H. A. Wilson. Two similar burners were used (*A* and *B*, Fig. 1), each consisting of a row of eight short quartz tubes set in a horizontal brass tube (*bb*) with De Khotinsky cement. The quartz tubes were of approximately 0.5 cm. internal diameter. In each flame were two electrodes (*EE*) of thin sheet platinum, 1.5 cm. square and 6.5 cm. apart, and two horizontal platinum wires (*WW*) 2.5 cm. apart. The electrodes in the two flames were connected directly in series with a battery (B_5)

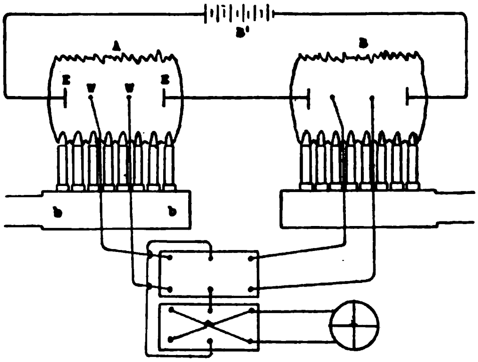

Fig. 1.

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[1] Phil. Trans., A, Vol. 193, p. 89, 1900.

[2] Phil. Trans., A, Vol. 216, p. 63.

of 15 volts, so that the same current passed through both flames. No current measuring instrument of any kind was used. The wires (*WW*) in both flames were connected through a commutator and reversing key to the quadrants of a quadrant electrometer, in such a manner that either pair of wires could be connected to the electrometer in turn. A rather heavy quartz fiber electrometer suspension was used. The needle of the electrometer was continuously connected to a source of potential, usually about 86 volts, and the sensitivity of the electrometer with this potential on the needle was 52 mm. per volt. The electrodes (*EE*), the wires (*WW*) and the battery (*B'*) were all supported on ebonite rods which afforded good insulation. The electrometer was insulated on ebonite blocks and its case was connected to one pair of quadrants. The commutator and reversing key were made of paraffin blocks.

Each burner was connected to a sprayer which sprayed a solution into the flame. Fig. 2 shows one of the burners connected to its sprayer with a short piece of large rubber tubing. Gasoline gas was admitted at *G* and air at *A*. The air supply, taken from the compressed air mains of the building, was maintained at constant pressure by allowing part of it to escape through a mercury trap and by passing it through two large carboys in series to remove small fluctuations. The air pressure was measured with a water manometer. Connections to the two sprayers from the common air and gas supplies were made with equal lengths of large rubber tubing, so that the air and gas pressures, respectively, at the two sprayers were the same. It was not found practicable to maintain the gas pressure constant from day to day, due to the varying

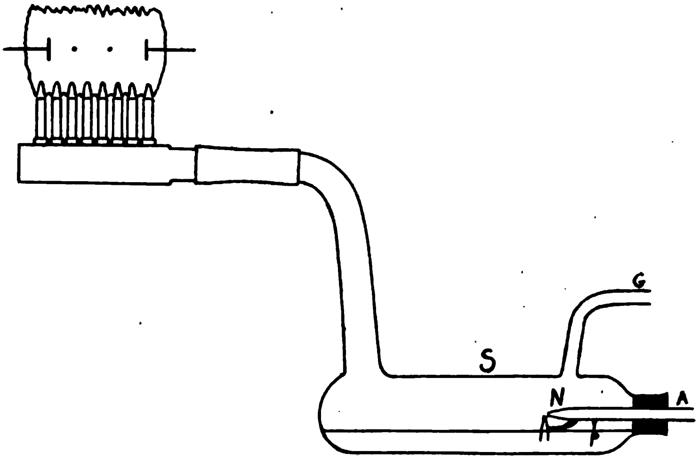

Fig. 2.

quality of the gas, and so the practice was to adjust the gas pressure each time to get equal non-luminous flames with well defined cones. The spraying nozzle (*N*) was fixed into the body of the sprayer through a rubber stopper and was held firmly by supports outside the sprayer, since it was found that if the nozzle tilted up very slightly toward the gas inlet the flow of gas was greatly increased due to the "injector" action of the stream of air. A small glass point (*P*) was fixed to the nozzle to mark the level to which the sprayer was always filled with solution. The two sprayers were made as nearly alike as possible and the nozzles were adjusted to deliver almost equal amounts of liquid and of air per second on equal air pressures. The sprayers were tested and it was found that sprayer *A* required 8.6 c.c. to refill after going 40 minutes, while sprayer *B* required 8.4 c.c. to refill after the same time. Sprayer *A* required 4.4 c.c. to refill after going 20 minutes, roughly half as much as before, indicating that the rate of delivery of solution to the burners is uniform and not much affected by the lowering level of solution in the sprayer during a long period of time. The air supplies were found to be equal to within one per cent. But it is not necessary that the two sprayers be exactly alike and is not necessary to assume that they deliver equal amounts of solution. A conical hood was fixed above each flame and connected to a suction pump to remove the flame gases and salt vapors and prevent their escape into the room.

The method of measuring conductivities with this apparatus will now be explained. Flame *A* serves merely as a standard with which the conductivity of flame *B* is compared, thus making it possible to obtain the conductivities of various solutions in *B* relative to one another. The conductivity of flame *B* is taken as unity when its sprayer is spraying distilled water. The electrometer is alternately connected, by means of the commutator, to the wires *EE* in flame *A* and to those in flame *B*, and the ratio *R* of the electrometer deflection for flame *A* to that for flame *B* is determined. Since the electrometer deflection is inversely proportional to the conductivity of the flames, *R* will be inversely proportional to the conductivity of flame *B* when that of flame *A* is kept constant. Hence if the ratio of the deflections is R_1 with water only in *B* and R_2 with a salt solution in *B*, the solution in *A* being the same in each case, then R_2/R_1 will be equal to the conductivity of flame *B* with the salt solution in it, since the conductivity of *B* is taken equal to unity when only water is in it. To get accurate results it is necessary to keep the ratio *R* less than three and greater than one third. To do this when solutions of increasing concentration are used in *B* it is necessary to increase the concentration of the solution in *A*. When the solution in *A* is changed the same solution must be kept in *B* for two readings, in order that the conductivity of the new standard may be determined. To illustrate the procedure a number of readings are given in Table I. below, whereby the conductivities of a number of CsCl solutions were determined. Under *A* and *B* are given the solutions which were sprayed into flames *A* and *B* respectively. Under *R* are given the ratios of electrometer deflections for flame *A* to those for flame *B*. Column C_A gives the conductivities of flame *A*. These values are given for convenience in calculation only and must not be taken as the true conductivities for the various solutions. In column C_B are given the true conductivities for the solutions which have been used in flame *B*. The concentrations of the solutions are given in grams per liter.

Table I.

A	B	R	C_A	C_B
13.001 K_2CO_3	H_2O	.806	1.24	1.0
.001 K_2CO_3	.004 CsCl	2.19	1.24	2.72
.05 K_2CO_3	.004 CsCl	.288	9.44	2.72
.05 K_2CO_3	.04 CsCl	1.30	9.44	12.25
.05 K_2CO_3	.04 CsCl + .10 K_2CO_3	2.63	9.44	24.8
2.00 K_2CO_3	.04 CsCl + .10 K_2CO_3	.33	74.4	24.8
2.00 K_2CO_3	.40 CsCl	.739	74.4	55.0
2.00 K_2CO_3	4.00 CsCl	2.09	74.4	155.5

The advantages of this experimental method will now be considered. The potential along a flame was found by H. A. Wilson to fall in the way shown in the curve in Fig. 3, which is taken from his paper.[1] There

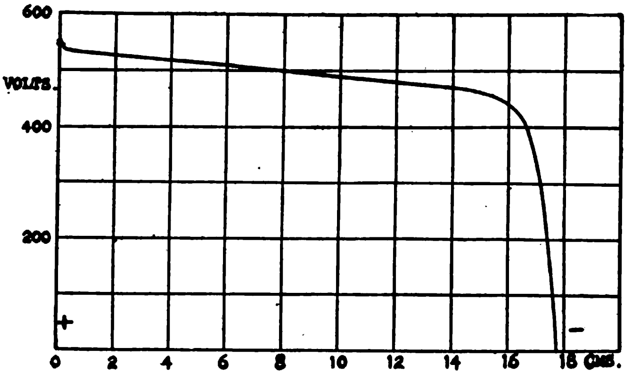

Fig. 3.

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[1] Proc. of the Phys. Soc. of London, Vol. xx., p. 476.

is a slight drop at the positive electrode, then a uniform gradient through the greater part of the flame's length and finally a very large drop close to the negative electrode. The total potential difference is 550 volts and of this there is a drop of 450 volts within one centimeter of the cathode. The relation between potential and current may be represented by the equation,

$$V = ACd + BC^2. \tag{1}$$

The term BC^2 represents the potential drops at the electrodes and is usually much larger than the ACd term, especially if the electrodes are close together. Thus if only two electrodes are used in the flame and the value of C/V is taken as a measure of the conductivity of the flame. as was done by Arrhenius, this value will not be constant but will decrease as the current is increased. But if the potential difference between two points in the middle of the flame, away from either electrode, is used, then the value of C/V will be independent of C and will furnish a better measure of the flame's conductivity, since Ohm's Law holds in the center of the flame. This is what is done in the method just described. The potential difference between the wires EE was measured with an electrostatic instrument so that no current was taken from the flame and the manner in which the potential falls along the flame was not changed by the presence of the wires.

It would be possible to measure the current through the flame directly and use the ratio of this current to the potential difference between the wires EE as a measure of the flame's conductivity, but there are several objections to this. It is difficult to keep the air and gas supplies, the electrometer sensitivity and all other conditions absolutely constant for any length of time, and so what is done instead is to put the flame A in series with flame B and use it as a standard for comparison. Then any changes affecting flame B will affect A equally and the ratios will not be changed.

The current through a flame can be greatly increased by applying K_2CO_3 to the cathode. The vaporizing K_2CO_3 will make the resistance of the flame very small in the space next to the cathode and since a large part of the flame's total resistance is in this space, as may be seen from Fig. 3, the total resistance will be decreased by a large percentage and the current correspondingly increased. It was frequently necessary to increase the current in this way in the present work in order to get electrometer deflections large enough to be read accurately. CaO, obtained by coating the cathodes with sealing wax and then heating them, was sometimes used instead of K_2CO_3.

III. Results.

In Table II. below are given the conductivities for solutions of several concentrations of K_2CO_3, Na_2CO_3, and CsCl.

Table II.

Solution Sprayed.	Conductivity.
.01 K_2CO_3	5.52
.10 "	22.9
1.00 "	77.7
10.00 "	225.0
.005 Na_2CO_3	1.69
.02 "	3.31
.10 "	6.36
1.0 "	14.55
10.0 "	37.7
50.0 "	75.0
.004 CsCl	2.72
.04 "	12.25
.4 "	55.0
4.0 "	155.5

The values for K_2CO_3 and Na_2CO_3 are averages of a number of readings. These averages were taken in the following way. A number of sets of readings such as that given in Table I. were taken and from each of these sets a value was obtained for the ratio of the conductivity for .01 K_2CO_3 solution to that for water. The average of these ratios was used to calculate the conductivity for .01 K_2CO_3, assuming the conductivity for water to be unity. Similarly a value was obtained from each set of readings for the ratio of the conductivity for 0.10 K_2CO_3 to that for .01 K_2CO_3, and the average of these ratios was used to calculate the conductivity for the .10 K_2CO_3 solution, and so on. This method of averaging prevents an error in the first part of a set of readings from affecting all the later values. In Table III. are given the values of these ratios for different solutions of K_2CO_3. Those numbers marked with asterisks are omitted in taking the averages. For the others the average variation from the mean values is about four per cent. This gives a fair idea of the accuracy of the results.

According to the theory given by H. A. Wilson[1] for the electrical conductivity of flames containing salt vapors, the relation between the conductivity C and the concentration K of salt solution sprayed is given by the equation,

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[1] Phil. Trans., A, Vol. 216, p. 79.

TABLE III.

$\frac{.01\ K_2CO_3}{H_2O}$	$\frac{.10\ K_2CO_3}{.01\ K_2CO_3}$	$\frac{1.00\ K_2CO_3}{.10\ K_2CO_3}$	$\frac{10.\ K_2CO_3}{1.\ K_2CO_3}$
4.48*	3.52*	3.85*	2.85
5.22	4.46	3.64	2.84
5.85	5.85*	3.27	2.53*
5.36	4.39	3.31	2.87
5.42	4.09	3.29	3.01
5.75	3.83		
	4.02		
5.52	4.16	3.38	2.89

$$\frac{10^4KC}{C^2 - 1} = b + aC, \tag{2}$$

where a and b are constants. When C is large so that bC may be neglected in comparison with aC^2, then the above equation may be written,

$$10^4K = aC^2. \tag{3}$$

That is, the conductivity is proportional to the square root of the concentration, as was found to be approximately true by both Arrhenius and H. A. Wilson.

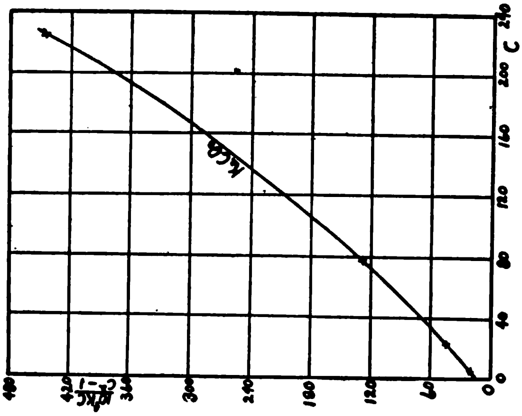

Fig. 4.

When $(10^4KC/(C^2 - 1))$ is plotted against C, a straight line should be given according to equation 2. In Figs. 4, 5 and 6 are given these curves, plotted for K_2CO_3, Na_2CO_3 and $CsCl$. They are not straight lines but are all slightly concave upward, indicating that the theory as given by Wilson does not accurately fit the experimental results. It has been found empirically that C is approximately a linear function of $K^{.435}$. In Figs. 7, 8 and 9, C is plotted against $K^{.435}$ for the data in

Table II. and it will be seen that a good straight line is given in each case. Hence the relation between concentration and conductivity may be expressed by the equation,

$$K^{.435} = AC + B, \tag{4}$$

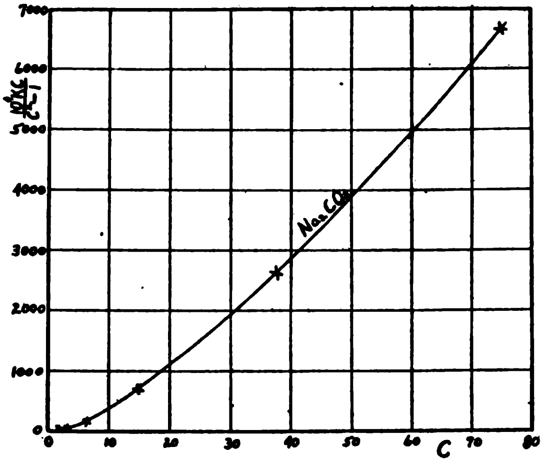

Fig. 5.

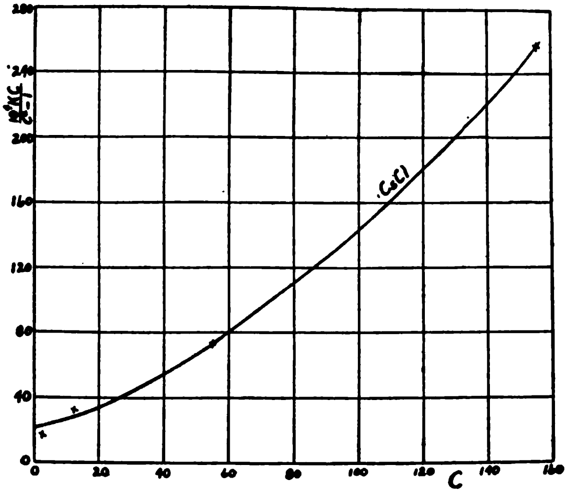

Fig. 6.

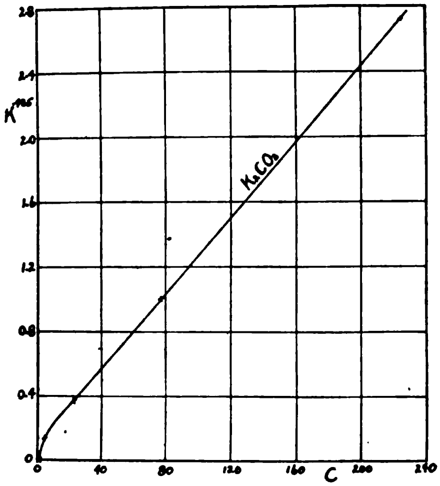

Fig. 7.

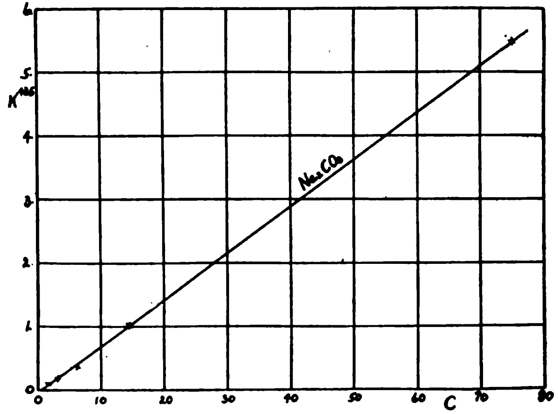

Fig. 8.

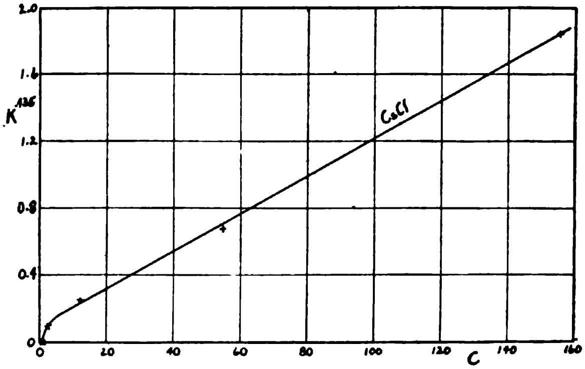

Fig. 9.

where A and B are constants. This equation holds for the larger conductivities only. For the smaller values the relation between C and $K^{.435}$ is not linear and an equation of the form,

$$K^{.435} = (C - 1)\left(A + \frac{B}{C^n}\right) \tag{5}$$

will better fit the experimental curves.

In Table IV. are given the experimentally determined and the calculated conductivities for several mixtures of salts.

Table IV.

Solution Sprayed.	Experimental Conductivity.	Calculated Conductivity.
0.1 K_2CO_3 + 1.0 Na_2CO_3	27.1	26.2
0.1 " + 10.0 "	40.2	42.6
1.0 " + 10.0 "	85.0	83.6
1.0 " + 20.0 "	88.1	88.1
0.1 " + 0.04 CsCl	24.8	25.1
0.1 " + 0.1 "	33.5	33.5
1.0 " + 0.4 "	89.7	91.2
1.0 " + 1.0 "	115.0	109.5

The following equation is also given by H. A. Wilson[1] to represent the relation between conductivity and concentration when a mixture of two salts is used.

$$\frac{C^2 - 1}{C} = \frac{10^4 K_1}{b_1 + a_1 C} + \frac{10^4 K_2}{b_2 + a_2 C}, \tag{6}$$

where K_1 and K_2 are the numbers of grams of the two salts in a liter of the solution which produces a conductivity C. a_1, b_1, a_2 and b_2 are constants depending on the salts used. When C is large b_1 and b_2 may be neglected and the equation may be written:

$$C^2 = \frac{10^4 K_1}{a_1} + \frac{10^4 K_2}{a_2}. \tag{7}$$

But from equation (3),

$$\frac{10^4 K}{a} = C^2,$$

when a single salt is used, and therefore, if C_1 and C_2 are the conductivities for concentrations K_1 and K_2 of two salts taken separately, then

$$\begin{aligned} C^2 &= C_1^2 + C_2^2, \\ C &= \sqrt{C_1^2 + C_2^2}. \end{aligned} \tag{8}$$

This equation could be used to calculate what the conductivity due to a mixture of two salts should be, knowing the conductivities which the two salts produce separately. But since in the present work C has been found to be more accurately proportional to $\sqrt[2.3]{K}$ than to $\sqrt{K}$ for large concentrations, a more accurate formula will be,

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[1] Phil. Trans., A, Vol. 216, p. 87.

$$C = \sqrt[2.3]{C_1^{2.3} + C_2^{2.3}}. \tag{9}$$

This formula has been used to obtain the calculated values of conductivity in Table IV. above. It will be seen that the agreement between experimental and calculated values is fairly good in every case.

The conductivity was also measured for several concentrations of $CaCl_2$ solution. The curve in Fig. 10 shows the results obtained. At concentrations of above 15 grams per liter a white deposit of oxide is formed on the electrodes and burner tubes. The whole flame is itself filled with small particles. This was shown by passing a beam of sunlight through the flame. The path of the beam through the flame could be seen on account of the light scattered by the small particles and this scattered light was found to be polarized.

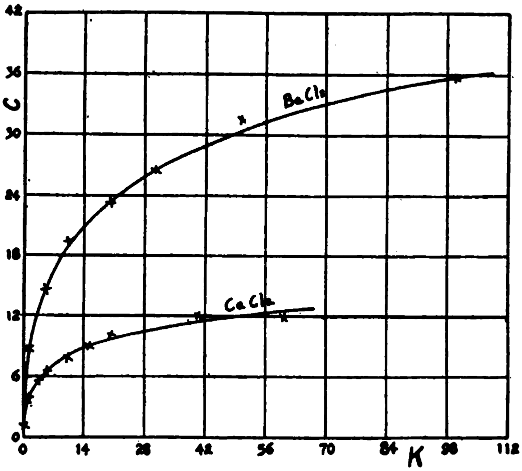

Fig. 10.

A $CaCl_2$ solution was found to make the most satisfactory standard solution for flame *A* because the oxide deposit produced on the electrodes kept down the potential drop at the cathodes and did not require renewal, as was the case when K_2CO_3 or lime was used on the cathodes.

The curve obtained for $BaCl_2$ is also shown in Fig. 10. It is seen that at a given concentration $BaCl_2$ gives about twice as much conductivity as $CaCl_2$. $BaCl_2$ does not produce any noticeable deposit on the electrodes.

If (log *K*) is plotted against (log *C*) for $BaCl_2$ and $CaCl_2$ as in Fig. 11, straight lines are obtained and from the slopes of these lines it is found that the conductivity is roughly proportional to the cube root of the

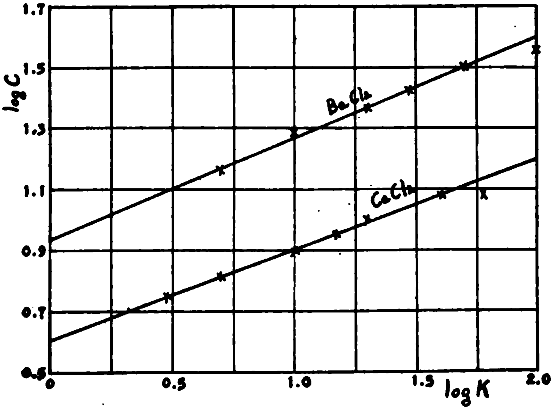

Fig. 11.

concentration. The following theory for this variation of C with K was worked out as the result of a suggestion by Professor H. A. Wilson.

The solution is sprayed into the flame in minute drops, each of which evaporates and leaves a small, non-volatile, salt particle as it enters the flame. The number of drops entering the flame should depend only on the sprayer and should be independent of the concentration of the solution. Hence the number of particles per cubic centimeter in the flame, formed by the evaporation of small drops of solution, should be constant. But the volume of each of these particles should be proportional to the concentration of the solution used. These particles will emit electrons and become positively charged.

Let N = number of positively charged particles per cubic centimeter in the flame.
r = radius of one of these particles.
E = the positive charge on each particle.
n = number of electrons per cubic centimeter which come from the salt particles.
m = number of electrons per cubic centimeter which are produced by ionization of flame gas molecules.
F = number of flame gas molecules per cubic centimeter capable of being ionized.
k = velocity of an electron in unit electric field.

The rate of emission of electrons by the positive particles in one cubic centimeter will be proportional to N and to the surface area of each particle, or equal to $BN4\pi r^2$ where B is a constant.

The electric intensity at a distance d from a positive particle will be E/d^2. The rate of absorption of electrons by a positive particle will be equal to $\{E/d^2 \cdot k 4\pi d^2 (n + m)\}$. The rate of absorption per cubic centimeter will therefore be $\{NEk4\pi(n + m)\}$ and must be equal to the rate of emission when a state of equilibrium is reached. Therefore,

$$NEk4\pi(n + m) = BN4\pi r^2, \tag{10}$$

$$r^2 = \frac{Ek(n + m)}{B}.$$

Since the total positive charge on the particles in a cubic centimeter must be equal to the total negative charge on the electrons which have been emitted by these particles, we have,

$$NE = ne,$$

where (e) is the charge on one electron,

$$E = \frac{ne}{N},$$

$$r^2 = \frac{nek}{NB}(n + m),$$

$$n = \frac{Ar^2}{(n + m)},$$

where

$$A = \frac{NB}{ek}. \tag{11}$$

The rate at which electrons are produced by ionization of the flame gas molecules is proportional to F. The rate at which electrons are lost through recombination with the flame gas positive ions is proportional to the number of these ions m and to the number of electrons per cubic centimeter. Since the rate of production must be equal to the rate of recombination when a state of equilibrium is reached, we have,

$$F = \beta m(n + m),$$

$$m = \frac{F}{\beta(n + m)}, \tag{12}$$

where β is a constant. Adding (11) and (12), we have,

$$n + m = \frac{Ar^2\beta + F}{\beta(n + m)},$$

$$n + m = \sqrt{Ar^2 + \frac{F}{\beta}}. \tag{13}$$

As has been said, the volume of a particle must be proportional to the concentration of the solution, or

$$\frac{4}{3}\pi r^3 \propto K,$$

$$r^2 \propto K^{2/3},$$

where K is the concentration in grams per liter. Then (13) becomes

$$n + m = \sqrt{A_1K^{2/3} + \frac{F}{\beta}}, \tag{14}$$

where A_1 is a new constant.

We assume that the positive particles are so heavy that they remain practically stationary and the electrons carry all the current. The conductivity C will be proportional to the total number of electrons per cubic centimeter.

$$C = A_2(n + m).$$

Substitute for $(n + m)$ from (14),

$$C = A_2\sqrt{A_1K^{2/3} + \frac{F}{\beta}}. \tag{15}$$

Now assume $C = 1$ when $K = 0$, that is, when there is no salt in the flame. Then (15) becomes,

$$1 = A_2\sqrt{\frac{F}{\beta}},$$

$$\frac{F}{\beta} = \frac{1}{A_2^2}.$$

Substitute this value back into (15),

$$C = A_2\sqrt{A_1K^{2/3} + \frac{1}{A_2^2}},$$

$$C^2 - 1 = A_3K^{2/3}, \tag{16}$$

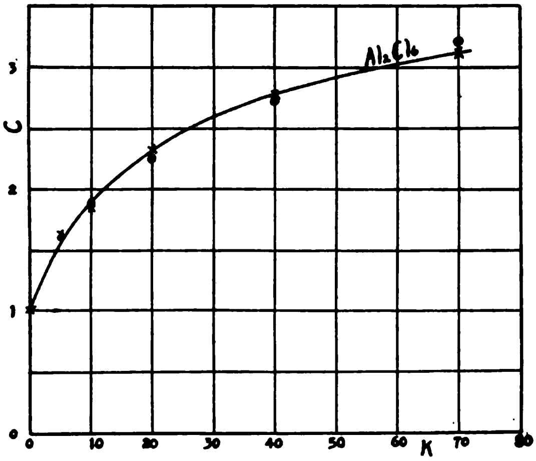

Fig. 12.

between conductivity and concentration. When C is large compared with one, the conductivity is directly proportional to the cube root of the concentration, as was observed for $CaCl_2$ and $BaCl_2$. No measure-
where A_3 is a new constant. This equation gives the final relation

ments were made with very small concentrations of these salts and consequently the above equation cannot be further tested with the data on $BaCl_2$ and $CaCl_2$.

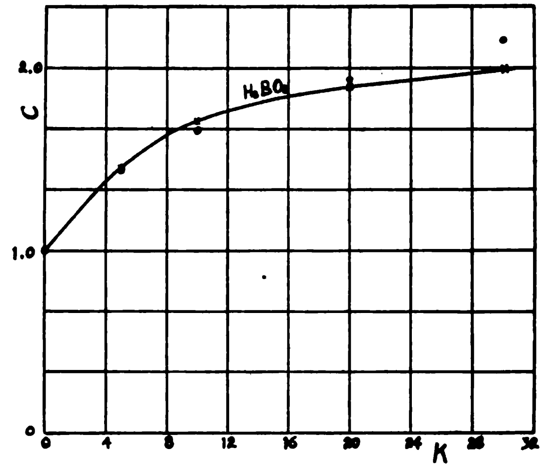

Fig. 13.

In Figs. 12 and 13 are shown curves obtained for Al_2Cl_6 and H_3BO_3. These curves may be used to test equation (16) because for these two salts the conductivities are small. The crosses in Figs. 12 and 13 indicate the experimental points.

TABLE V.

Al_2Cl_6			H_3BO_3		
K	C	A_3	K	C	A_3
5.0	1.63	0.568	5.0	1.46	0.387
10.0	1.84	0.512	10.0	1.71	0.407
20.0	2.32	0.594	20.0	1.90	0.380
40.0	2.77	0.567	30.0	1.99	0.306
70.0	3.12	0.513			

In Table V. K is the concentration in grams per liter, C the experimentally found conductivity and A_3 the constant of equation (16), calculated from the values of C and K. The variation of A_3 for Al_2Cl_6 is about 14 per cent. and for H_3BO_3 about 25 per cent.

In Figs. 12 and 13 the circled points represent values of conductivity calculated from equation (16), using the given values of K and average values of A_3. The agreement with the experimental curve is about as good as could be expected, since it is not possible to measure these small conductivities with great accuracy.

A single measurement was made with uranium. The conductivity produced by a solution containing 10 grams of $UO_2(NO_3)_2$ per liter was found to be 3.19. When this solution was being sprayed the whole flame had a distinct sodium yellow color and a bright sodium line was seen when the flame was observed through a spectroscope. The flame also appeared to be filled with small luminous particles moving rapidly upward, giving it a streaked appearance. It is probable that the small conductivity observed was due to sodium impurity and that the $UO_2(NO_3)_2$ itself produced little or no conductivity.

In conclusion, I wish to express my sincere thanks to Professor H. A. Wilson for his valuable advice and kindly interest during the course of this work.

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A GENERALIZATION OF ELECTRODYNAMICS WITH APPLICATIONS TO THE STRUCTURE OF THE ELECTRON AND TO NON-RADIATING ORBITS.

BY LEIGH PAGE.

SYNOPSIS.

Generalization of Electrodynamics.—If every element of charge is assumed to be the center of equal electric and magnetic fields, each of which is capable of rotation, the electromagnetic equations assume a symmetrical form, containing Lorentz's formulation of Maxwell's equations as a special case.

Structure of the Electron.—If the electric field of each element of charge on the surface of the Lorentz electron is supposed to rotate about the normal to the surface, a magnetic field is produced which just annuls the intrinsic field due to the magnetic charge assumed to be distributed over the surface, and which exerts an inward stress equal to the outward stress of electrostatic repulsion. Not only is the surface in equilibrium, but the equilibrium is found to be stable for small volume changes.

Non-Radiating Electronic Orbits.—A suitable rotation of the fields of the elementary charges constituting the electron about an axis parallel to the acceleration would cause the radiation term to vanish. This suggests the possibility of radiationless orbits.

SEVEN years age the author of this paper proposed a kinematical interpretation of the electromagnetic equations[1] which both H. Bateman[2] and he[3] have since developed and generalized. In this interpretation electric charges are assumed to be the fundamental constituents of matter, magnetic poles existing only as secondary entities. Each element of electricity is supposed to emit uniformly in all directions with the velocity of light continuous streams of *moving elements*, or, as Bateman has termed them, *light particles*. Each moving element travels out from its source in a straight line uninfluenced by the subsequent motion of the source or by the presence or motion of neighboring moving elements. The nature of these moving elements is immaterial for the purposes of the theory other than that each must be susceptible of continuous identification and must move in a straight line with the velocity of light. A *line of electric force* is defined as the locus of a stream of moving elements emerging from a single source. To employ an analogy, a source may be likened to a machine gun firing bullets with the velocity of light. The bullets correspond to moving elements. If they are supposed to be strung along an endless perfectly elastic thread, such a thread constitutes a line of force. A charged particle, then, consists of a group of such guns pointing uniformly in all directions, each firing a continuous stream of bullets. If the charge is in motion, a line of force emerging from it will not in general have the same direction as the velocities of the moving elements which constitute the line. Thus arises a magnetic field, which has a direction at right angles to the electric lines of force and to the direction of motion of the moving elements constituting these lines, and an intensity proportional to the sine of the angle between these two vectors.

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[1] Am. Jour. Sci., 38, p. 169, 1914.

[2] Phil. Mag., 41, p. 107, 1921.

[3] Proc. Nat. Acad. Sci., 6, p. 115, 1920.

The number of lines of force emerging from an element of electric charge must be supposed to be very great. These lines may be grouped into bundles, each of which may be called a *tube of force*. The *electric intensity* **E** is defined as the number of tubes of force per unit cross section. If the light-velocity of the moving elements is denoted by the vector **c**, the magnetic intensity **H** of an elementary field is defined by

$$\mathbf{H} = \frac{1}{c}(\mathbf{c} \times \mathbf{E}).$$

The total electric or magnetic intensity at a point is the vector sum of the electric or magnetic intensities respectively of all the elementary fields which extend to that point.

Coulomb's and Ampère's laws follow at once as necessary consequences of this representation, and if the Lorentz-Einstein space and time transformations are used, Faraday's law and the law expressing the fact that the divergence of the magnetic intensity is zero, can be shown to hold exactly, provided the sources of moving elements do not rotate.[1] Thus the proposed representation contains the four field equations of electromagnetism. Furthermore, if it is assumed that the force on a charge e relative to an observer at rest with respect to the charge is given by the product $e\mathbf{E}$, the fifth equation of electrodynamics is easily deduced. In fact, the abundantly proven validity of this last relation, as interpreted on the basis of the representation under discussion, constitutes better experimental confirmation of the restricted relativity principle, perhaps, than any other evidence which has yet been adduced.

The generalization of the classical equations of electromagnetism which is obtained by taking into account the possibility of rotation of the sources of moving elements[2] is particularly suggestive. The field equations assume the symmetrical form (in Heaviside-Lorentz units)

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[1] See L. Page, "An Introduction to Electrodynamics," Chap. II., Ginn & Co., 1921.

[2] Proc. Nat. Acad. Sci., 6, p. 120, 1920.

$$\nabla \cdot \mathbf{E} = \rho, \tag{1}$$

$$\nabla \times \mathbf{E} = -\frac{1}{c}(\dot{\mathbf{H}} + \epsilon \mathbf{u}), \tag{2}$$

$$\nabla \cdot \mathbf{H} = \epsilon, \tag{3}$$

$$\nabla \times \mathbf{H} = \frac{1}{c}(\dot{\mathbf{E}} + \rho \mathbf{v}), \tag{4}$$

where the velocity u of the free magnetism produced by a rotating field is in general equal to the velocity c of light. If the entire field rotates with the same angular velocity ω, then at a time r/c

$$\epsilon = \frac{e}{2\pi r^2 k^2 c^2} \frac{\omega \cdot \mathbf{c} - \omega \cdot \mathbf{v} k^2 \left(1 - \dfrac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)}{\left(1 - \dfrac{\mathbf{c} \cdot \mathbf{v}}{c^2}\right)^4}, \tag{5}$$

at a distance r from a charge e which is moving as a whole with velocity $\mathbf{v}$ at the time 0. The letter k stands for the reciprocal of $\sqrt{1 - \beta^2}$, β being the ratio of the velocity of the charge to that of light. If, now, the energy radiated for each complete rotation of the field is computed, it is found to have very closely the value

$$\frac{1}{32} h\nu,$$

where h is Planck's constant, and ν the frequency of rotation. This numerical coincidence suggests that quantum phenomena may be somehow connected with rotating electric fields.

The object of this paper is to propose a further generalization of the field equations, and to make applications to the structure of the electron and to non-radiating orbits.

GENERALIZED ELECTRODYNAMICS.

In the discussion immediately following, the terms *electric charge* and *magnetic charge* refer to an *element* of an electron or hydrion (positive electron), and not to charges so gross as even the entire charge on the electron. The postulates of the generalized theory are as follows:

1. With every electric charge is indissolubly associated an equal magnetic charge. By a charge is meant a center of uniformly diverging lines of force,—lines of electric force in the case of electric charges, and lines of magnetic force in the case of magnetic charges. Hence, from every element of the electron or hydrion radiate two sets of lines, one electric and the other magnetic. Since the Heaviside-Lorentz unit of charge is used in the following analysis, the magnitude e of the elementary charge is equal numerically both to the number of tubes of electric force and to the number of tubes of magnetic force radiating from it. The

electric intensity at any point is equal to the number of tubes of electric force per unit cross-section, and the magnetic intensity to the number of tubes of magnetic force per unit cross-section.

At first sight it may seem that this association of magnetism with electricity in the electron, for instance, is absurd in that it must give rise to a particle which has the properties of a magnetic pole as well as an electric charge. That such is not the case, however, will appear later on.

2. If $\mathbf{E}_1$ and $\mathbf{H}_1$ are the components of the electric and magnetic fields respectively at right angles to $\mathbf{c}$ due to an elementary charge e,

$$\mathbf{H}_1 = \frac{1}{c}(\mathbf{c} \times \mathbf{E}), \tag{6}$$

$$\mathbf{E}_1 = -\frac{1}{c}(\mathbf{c} \times \mathbf{H}), \tag{7}$$

just as in the classical theory. These are not independent relations, as either may be obtained from the other by forming the cross product with $\mathbf{c}$.

3. As in the classical theory, the force on an elementary charge e as measured by an observer relative to whom the charge is at rest at the instant considered, is

$$\mathbf{K} = e(\mathbf{E} + \mathbf{H}), \tag{8}$$

where $\mathbf{E}$ and $\mathbf{H}$ are the total field strengths. It follows from this that the force as measured by an observer relative to whom e is moving with velocity $\mathbf{v}$ is

$$\mathbf{K} = e\left(\mathbf{E} + \frac{1}{c}\mathbf{v} \times \mathbf{H} + \mathbf{H} - \frac{1}{c}\mathbf{v} \times \mathbf{E}\right), \tag{9}$$

the method of proof being the same as that used in obtaining the corresponding expressions in the paper[1] already referred to.

The expressions for the retarded electric and magnetic intensities due to an elementary charge e which has velocity $\mathbf{v}$ and acceleration $\mathbf{f}$ relative to the observer under consideration are obtained in the same manner as the corresponding expressions in the papers[1,3] referred to previously. They are

$$\mathbf{E} = \frac{e}{4\pi r^2 k^2 c\left(1 - \frac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)^3}\left\{\mathbf{c} - \mathbf{v} + \frac{1}{c}\mathbf{c} \times \mathbf{v} + \frac{rk^2}{c^3}\left[\left((\mathbf{f} - c\omega_H) \times (\mathbf{c} - \mathbf{v})\right\} \times \mathbf{c} + \left(\left\{\left(\omega_E + \frac{1}{c}\mathbf{f}\right) \times (\mathbf{c} - \mathbf{v})\right\} \times \mathbf{c}\right) \times \mathbf{c}\right]\right\}, \tag{10}$$

$$\mathbf{H} = \frac{e}{4\pi r^2 k^2 c\left(1 - \frac{\mathbf{c}\cdot\mathbf{v}}{c^2}\right)^3}\left\{\mathbf{c} - \mathbf{v} - \frac{1}{c}\mathbf{c} \times \mathbf{v} + \frac{rk^2}{c^3}\left[\left\{(\mathbf{f} + c\omega_E)\right.\right.\right.$$

$$\times (\mathbf{c} - \mathbf{v}) \Big\} \times \mathbf{c} + \left(\left\{ \left(\boldsymbol{\omega}_H - \frac{1}{c}\mathbf{f} \right) \times (\mathbf{c} - \mathbf{v}) \right\} \times \mathbf{c} \right) \times \mathbf{c} \Big] \Big\}, \tag{11}$$

where $\boldsymbol{\omega}_E$ and $\boldsymbol{\omega}_H$ refer to the electric and magnetic fields respectively. If $\boldsymbol{\omega}_0$ is the angular velocity of rotation of a field relative to an observer with respect to whom the charge producing the field is momentarily at rest, then the components of $\boldsymbol{\omega}$ to be used in these equations by an observer relative to whom the charge has a velocity $\mathbf{v}$ in the x direction are related to the components of $\boldsymbol{\omega}_0$ by the following transformations,

$$\omega_x = \frac{1}{k^3}\,\omega_{0x},$$

$$\omega_y = \frac{1}{k^2}\,\omega_{0y},$$

$$\omega_z = \frac{1}{k^2}\,\omega_{0z},$$

which are the same as the transformations of the components of acceleration.

It is observed that (10) and (11) satisfy conditions (6) and (7). Furthermore, *the radiation field vanishes* provided

$$\boldsymbol{\omega}_E = -\frac{1}{c}\,\mathbf{f}, \tag{12}$$

$$\boldsymbol{\omega}_H = \frac{1}{c}\,\mathbf{f}. \tag{13}$$

Therefore radiationless orbits are possible. It is to be noticed that the rotation of the field of an element of an electron does not in any manner imply rotation of the electron as a whole, and that the electric and magnetic fields may rotate at different rates and about different axes.

The field equations of the generalized theory are the equations (1) to (4) already given. The divergence ρ of the electric vector may conveniently be split into two parts, the density ρ_0 of electric charge and the divergence ρ_H due to rotation of the magnetic lines of force. Similarly ϵ consists of the density of magnetic charge ϵ_0 and the divergence ϵ_E of the magnetic vector due to rotation of the electric lines of force. According to postulate (1) $\rho_0 = \epsilon_0$ always. The retarded expressions for ρ_H and ϵ_E due to a charge e moving with velocity $\mathbf{v}$, whose electric field has angular velocity ω_E and magnetic field angular velocity ω_H, are respectively

$$\rho_H = -\frac{e}{2\pi r^2 k^2 c^2}\,\frac{\boldsymbol{\omega}_H \cdot \mathbf{c} - \boldsymbol{\omega}_H \cdot \mathbf{v} k^2 \left(1 - \dfrac{\mathbf{c} \cdot \mathbf{v}}{c^2} \right)}{\left(1 - \dfrac{\mathbf{c} \cdot \mathbf{v}}{c^2} \right)^4}, \tag{14}$$

$$\epsilon_E = \frac{e}{2\pi r^2 k^2 c^2} \frac{\omega_E \cdot \mathbf{c} - \omega_E \cdot \mathbf{v} k^2 \left(1 - \frac{\mathbf{c} \cdot \mathbf{v}}{c^2} \right)}{\left(1 - \frac{\mathbf{c} \cdot \mathbf{v}}{c^2} \right)^4}. \qquad (15)$$

It should be noticed that while ρ_H and ϵ_E are divergences in the electric and magnetic vectors respectively, they are due to the rotations of the lines of force and are not to be considered as true charges. A charge is detected by its field, and the divergences under discussion lack the characteristic uniformly diverging field of a charge. They are merely the end points of broken off lines of force which originate on a charge.

The energy equation is obtained by the usual methods from the field equations (1) to (4). It is

$$\frac{d}{dt} \{ \tfrac{1}{2}(E^2 + H^2) \} + c\nabla \cdot (\mathbf{E} \times \mathbf{H}) + \mathbf{E} \cdot \mathbf{v}\rho + \mathbf{H} \cdot \mathbf{u}\epsilon = 0. \qquad (16)$$

The last two terms represent the rate at which work is done on the divergences of **E** and **H**, whether these divergences are true charges or end points of broken off lines of force;

$$\tfrac{1}{2}(E^2 + H^2)$$

is the field energy per unit volume; and

$$(c\mathbf{E} \times \mathbf{H})$$

the Poynting flux.

STRUCTURE OF THE ELECTRON.

The Lorentz electron is the simplest model which has yet been proposed. This electron is a uniformly charged spherical shell relative to an observer with respect to whom it is at rest at the instant considered. It may easily be shown that all points of the electron are at rest relative to a given observer at the same time. If the electron is in motion, the relativity contraction causes it to appear shorter in the direction of motion than if it were at rest. Its dimensions at right angles to the direction of motion are unaltered. Each element of charge of the electron is assumed to act on every other element according to the same law of force as governs the actions of gross charges on one another. Hence the repulsive force expressed by Coulomb's law produces an outward stress at the surface of the electron which would cause it to explode in the absence of some inward force not explicable in terms of the classical electrodynamics. This outward stress is the same for an electron moving with constant velocity as for one at rest, and normal to the surface in both cases. Hence Poincaré has been led to suggest that it is counterbalanced by a uniform hydrostatic pressure of the ether. This hypothesis is open to several objections. In the first place, it requires the existence of forces of a character quite different from those contained in the electromagnetic scheme, as well as a second type of interaction between ether and electricity. Secondly, it implies the absence of ether from the interior of the electron, which is inconsistent with the assumption that electromagnetic forces are propagated across the inside of the electron from one element of charge to another. Thirdly, it is at variance with evidence showing that the ether (if one may be allowed to use the term) cannot have many, if any, of the properties of material media.

When an electron is moving with any other type of motion than constant velocity, its field exerts a drag on it. This retarding force is proportional, to a first approximation, to the acceleration, and accounts for the mass of the electron. The mass formula involves the radius of the electron, its charge, and the velocity of light. The last two being known, the first may be calculated, and comes out to be $1.88(10)^{-13}$ cm., which is of the order of magnitude to be expected. If the electron is assumed to have a volume distribution of charge expressible by any arbitrary function of the distance from its center, instead of the surface distribution of the Lorentz model, the value of the radius comes out somewhat different, though of the same order of magnitude, and the ether, if Poincaré's hypothesis be accepted, must be supposed to penetrate into the interior with a density which falls off as the center is approached. However, the objections to this hypothesis cited above still hold. The object of this section of the present paper is to show that the proposed generalized electrodynamics makes it possible to account for both the equilibrium and the stability of the electron without the necessity of introducing extra-electromagnetic forces.

Lorentz's model will be adopted in that the electron will be supposed to be a uniformly charged spherical shell of charge e and radius a. Each element of charge is the origin of equal electric and magnetic fields. The magnetic fields will be supposed, normally, to be without rotation; the electric field of each element of charge, on the other hand, will be considered to have a constant angular velocity ω about the normal to the surface at the point considered. Consider, now, an electron permanently at rest relative to the observer. Expressions (10) and (11) for the electric and magnetic intensities respectively due to an element de of the electron's charge reduce to

$$d\mathbf{E} = \frac{de}{4\pi r^2 c}\left\{ \mathbf{c} - \frac{r}{c}\,\boldsymbol{\omega} \times \mathbf{c} \right\}, \tag{17}$$

$$d\mathbf{H} = \frac{de}{4\pi r^2 c}\left\{ \mathbf{c} + \frac{r}{c}\left[\frac{\boldsymbol{\omega} \cdot \mathbf{c}}{c^2}\,\mathbf{c} - \boldsymbol{\omega} \right] \right\}. \tag{18}$$

Symmetry shows that the resultant **E** and the resultant **H** are radial. Integrating over the surface of the electron, the intensities at a point P (Fig. 1) outside the electron and distant p from its center are found to be

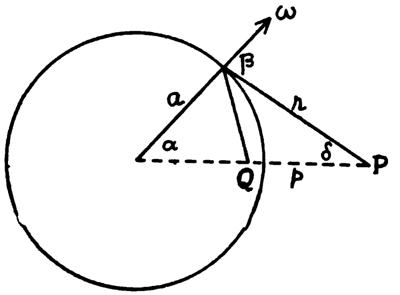

Fig. 1.

$$\begin{aligned}
E &= \frac{1}{4\pi}\int \frac{\cos\delta}{r^2}\,de \\
&= \frac{e}{16\pi a p^2}\int_{p-a}^{p+a}\left[1+\frac{p^2-a^2}{r^2}\right]dr \\
&= \frac{e}{4\pi p^2},
\end{aligned} \qquad (19)$$

$$\begin{aligned}
\mathbf{H} &= \frac{1}{4\pi}\int\left[\frac{\cos\delta}{r^2}+\frac{\omega}{cr}(\cos\beta\cos\delta-\cos\alpha)\right]de \\
&= \frac{e}{16\pi a p^2}\int_{p-a}^{p+a}\left[1+\frac{p^2-a^2}{r^2}\right]dr \\
&\quad+\frac{e\omega}{32\pi a^2 p^2 c}\int_{p-a}^{p+a}\left[r^2-2(p^2+a^2)+\frac{(p^2-a^2)^2}{r^2}\right]dr \\
&= \frac{e}{4\pi p^2}\left[1-\frac{2}{3}\frac{a}{c}\omega\right].
\end{aligned} \qquad (20)$$

Choose

$$\begin{aligned}
\omega &= \frac{3}{2}\frac{c}{a} \\
&= 2.40\,(10)^{23} \text{ per sec.}
\end{aligned} \qquad (21)$$

Then there is no magnetic field outside the electron, and the electric field follows Coulomb's familiar inverse square law.

At a point Q inside the electron, the same integral expressions for E and H hold, but the limits of integration are $a + p$ and $a - p$ instead of $p + a$ and $p - a$. Hence

$$E = 0, \qquad (22)$$

$$\begin{aligned}
H &= -\frac{e\omega p}{6\pi a^2 c} \\
&= -\frac{ep}{4\pi a^3},
\end{aligned} \qquad (23)$$

The electric field vanishes, and the magnetic field is directed toward the center and has a strength proportional to the distance p from the center.

The divergence of the electric vector is everywhere zero (except at the surface). The divergence of the magnetic vector is zero outside the surface, but (23) shows that inside it must have the constant value

$$\nabla \cdot \mathbf{H} = -\frac{e\omega}{2\pi a^2 c}$$
$$= -\frac{3e}{4\pi a^3}. \tag{24}$$

Thus the total volume distribution of divergence of the magnetic vector inside the electron is equal and opposite in sign to the surface magnetic charge, agreeing with the computed absence of magnetic field outside the electron. The divergences of **E** and **H** might have been obtained directly from (14) and (15).

At the surface of the electron, E and H have the same values but opposite directions. Hence the charge on the surface is in equilibrium. The outward stress due to the electric field is

$$S = \frac{e^2}{32\pi^2 a^4}, \tag{25}$$

per unit area, and the equal inward stress due to the magnetic field

$$S = \frac{e^2\omega^2}{72\pi^2 a^2 c^2}. \tag{26}$$

Let the radius of the electron increase to $a + x$, the angular velocity ω retaining the value given above. Then the net outward stress is

$$S = \frac{e^2}{32\pi^2 a^4}\left[-2\frac{x}{a} + 7\frac{x^2}{a^2}\cdots\right]. \tag{27}$$

The electron, therefore, is in stable equilibrium as regards changes in volume.

The generalized electrodynamics under discussion has been derived directly from the relativity principle, and hence all deductions to which it leads must be strictly in accord with this principle. Therefore the conclusions arrived at regarding equilibrium and stability of an electron permanently at rest apply equally well to an electron moving with constant velocity relative to the observer. In the latter case a magnetic field external to the electron is present, but it is the ordinary field due to motion of the electron as a whole, and not due to the rotations of the generalized theory. The divergences of both vectors are zero everywhere outside the electron.

These speculations as to the nature of the electron suggest that the positive electron or hydrion differs from the negative electron in the sense of the rotation ω. If, for instance, the rotation of the field of an element of charge on the surface of the negative electron is clockwise when viewed along the outward drawn normal, the rotation may be counterclockwise in the case of the positive electron. The fact that only two senses of rotation are possible is in accord with the existence of two and only two kinds of electricity.

NON-RADIATING ORBITS.

Integration over the entire surface of an unaccelerated electron has shown that **E** and **H** have the same values outside the surface as are predicted by Maxwell's classical equations. The rotations of the electric fields of the elements of charge constituting the electron annul one another in such a manner as to produce no resultant effect in so far as the electric intensity is concerned. An investigation of the external field and the dynamical equation of an accelerated electron is being undertaken in the hope that it may throw some light on quantum phenomena. For the results of Rutherford's experiments on the scattering of alpha rays are very excellent evidence, in the author's opinions that quantum phenomena cannot be accounted for by substituting for the inverse square electrostatic field of the nucleus amore complicatedfield involving repulsive as well as attractivef orces. Hence it would seem as though the quantum must be somehow or other connected with the electron.

The conditions for annulment of the radiation field have been already stated. The necessary small angular velocities in the direction of the acceleration must be supposed to be superimposed on the much larger angular velocity of the electric fields normal to the surface. In the case of an electron rotating in a circular orbit of quantum number n about a nucleus of atomic number Z,

$$\begin{aligned} \omega_H = -\omega_E &= \frac{1}{c}\mathbf{f} \\ &= \frac{16\pi^4 m e^6}{h^4 c}\frac{Z^3}{n^4} \\ &= 3.01\ (10)^{14}\frac{Z^3}{n^4}\text{ per sec.,} \end{aligned} \tag{28}$$

which is quite negligible compared to (21).

Note Added in Course of Publication.—Further calculation shows that the mass and the cofficient of the rate ot change of acceleration are each one and a half times as great as for the simple Lorentz electron of the same radius. Hence, the mass being known, the radius of the proposed electron comes out one and a half times as great as that of the Lorentz type, i. e., $2.82(10)^{-13}$ cm.

While the suggested superimposed rotations of the electric and magnetic fields of a revolving electron annul the radiation due to acceleration of the electron as a whole, there remains a radiation of the same order of magnitude due to the rotation about the normal to the electron's surface of the electric fields of each element of charge. If the superimposed rotations are such as to annul *entirely* the radiation field of each element of charge, the equilibrium of the electron's surface is interfered with, and the electromagnetic mass of the electron vanishes.

SLOANE PHYSICS LABORATORY,
YALE UNIVERSITY,
April, 1921.

AN ELECTRICAL DOUBLET THEORY OF THE NATURE OF THE MOLECULAR FORCES OF CHEMICAL AND PHYSICAL INTERACTION.

By R. D. Kleeman.

Synopsis.

Electrical doublet theory of molecular attraction.—The author has previously found from data on the internal heat of evaporation of liquids that the attraction between two atoms is proportional to the product of the square roots of their atomic weights. It is now suggested that a neutral atom with a positive nucleus surrounded by electrons is an electrical doublet; and it is shown that the force between two such doublets would on the average be an attraction proportional to the product of the moments of the two doublets. This result combined with the previous one leads to the conclusion that the *electrical moment of an atom* is proportional to the square root of its atomic weight. Similarly the electrical moment of a molecule comes out equal to the sum of the moments of the component atoms. This theory of molecular attraction is supported by the fact that the *stopping power of gases for alpha rays* is known to be proportional to the sum of the square roots of the atomic weights of the component atoms; for this law follows directly if we suppose the transfer of energy from the alpha particles to the gas molecules occurs solely as a result of electrical forces exerted by the charged rays on the molecular doublets.

Previous Investigations of the Molecular Forces

It will be instructive and useful to consider as an introduction to the subject of this paper the nature of the law of force between two molecules which can be deduced directly from physical data. I have shown[1] that the force of attraction between two molecules deduced from data on the internal heat of evaporation of liquids, is given by the expression

$$\frac{\phi\left(\frac{T}{T_c}, \frac{z}{x_c}\right)}{z^5} \cdot (\Sigma \sqrt{a})^2,$$

where z denotes the distance of separation of the two molecules in the liquid whose temperature is T, x_c the average distance of separation of molecules of the liquid at the critical point whose critical temperature is T_c, $\Sigma \sqrt{a}$ denotes the sum of the square roots of the atoms of a molecule, and $\phi(T/T_c, z/x_c)$ is a function whose form is left arbitrary by the data, and which for shortness will be denoted by (ϕ). It appears from the foregoing expression that the attraction between two molecules is a function of their temperature as well as of their distance of separation. We would expect, of course, that the attraction between two molecules might depend on the temperature, since a change in temperature is accompanied by a change in the internal molecular energy, which probably involves a change in atomic configuration, as is shown by the positive value of the specific heat. The foregoing law, it should be remarked, was obtained from latent heat data taking into account the following mathematical theorem: The nature of a function F of two independent variables x and y can be determined approximately in a definite manner only from data giving the values of the function corresponding to variations of each of the variables keeping the other constant, while if the form of the function is determined from data corresponding to the simultaneous variation of x and y it will contain an arbitrary function. A corollary of this theorem is, if the assumption is made that the function contains one variable only, and a form of the function is found agreeing with the facts, this does not signify that the assumption made is true. Now the density of a liquid changes on change of temperature, and the internal heat of evaporation is therefore a function of the two independent variables temperature and distance of separation of the molecules, which vary simultaneously. The law deduced from such data by itself should therefore contain an arbitrary function, and the nature of this function must accordingly be determined by considering in addition data relating to other quantities. This has been carried out by the writer[1] but the results need not concern us here, except that the function varies little with z and T. For our present purpose it is important to notice that the function involves quantities depending on the critical state of the substance to which it refers, which indicates that the nature and magnitude of the attraction between two molecules in the substance is influenced by the surrounding molecules, as we might expect.

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[1] Phil. Magazine, May, 1910, pp. 783–809; Jan., 1911, pp. 83–102. Proc. Camb. Phil. Soc., XVI., Pt. 7, pp. 583–598.

THE MOLECULAR FORCES ARE EXPLAINED BY CONSIDERING THE ATOMS AS ELECTRICAL DOUBLETS.

In this paper an attempt will be made to explain the nature of the molecular attraction under consideration (which give rise to the internal heat of evaporization of a liquid, its surface tension, intrinsic pressure, chemical interaction between its atoms and molecules, etc.) by the electric charges each atom is supposed to contain. It will be of interest and serve as an introduction to the main investigation, to consider the attraction which one of the electrical charges in an atom would exert on another atom, supposing that the latter atom behaves as a spherical conductor of electricity. The attraction in that case is given by the well known expression

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[1] *Loc. cit.*

$$\frac{e^2r_1^3(2f^2 - r_1^2)}{f^3(f^2 - r_1^2)^2},$$

where e denote the electric charge in the former atom, r_1 the radius of the latter atom, and f the distance between their centers. When r_1^2 is small in comparison with f^2 this expression reduces to

$$\frac{2e^2r_1^3}{f^5}.$$

Now we have seen that the function ϕ in the law of molecular attraction does not vary much with z and T, in which case the attraction between two molecules varies approximately inversely as the fifth power of their distance of separation, and thus corresponds to the foregoing expression. But the resemblance goes further. Traube has shown[1] that the apparent volume of an atom at the absolute zero is approximately proportional to the square root of its atomic weight, and hence r_1^3 is proportional to $\sqrt{a}$. Therefore if we make the assumption that the total electric charge of one sign associated with an atom is proportional to $\sqrt{a}$ (this is roughly the case) we obtain that the electric attraction between two atoms c and e is equal to

$$\frac{k\sqrt{a_c}\sqrt{a_e}}{f^5},$$

where a_c and a_e denote the atomic weights of the atoms, and k a constant. The attraction between two molecules of the same kind composed of various atoms would accordingly be given by

$$\frac{k(\Sigma\sqrt{a})^2}{f^5}.$$

This expression contains the fundamental factor $\Sigma\sqrt{a}$ contained in the law of attraction deduced from latent heat data.

The atom must however more properly be regarded in all cases as an assemblage of electrons and positive charges instead of a spherical conductor of electricity, whose effect on an external electron, or on another atom, is more or less equal to the total sum of the effect of each electrical charge. Now according to Rutherford[2] the atom consists of a number of electrons located near the surface of the atom, and a corresponding positive charge concentrated more or less at the center. It is very probable that the electrons and corresponding positive charges of an atom are not arranged in a symetrical way with respect to the center of the atom. Under these conditions the atom would as a whole behave approximately like an electrical doublet. Now it can be shown that the orientation of the axes of the atomic doublets would on the average be such as to produce attraction between the atoms. Thus suppose that the atoms are stationary in space and occupy their average positions, which correspond to the points of intersection of three sets of equidistant planes situated at right angles to each other. Suppose that the axes of the electrical doublets of a row of atoms lie on the straight line joining them, and that the charges are so arranged that opposite kinds face one another. Under these conditions the atoms will attract each other, tending to move along the straight line joining them. Let us also suppose that the atoms in an adjacent parallel row be similarly arranged except that the positive charge of each doublet faces in the opposite direction with respect to the negative charge, as shown in Fig. 1. It will be evident that we may suppose that the set of atoms consists of parallel rows of the foregoing kind lying in parallel planes, and that in each alternate row of the rows lying in a plane the positive charges face one way, and in the other rows face in the opposite way. Hence we may also look upon the atoms as being divided up into a number of parallel rows at right angles to the foregoing rows, in which the charges of opposite sign will be nearer to each other than the charges of the same kind. Thus the atoms would also exert an attraction upon each other along the directions of these rows. Hence the atoms would exert an attraction upon each other in all directions, tending to contract the set of atoms as a whole.

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[1] Physik. Zeit., Oct., 1909, p. 667.

[2] Phil. Mag., 21, p. 669, 1911.

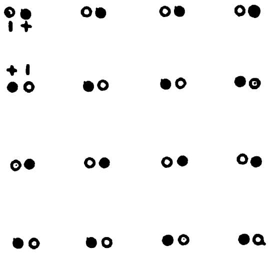

Fig. 1.

In practice however, the atoms or molecules, are not stationary in space, but undergo motion of translation, which gives rise to collisions with each other, which have the effect of abruptly changing the orientations, or the directions of rotation, of the axes of the electrical doublets. But according to the principle that *the potential energy of a system tends to become a minimum*, the axes of the doublets would tend to point on the average in such directions as will give rise to an attraction between the atoms. A closer examination will show how this may be brought about. The axis of a doublet points successively in different directions in space through molecular collisions. Each direction will occur equally frequently since the substance as a whole has no electric polarity. But the successive directions of one atomic doublet are connected with the successive directions of an adjacent atomic doublet. For the axes of the doublets would tend to point parallel to each other, through the forces of attraction and repulsion which the charges of the doublets exert upon each other. Hence the axis of an atomic doublet would be oftener parallel to that of another atomic doublet than at right angles to it. This would give rise to attraction tending to be the outstanding force between the doublets. The extreme case of such a connection between the positions of the atomic doublets with respect to each other would be if the atomic doublets in Fig. 1 were to rotate round their centers in any manner but with their axes remaining parallel to each other.

It may happen, of course, that for certain densities of the substance the effect of molecular collision on the orientation of the axes of the electrical doublets may be such that repulsion is produced between the atoms instead of attraction. According to the foregoing principle connected with potential energy this repulsion would tend to be as small as possible. Thus atomic repulsion, whose existence is shown by the Joule Thomson effect, may be explained as well as attraction by the existence of electrical doublets.

In the case of two molecules the forces of attraction and repulsion would be due to the effect of the atomic doublets of one molecule on those of the other. The effect of each atomic doublet on another doublet would very probably be independent of the presence of the other doublets, and the electrical effect of a molecule would therefore in a sense be an additive quantity of its atoms.

It will be of interest next to examine the algebraical nature of the law of force according to the foregoing supposition. It can be easily shown that the force between two electrical doublets of magnitude M' and M'' is

$$\frac{M'M''}{r^4} 3\{2 \cos\cdot\theta\cdot\cos\theta' - \sin\theta\cdot\sin\theta'\} = \frac{M'M''}{r^4} C_e,$$

where r denotes the distance between the centers of the doublets and θ and θ' the angles the doublets make with the line joining their centers. In the case in which the doublets belong to two atoms of a mon-atomic substance in the liquid or gaseous state the quantity C_e would be a function of the two average angles the doublets make with the line joining them. These angles would evidently depend on the frequency of atomic collision in the substance, and on the nature of a collision, and hence would depend on the temperature and density of the substance, and on its nature, *i.e.*, on its temperature, distance of atomic separation, atomic weight, and critical constants. In the case that the substance consists of molecules, M' would consist of the sum $(M_1' + M_2' + \cdots)$ of the atomic doublets of a molecule, and M'' of the sum $(M_1'' + M_2'' + \cdots)$

of the doublets of another molecule. When repulsion exists, as we have seen may be the case, the sign of C_e is negative.

The foregoing results are borne out by experiment. According to what has gone before the quantity C_e would be proportional to

$$\frac{1}{z}\cdot\phi\left(\frac{T}{T},\frac{z}{x_c}\right),$$

and thus a function of the temperature of the substance, the distance of molecular separation, and the critical constants. It appears also that the moment of each electrical doublet is proportional to $\Sigma\ \sqrt{a}$, the sum of the square roots of the atomic weights of the atoms. This result will be confirmed by other evidence later.

It will be of interest next to consider more closely the distribution of the electrons and positive charges in an atom. According to the experiments of Rutherford mentioned the positive charge is concentrated in a sphere of radius less than 10^{-12} cm. From considerations of symmetry we would expect this charge to be the electrical center of the atom. Since the number of electrons is finite it is evident that they cannot be arranged around the positive charge in such a way that the electric force is zero on any point on a line passing through the positive charge. If the point selected is at a distance from the positive charge which is large in comparison with the distance of each electron, the effect is approximately equivalent to the sum of the effects of a number of doublets each of which corresponds to an electron in the atom, *i.e.*, the atom as a whole behaves approximately like an electrical doublet. This corresponds to the state of affairs in practice, where the forces we are dealing with correspond to an average distance of separation of the atoms or molecules which is much larger than the atomic and molecular diameters. Of course, on rotating an atom round its center the moment of the average doublet will change somewhat, and we must therefore associate an average doublet with an atom which includes the consideration of direction from the atomic center. It is, of course, also not improbable that the positive charge is not located at the center of the atom.

If the two electrical charges of the representative doublet of an atom were separated by the same distance in all cases, each charge would be proportional to the square root of the atomic weight. But this is not likely to be the case. Moreover there is good reason to believe from various evidence that the number of unit positive charges in an atom is equal to A its atomic number. Therefore if x denote the distance of separation of the charges of the representative doublet of an atom, we have

$$xA = \sqrt{a}.$$

Now the atomic number increases with the atomic weight, and for high atomic weights is approximately half of the atomic weight. Thus it appears that the distance x decreases with the atomic weight of the atom. This is probably due to the fact that the larger the number of electrons in an atom, the nearer can their distribution be made symmetrical around the positive charge.

Since the nature of the quantity C_e in the expression for the force of two atomic or molecular doublets upon each other, which corresponds to the function (ϕ) in the law of molecular attraction, depends on the average orientation of the atomic or molecular doublets with respect to each other in a substance, the function (ϕ) would depend, as has already been pointed out, on the nature of the atomic and molecular interaction. It should therefore be a function of the temperature and the density of the substance, as we have already seen is the case. In general, it will be evident that any process that will change the nature of the molecular interaction will change the nature of the function (ϕ).

Thus for example, in the case of a mixture of molecules a and c the nature of the interaction of a molecule a, or c, with other molecules would be different from that when the constituents are in the pure state. The attraction between two molecules a would then be given by the general expression

$$\psi(R_a', \rho)\cdot(\Sigma\sqrt{a})^2,$$

where R_a' denotes the ratio of the different constituents of the mixture, and ρ its density. The nature of the function ψ could evidently not be predicted from the form of (ϕ) applying to a pure substance. Similar remarks apply to the nature of the force between two molecules a and c, or two molecules c. The nature of the function ψ could be determined only from data relating to mixtures.

The nature of the molecular interaction of the molecules of a liquid would evidently be changed by the application of an electric field, since it would tend to set the doublets parallel to each other, and a corresponding change in the law of molecular attraction would be introduced. This might therefore be attended by a change in the volume of the substance. Since molecules and atoms possess magnetic properties a magnetic field would act in the same way.

The motion of a substance through space would tend to make the electrical atomic doublets set themselves with their axes at right angles to the motion, an effect which would become very marked when the velocity of the substance approaches that of light. This would obviously change the nature of molecular interaction, and accordingly a change in the functional nature of the law of molecular attraction be introduced.

Hence on giving a motion to a liquid it would undergo a contraction or an expansion as the case may be. When the substance is a solid the change in length in the direction of the motion might be different from that at right angles to it.

Since light consists of electromagnetic waves, or of waves of electric and magnetic force, it would have the effect of changing the nature of molecular interaction to a certain extent and hence the nature of the forces of interaction. The chemical changes induced by light in liquids and gases could be well and strikingly explained in this way.

THE PASSAGE OF THE α PARTICLE THROUGH MATTER.

The behavior of a charged particle passing through matter is intimately connected with the foregoing considerations. Bragg and the writer[1] have carried out some measurements on the range of the α particle in different gases, that is, the distance it traverses during which its energy decreases to a certain fraction of its original value. The α particle, it should be mentioned, is according to Rutherford a helium atom carrying a positive charge equal to $2e$. It was found that the range is inversely proportional to the sum of the square roots of the atomic weights of the atoms of a molecule of the gas. The passage of an electron through matter (the electron carries a negative charge equal to e), one would expect, should be subject to the same law. The writer has obtained some evidence that this is the case for velocities of the electron of the same order as those of the α particle.[2]

Let us suppose that the α particle mainly spends its kinetic energy in increasing that of the molecules along its path by means of the forces of attraction and repulsion it exerts upon them. Now Maxwell has investigated the motion of a molecule that expends its energy in a gas in this way for the case when the molecule is attracted or repelled by a molecule of the gas according to a law of the form K/z^n, where z denotes the distance of separation of the interacting molecules, and K and n are constants.[3] Thus the decrease of the velocity V of a molecule of mass m_2 in passing over a distance dx in a gas whose molecules are each of mass m_3, is according to this investigation given by

$$\frac{dV}{dx} = \frac{-m_3}{m_2 + m_3}\left(\frac{K(m_2 + m_3)}{m_2 m_3}\right)^{2/(n-1)} N A_c V^{(n-5)/(n-1)},$$

where N denotes the number of molecules in the gas per cubic cm., and A_c a constant which is independent of m_2, m_3 and V. The force between a neutral gas molecule and a charged α particle, according to the preceding Article, corresponds to the force between the electrical doublet of the neutral molecule, and the electrical charge and the electrical doublet of the α particle. It will not be difficult to see that the force between the two doublets is small in comparison with that due to the doublet of the neutral molecule and the charge on the α particle, and hence the latter force only need be considered. It can easily be shown that if a charged particle is situated on the prolongation of the axis of a doublet the force between the two is $M2e/r^3$, where M denotes the moment of the doublet, $2e$ the charge on the particle, and r their distance of separation. This force may be either positive or negative in sign as the case may be. The doublet of a molecule is however continually undergoing varying oscillations and rotations. The average force between a neutral molecule and the α particle during its motion through the gas, may thus be written $C_e' \cdot M/r^3$, where C_e' is a constant. It might be conjectured at first sight that C_e' is a function of the temperature and density of the gas. But this is evidently not the case since the various positions of the doublets of the neutral molecules *with respect to the rapidly passing α particle* must be symmetrically distributed in space because the gas as a whole has no polarity, and hence these positions are independent of the concentration and temperature of the molecules. The moment M of the doublet of a molecule is according to the preceding Article proportional to the sum of the square roots of the atoms of a molecule, or proportional to $\Sigma \sqrt{a}$. The law of force between the α particle and a neutral molecule may accordingly be written $C_x \Sigma \sqrt{a}/z^3$, where C_x is a constant. The above equation then reduces to

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[1] Phil. Mag., 10, 1905, p. 318.

[2] Proc. Roy. Soc., A, vol. 84, p. 20, 1910.

[3] Conduction of Electricity through Gases, by J. J. Thomson, 2d ed., p. 371.

$$\frac{dV}{dx} = \frac{C_x}{m_2} \Sigma \sqrt{a} \cdot \frac{NA_e}{V},$$

which on multiplying by $m_2 V \cdot dx$ and integrating becomes

$$m_2(V_1^2 - V_2^2) = C_x(x_1 - x_2) \Sigma \sqrt{a}.$$

This equation states that an α particle whose energy falls from $\frac{1}{2} m_2 V_1^2$ to $\frac{1}{2} m_2 V_2^2$ traverses a distance $x_1 - x_2$, which is inversely proportional to the square roots of the atomic weights of the atoms of a molecule of the gas. This is the law found by experiment, and which is thus in a striking way explained by the existence of electrical doublets in atoms and molecules.

It is interesting to note that according to the foregoing investigation the rate at which an α particle spends energy on a molecule is independent of the density and temperature of the gas through which the α particle passes. This is borne out by the experimental results. It also follows from the investigation that the α particle will cease spending its energy in the way deduced when it has lost its charge.

In the foregoing investigation it was assumed that the α particle spends its energy mainly in interacting with neutral molecules. This will appear highly probable when it is considered that less than ten molecules of ten thousand molecules lying in the path of the α particle are likely to get ionized.

Physical Laboratory,
Union College,
Schenectady, N. Y., March 23, 1921.

PROCEEDINGS

OF THE

AMERICAN PHYSICAL SOCIETY.

MINUTES OF THE BERKELEY MEETING, AUGUST 4, 1921.

THE 110th meeting of the American Physical Society was held in Room 113, South Hall, University of California, Berkeley, on Thursday, August 4, 1921. Professor W. B. Anderson presided. About 75 persons were in attendance. At the morning and afternoon sessions, beginning at 9:30 and 2 o'clock respectively, the following program was presented:

The Moon's Electric Charge. FERNANDO SANFORD.

Earth Currents and the Sun's Electrical Induction. FERNANDO SANFORD.

The Effect of Temperature on the 3883 Cyanogen Band. RAYMOND T. BIRGE.

Some X-Ray Isochromats. D. L. WEBSTER.

The Width of X-Ray Spectrum Lines. ARTHUR H. COMPTON.

The Effect of Damping on the Width of X-Ray Spectrum Lines. G. E. M. JAUNCEY.

Experiments on Polarization of X-Rays. PAUL KIRKPATRICK.

An X-Ray Spectrometer for the Determination of Absorption Coefficients. ELMER DERSHEM.

A Photographic Method of X-Ray Crystal Analysis. ELMER DERSHEM.

Secret Photoelectric Signaling. ELMER DERSHEM.

An Harmonic Curve Writer. WILLIAM J. RAYMOND.

The Variation of the Thermal Conductivity of Iron with Temperature. E. E. HALL, O. R. HULL, and O. S. IMHOFF.

The Index of Refraction of Water, Ethyl Alcohol, and Carbon Bisulphide at Various Temperatures. A. R. PAYNE and E. E. HALL.

Preliminary Report on Spectroscopic Investigations in the Extreme Ultra-Violet. J. J. HOPFIELD.

The Effect of Pressure on the Band Spectrum of Nitrogen. JEAN HUDDLESTON.

The Reaction of Iron with Nitric Acid. JOSEPH G. BROWN.

A Braun-Tube, Undamped-Wave, Precision Method of Determining Dielectric Constants of Gases. L. T. JONES and H. G. TASKER.

The Effect of Ultra-Violet Light on the Viscosity of Rubber Colloids. RICHARD HAMER.

The Simplest Mercury Vapor Pump. L. T. JONES.

A New Rotary Mercury Pump. L. T. JONES.

Atomic Radii. MAURICE L. HUGGINS.

Structure of the L-Series of Tungsten and Platinum. FRANK C. HOYT.

The Sun's Charge. A Criticism of Method. F. J. ROGERS.

Preliminary Determination of the Critical Potentials of the M Lines in the Lead X-Ray Spectrum. P. A. ROSS.

Recent Observations of Absorption Spectra. A. S. KING. (Read by title.)

A Possible Origin of the Defect of the Combination Principle in X-Rays. ARTHUR H. COMPTON.

At the conclusion of the meeting the audience listened to a concert at the Fairmount Hotel, San Francisco, transmitted by wireless telephone, arranged by Mr. H. G. Tasher.

E. P. LEWIS,
Local Secretary for the Pacific Coast.

THE MOON'S ELECTRIC CHARGE.

BY FERNANDO SANFORD.

AT the January, 1921, meeting of this society I reported some observations on the diurnal change in electric potential-difference between the earth and an uncharged, insulated conductor kept inside an earthed metal cage. These variations were attributed to the electrostatic induction of the sun's negative charge. Photographic records of these variations have now been kept for a year.

In the meantime the diurnal variations due to the moon have been computed for five lunation periods of 29 days each. Three of these sets were made by distributing the data derived from the measurements of the photographic records for each hour of the solar day with reference to the position of the moon. Thus, the measurement made nearest the time of the moon's upper culmination was taken as the middle of the lunar day, and the twelve hourly measurements preceding and the twelve following this one were distributed in their proper order with reference to it.

Two of the sets were made by Chapman's method of subtracting the mean of the measurements for a given hour for the whole period from each of the single measurements made at that hour of the day, and distributing the residuals, either positive or negative, to the proper hour of the lunar day, just as the total measurements at the given hours were distributed in the previous cases. This method seems to possess no advantages over the other in determining the moon's electrostatic effect.

The data derived from the five series are given in Table I.

It will be seen from Table I. that while the diurnal variation is plainly shown in all the series the maximum positive variation came much later in Series 5 than in any of the others. No reason for this is known. The maximum positive solar variation for the same period came at noon.

TABLE I.

Mean Diurnal Variation in Potential-Difference, Expressed in Millivolts, between the Earth and an Uncharged, Insulated Conductor Due to the Moon's Electrostatic Induction.

Hour.	Series 1.	Series 2.	Series 3.	Series 4.	Series 5.	Mean of 5.
1	+ 0.5	− 4.7	− 0.6	− 5.3	− 3.6	− 2.7
2	− 0.3	− 5.2	− 0.7	− 7.3	− 4.2	− 3.5
3	− 1.7	− 5.3	+ 0.2	− 7.0	− 3.6	− 3.5
4	− 1.6	− 4.7	+ 0.8	− 7.1	− 5.5	− 3.6
5	+ 0.9	− 1.1	− 2.2	− 4.6	− 3.6	− 2.1
6	+ 1.6	+ 0.5	− 2.4	− 3.7	− 1.0	− 1.0
7	+ 3.3	+ 2.8	− 2.5	+ 0.4	− 1.1	+ 0.6
8	+ 4.6	+ 4.3	+ 0.6	− 0.4	− 0.5	+ 1.7
9	+ 5.4	+ 5.6	− 0.5	+ 4.5	− 1.1	+ 2.8
10	+ 5.2	+ 6.7	+ 2.9	+ 7.9	− 3.0	+ 3.9
11	+ 4.1	+ 6.8	+ 2.0	+ 7.6	− 6.5	+ 2.8
U.C.	+ 3.7	+ 8.2	+ 2.6	+ 8.0	− 3.0	+ 3.9
13	+ 2.8	+ 7.3	+ 5.0	+ 5.2	− 0.8	+ 3.9
14	− 0.4	+ 4.0	+ 4.1	+ 4.0	+ 1.0	+ 2.5
15	+ 0.4	+ 3.1	+ 3.0	+ 4.8	+ 2.5	+ 2.8
16	− 1.6	+ 0.8	+ 0.5	+ 3.2	+ 6.5	+ 1.9
17	− 3.2	− 1.8	+ 2.2	+ 3.6	+ 6.4	+ 1.4
18	− 3.8	− 3.2	+ 0.3	+ 0.1	+ 8.7	+ 0.4
19	− 3.4	− 3.3	− 2.0	− 2.2	+ 6.0	− 1.0
20	− 2.1	− 2.6	− 6.2	+ 0.6	+ 7.8	− 0.5
21	− 2.2	− 3.1	− 5.3	− 2.2	+ 5.3	− 1.5
22	− 2.6	− 5.7	− 2.2	− 3.3	− 0.7	− 2.9
23	− 3.8	− 7.7	− 0.6	− 1.8	− 2.1	− 3.2
24	− 1.4	− 6.4	− 0.7	− 1.4	− 5.1	− 3.0

Curves have been plotted showing the mean diurnal variation for the five series and the mean diurnal variation of potential-difference for the solar days from August 1, 1920, to May 31, 1921. It will be seen that the moon has about one-seventh the effect of the sun in producing the observed variations. It is for this reason that the irregular disturbances due to unknown causes make the lunar curve much more irregular than that due to the sun.

If the variations described are due to the electrical induction of the sun and moon, the moon must have a negative charge of an intensity somewhat less than one millionth that of the sun's charge. It seems almost certain that if the sun and moon have the enormous charges indicated by these observations the earth must also have a very great negative charge.

When these variations have been reported at previous meetings of the Society they were compared with the diurnal variations in the North magnetic component of the earth's field. It has been known for a long time that there are also variations in the earth's magnetic elements due to the moon's influence. Thus Chapman says in Phil. Trans., 213, 279–321 (1913): "The magnetic elements show regular periodic changes depending on the lunar hour angle, just as on the solar hour angle; the latter variations are considerably the greater of the two, and almost entirely mask the lunar variations."

The above statement applies equally well to the variations in electrical potential-difference now under consideration.

STANFORD UNIVERSITY.

EARTH CURRENTS AND THE SUN'S INDUCTION.

BY FERNANDO SANFORD.

By means of a comparison between the records of earth currents and of magnetic disturbances begun at the Greenwich Observatory in 1865, Airy was able to decide that the irregular magnetic disturbances were certainly due to earth currents and that the regular, diurnal magnetic variations were probably due to the same cause.

The relation between the diurnal variations in the two phenomena is obscured by the fact that the changes in the earth currents precede the corresponding variations in the magnetic elements by considerable periods of time. This fact is shown in Fig. 1,[1] in which the continuous curve shows the mean diurnal variation in the N–S earth current at Tortosa, Spain, for the year 1910, and the broken line shows the mean diurnal variation in the magnetic declination for the same period. The curves are drawn from the data given in the annual report for that year of the Observatorio del Ebro, at Tortosa. It will be seen that while the curve for variation in magnetic declination is very similar in shape to the earth current curve, it lags behind it in time by about three hours.

A similar lag in the diurnal variation in potential-difference between the earth and an uncharged, insulated conductor which is kept in an earthed metal cage and the diurnal variation in the north magnetic component is shown in the simultaneous records of the two phenomena made at Palo Alto during the month of March of the present year. In Fig. 2, the continuous line shows the mean diurnal variation in potential-difference between the earth and the insulated conductor, and the broken line shows the simultaneous variation in the North magnetic component.

In the extensive study of earth currents made at Berlin from 1884 to 1888, Weinstein was able to show that "almost the whole of the variation which we are able to observe by magnetometers is due to earth currents." Secular changes in the magnetic elements have not been related to earth currents.

Weinstein made another observation, the importance of which he seems not to have recognized. He stated that the general direction of the resultant earth current at Berlin is toward the sun. This must mean that the electrons in the earth regularly flow from that region of the earth nearest the sun.

In Fig. 3 the mean hourly values of the resultant *Negative* summer current at Berlin are represented in their relative directions and magnitudes by the arrows starting at distances of 15 degrees apart from the parallel of latitude of Berlin. That is, they are drawn in the directions and relative magnitudes which they had at Berlin at the hours specified on the chart. If the currents could remain stationary in the positions represented without being carried around by the rotation of the earth, the observed diurnal variation at Berlin would result. This is approximately what must occur if the currents are caused by the sun's electrostatic induction. In that case the chart represents approximately the direction and relative magnitude of the total electronic current around the 52nd parallel when it is noon at Berlin.

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[1] Curves not here included.

It will be seen that the electrons are everywhere flowing away from the part of the earth nearest the sun, and that their movement away from this region is greater on the side which is approaching the sun than on the side which is receding from the sun. The electrons are apparently repelled by the sun, and hence follow the sun around the earth, keeping as nearly as may be on the opposite side.

Since in the summer time, when these currents were observed, the part of the earth nearest the sun was about 40 degrees south of Berlin, the north component of the current predominates at noon, while at midnight, when the sun is on the opposite side of the earth and the nearest direction to its vertical rays is northward, the direction of the current is due south.

If the earth currents are due to the sun's electrostatic induction, there should be a correspondence between the diurnal variations in magnitude of the two phenomena. Since simultaneous measurements of the two phenomena have not been made at the same place, the only comparison that can now be made is between the diurnal variation of the sun's induction at Palo Alto and the diurnal variation in the total earth current at some other place.

This is done in Fig. 4, where the diurnal variation in magnitude of the total summer earth current at Berlin is represented by the continuous line and the diurnal variation in potential-difference between the earth and the insulated conductor for the month of August, 1920, is represented by the broken line.

Stanford University.

The Sun's Charge: A Criticism of Methods.

By F. J. Rogers.

In abstracts[1] of papers presented before the American Physical Society, Prof. Sanford has given experimental results obtained in an attempt to measure the diurnal variation of the earth's potential, presumably caused by the rotation of the earth in an assumed electrical field due to the sun.

I. If the earth and sun were charged alike then the electrical field near the earth would be subject to a diurnal variation, being weaker in the day-time and stronger at night. The *potential* of the earth, however, assuming it to be a conductor, would not be changed by its rotation in the field of the sun.

II. Professor Sanford's method consisted in measuring the variation in the potential difference between the earth and an insulated conductor "by means of a quadrant electrometer, one pair of whose quadrants is connected to the water system of Palo Alto while the other pair is joined to an insulated capacity enclosed in an earthed metal cage." It was apparently expected that the potential of the insulated conductor would not change by the same amount as the expected change in the potential of the earth and the earthed screen.

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[1] Phys. Rev., Vol. 17, pp. 243, 517, 1921.

If the insulated system consisting of the quadrants and the conductor connected to them is surrounded by an efficient screen, then no change in potential of the screen and the other quadrants connected to it will produce any field or difference of potential inside the screen. This conclusion follows as a necessary result from Gauss's theorem or from one of Faraday's famous experiments. The writer tested this conclusion experimentally as follows: The screen containing the electrometer, battery, and insulated conductor was set upon insulating supports and its potential was suddenly raised by 30,000 volts by means of sparks from an electrical machine, no perceptible motion of the electrometer needle could be observed. Of course this experiment will not succeed unless the screen is an efficient one, all parts of which change potential by the same amount. Reason will be given later to show that it would not succeed with Prof. Sanford's set up of apparatus.

III. An electrometer and an insulated conductor connected as described does give a diurnal variation of the electrometer deflection. There are at least three causes for such a variation, which are located inside the screen.

1. When the quadrants are insulated the needle "drifts" towards a "false zero"; this zero is not stable but itself varies. The cause of the drifting is a minute ionization current from the charged needle to the quadrants and a leakage current from quadrants to ground.

2. If the potential of the needle changes while one pair of the quadrants is insulated, there will be a proportionate change in the potential difference between the quadrants. If e is the potential difference between the quadrants (starting from zero) and ΔE is the *change* in potential of the battery charging the needle, it may be easily proved that

$$e = \frac{C}{C + C_0} \Delta E$$

in which C_0 is the capacity of the insulated quadrants and the conductor connected to them and C is the capacity between the insulated quadrants and the needle. The e.m.f. of the battery necessarily has a diurnal variation on account of the diurnal change of temperature.

3. The insulated quadrants and the conductor connected to them form one coat of a condenser the other being the electrometer case and the earthed screen. When the quadrants are earthed this condenser becomes charged by the contact e.m.f. between the two coats, producing a weak electrical field between the opposing surfaces. Upon insulating the quadrants this field immediately begins to disappear on account of residual conductivity induced by radio activity or some other cause. This change of potential depends primarily upon the value of the contact e.m.f. but the *rate* of change depends

greatly upon the condition and temperature of the opposing surfaces. In particular if the conductor is freshly sand-papered aluminum the change of potential after insulation is at first quite rapid but as the aluminum tarnishes the rate of change (starting from zero each time) becomes less and less, and finally becomes very small. An aluminum plate which had never been sand-papered (or otherwise "sophisticated") changed its potential very slowly after being insulated, but this rate of change was enormously increased by a considerable rise in temperature, quickly returning to its original value as the original temperature was regained. It is quite obvious that such a condition provides a cause for change of potential which, at least in part, explains Prof. Sanford's experimental results.

IV. It has been assumed so far that Prof. Sanford's screen is an efficient one. That this is not the case is demonstrated by the fact that a passing street car (too distant to produce any mechanical effect) makes a perceptible deflection of the electrometer needle. It appears that the metallic screen surrounding the insulated conductor does not extend *under* it. The ground itself is depended upon to be sufficient. Now the ground and the water pipes are not at the same potential and what is more important, the potential of these two 'grounds" do not change at the same rate or by the same amount. Any change in the relative potential of ground and water pipes, produced by a passing street car or other earth currents will produce an electrometer deflection. Thus the imperfection of Prof. Sanford's screening allows him to obtain an effect in part due to potential differences in the ground, which are directly associated with earth currents. I believe that, in general, these effects are extremely small as compared with the disturbances due to the other causes already mentioned.

All the statements above made have been substantiated by profuse experimental results which will probably be published later.

Stanford University.

The Effect of Temperature on the 3883 CN Band.

By Raymond T. Birge.

A study of furnace spectrograms of the 3883 CN band,[1] kindly lent to me by Dr. King, and of ordinary arc exposures of my own, has furnished some new evidence in connection with the quantum theory of band spectra, as developed by Heurlinger and others. The chief point in this theory is the similarity of the positive and negative branches of the series forming each sub-head of the band, the common origin of the two branches being taken at the point of zero intensity of the "singlet" series. In the A_1 series this is at λ 3875.342, for which $m' = 28$ by the usual nomenclature, or $m = 0$ by the new nomenclature. The A_2 series is theoretically a portion of the negative branch and is simply a continuation of the A_1 series.

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[1] Astrophysical Journal, 53, 161, 1921.

By comparing lines of equal photographic blackening, on arc exposures of varying length, there has been obtained, for the first time, a quantitative curve of the intensity distribution in each branch of the A series. Using only "free" lines, the curve is *smooth* and *regular* throughout, even for perturbed lines, and is quite different in shape from Haferkamp's plot.[1] Within limits of error, the *shape* of the curve is quantitatively the *same*, for the positive and negative branches, but the negative branch is about 15 per cent. stronger. The phenomena is similar to that shown by infra-red bands, for which a theoretical intensity formula has been derived by Kemble.[2] This formula ($I = c_1 m e^{-c_2 m^2}$) does *not* hold for the author's curve. But the modified formula $I = c_1 m^{3/2} e^{-c_2 m^{3/2}}$ holds within limits of error (about 10 per cent.) from $m = 4$ to $m = 137$. This covers an intensity variation of 60,000 per cent. The maximum intensity is at $m = 30.4$ and hence the especially strong head of this band is due to the accidental coincidence of the maximum of intensity of the negative branch and the vertex of the $\nu : m$ curve.

The arc spectrograms of King, taken with 170 volts, 4 amperes, show a definitely different intensity curve from those of the author, taken with 170 volts, 13 amperes. The maximum is shifted to about $m = 28$, but the equation quoted still seems to hold. In the case of the *furnace* spectrograms (taken at 2700° C., 2500° C., and 2300° C.) there is a *radical* redistribution of energy, the maximum being shifted to $m = 22$ or less, and the higher values of m having far less intensity. This is to be expected on quantum theory, as the value of m is proportional to the number of quanta of angular momentum possessed by the radiating molecule, and this number should vary with temperature. *In all cases*, the positive and negative branches seem to have the *same* shape, the negative being however slightly stronger. It is difficult to get any accurate criteria for judging the relative intensity of the self-reversed lines of the furnace spectra, and it is not certain, thus far, whether the formula quoted holds for these spectra.

The enhancement of the A_1 series over the A_2, as noted by King, is thus due to a *similar* redistribution of energy in the positive and negative branches of the general A series. Similar phenomena are shown by the B and C series. Because of these energy changes, it is possible to further analyze certain perturbations. The results for the A series show that in all cases the perturbations occur in pairs at m and $-(m+1)$, and are similar, agreeing with Bachem's recent results from magnetic effects.

A careful study of the head of the band shows that it is possible to assume that the *doublet separation* is the *same*, for *corresponding* members of the two branches, and that this separation decreases linearly to zero, at $m = 0$, from the last resolvable doublet at $m = 28$. The first members of the A_1 and A_2 series coincide to less than 0.002 Å. at the "head" of the band (3883.4), and the value of $\Delta\nu$ for A_1 and A_2 is the same at this point, showing that A_2 is a smooth continuation of A_1, in every respect, as demanded by theory. However, the values of A_2-A_1, using the *mean* frequency of each doublet, and using a newly discovered A_2 doublet $m = 77$, show that the complete $\nu : m$ curve is *not* symmetrical with respect to a line $m = c$, as demanded by Thiele's theory. Imes has found similar dissymmetry for the infra-red bands.

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[1] Zeit. wiss. Photographie, 9, 19, 1910.

[2] PHYS. REV., 8, 689, 1916.

A series of very faint doublets, as mentioned by King, appears in the furnace spectra, but not in any arc spectra I have seen. (However, Kayser and Runge seem to have obtained these lines in the arc, and measured them, as single lines. Uhler and Patterson did not obtain them.) I have succeeded in measuring six of these doublets, and using the mean frequency of each doublet they fit, with an average error of 0.0004 Å (purely accidental accuracy) the Deslandres' formula $\nu = 25,750.175 + 0.017143\,(m + 0.3232)^2$ where $m = 25$ to 30. This gives the vertex of the parabola at 3882.37 Å, but if the series deviates from Deslandres' law as does the A_1 series, the "phase" would be practically zero, and the head at about 3881.47 Å. The doublet separation is the same as that of neighboring A_2 lines. This new series is seemingly not accounted for, in the quantum theory. The theoretical "zero" branch of the A series should have the vertex of the parabola at λ 3874.906.

This investigation is still in progress and some of the above results are to be considered as provisional.

PHYSICAL LABORATORY,
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SOME X-RAY ISOCHROMATS.

BY DAVID L. WEBSTER.

THIS paper is a preliminary report on a study of the x-ray isochromats of different metals in different directions, that is being made as a first step toward the law of intensity as a function of potential, frequency and direction, by a method previously described. The present report deals with wave-lengths 1.0 to 0.3 Å, as obtained from tungsten at 50° and 140° from the line of the cathode rays, and from copper at 55° and 145°, the latter metal being the target in a special Coolidge tube furnished by the General Electric Co., through the kindness of Dr. Coolidge.

The graphs are of the same general type as those previously published for other conditions, with only slight deviations from the author's empirical formula,

$$I(V, \nu) = k(\nu)\left\{(V - H\nu) + H\nu p(\nu)\left(1 - e^{-q(\nu)(V/H\nu)-1}\right)\right\},$$

but no exact statements can be made until several sources of error are eliminated. One of these errors, due to absorption in the target, is worth noting here, because the only direct experimental evidence previously available on that point was in the form of spectrum graphs. These graphs show no discontinuities in the general radiation at the absorption limits, except in

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[1] See Bulletin of the National Research Council, Vol. 1, Part 7, Dec., 1920.

special cases, where the rays used were ones leaving the target at a small angle to its surface, and the target was known to be rough. From this one would infer that absorption in the target is relatively unimportant except in those special cases, and that even there it is likely to be an almost constant factor, depending on the shapes of the minute irregularities in the surface. But some evidence to the contrary now appears in some isochromats taken at different angles from the surface of the target but at the same angle from the cathode rays. These isochromats show differences of a sort suggesting variable absorption in the target, at least when the angle of the x-rays from its surface is small. The theory of this absorption is complicated by the fact that the cathode rays do not go straight within the target, as well as by the most minute roughness in the target, but it ought to be possible to determine the absorption factors experimentally, by reference to the absorption limits as shown in the emission spectra under different conditions.

STANFORD UNIVERSITY.

THE WIDTH OF X-RAY SPECTRUM LINES.

BY ARTHUR H. COMPTON.

THE width of x-ray spectrum lines is ascribed to (*a*) the width of the slits employed, (*b*) angular imperfections of the crystal, (*c*) finite resolving power of the crystal grating, and (*d*) lack of homogeneity of the x-rays. Experimental values of the width of the tungsten lines $\lambda = 1.242$ and 1.279 Å.U. in the first four orders from the cleavage faces of rock-salt and calcite are given. Analysis of the data indicates that each of the above factors contributes measurably to the observed width. In particular the average lack of homogeneity of the lines is found to be $\delta\lambda/\lambda > 0.0007 \pm .00013$. This result is in accord with the complexity of the lines as predicted by Sommerfeld's theory of elliptic electronic orbits.

WASHINGTON UNIVERSITY,
ST. LOUIS, MO.

THE EFFECT OF DAMPING ON THE WIDTH OF X-RAY SPECTRUM LINES.

BY G. E. M. JAUNCEY.

IT is assumed that the ether vibrations due to an oscillator giving out x-rays can be represented by

$$y = e^{-kt} \sin (pt + A).$$

Developing this as a Fourier integral, a relation between the damping factor k and the width of the resulting spectrum line is found. To obtain the experimental widths of x-ray spectrum lines found by A. H. Compton, k must exceed $5.3 \times 10^{+5}$ sec.$^{-1}$ for the wave-lengths used, while on electrical theory the damping factor of an oscillator giving out radiations is 7.2×10^{14} — sec.$^{-1}$ for this wave-length. The damping therefore accounts for something less than one seventh of the width found. The width due to damping for any wavelength is 1.2×10^{-4} Å.U.

Washington University,
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Experiments on Polarization of X-Rays.

By Paul Kirkpatrick.

Formulæ expressing the intensity of reflection of x-rays from crystals contain a term sometimes referred to as the "polarization factor." It is of the form $(1 + \cos^2 2\theta)$, where θ is the angle of reflection. This term is the summation of the squares of the amplitudes of the components of the electric vector respectively normal to and in the plane of reflection, the normal component of the vibration of the radiating oscillator being equally effective for all values of θ, while only a projection of the component in the plane of reflection is effective, this projection diminishing with the cosine of the angle of deviation, or 2θ. It is seen that with θ small, as it ordinarily is, the degree of polarization of the reflected beam is also quite small.

However the polarization factor as stated above becomes incapable of predicting the degree of polarization in any experimental case for two reasons. The original ray from the ordinary x-ray tube is not free from polarization. In consequence we must write the polarization factor $(1 + A \cos^2 2\theta)$, A becoming unity only when the line joining the electrodes within the tube makes an angle of 45° with the plane of reflection. Further, the ordinary form of the polarization factor ignores the possibility of multiple reflection within the crystal, in consideration of which we should write $(1 + \cos^{2n} 2\theta)$, wherein is the number of reflections undergone before emergence from the crystal. This form of the factor leads us to expect a higher per cent. polarization than that predictable from the simpler form.

Apparatus has been constructed for measuring polarization by successive reflection of x-rays from two crystals variously oriented. The method is analogous to that used for exhibiting polarization of light by reflection from two glass plates. The twice reflected ray is received in an ionization chamber and the total ionization is recorded by a string electrometer as the second crystal is swept across its position of reflection. Rough preliminary measurements show a tube polarization in the expected direction. In investigations of crystal polarization the intensities in the parallel and crossed positions have been found sensibly the same for angles of reflection less than eight degrees.

Department of Physics,
University of California.

AN X-RAY SPECTROMETER FOR THE DETERMINATION OF ABSORPTION COEFFICIENTS

BY ELMER DERSHEM.

THIS x-ray spectrometer is equipped with a high grade circle and micrometer eyepieces such as to make readings possible to one or two seconds of arc. A string electrometer is attached to the ionization chamber and rotates with it, thus eliminating changes of capacity. The electrometer has a sensitivity of about 75 divisions per volt while the electrical capacity of ionization chamber and electrometer combined is only 13 centimeters. By the use of amber insulation and specially purified and dried methyl iodide gas the blank leak is made entirely negligible in comparison with the rate of motion of string during readings.

When used in connection with a carefully regulated Coolidge x-ray tube one is able to measure absorption coefficients of any material for homogeneous x-rays with a probable error of less than 0.05 per cent.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA.

A PHOTOGRAPHIC METHOD OF X-RAY CRYSTAL ANALYSIS.

BY ELMER DERSHEM.

WHENEVER the x-ray analysis of crystals by the photographic, crystal rotation method is undertaken, many curved lines or streaks, some of them very sharply defined, appear on the plate if it is sufficiently large. The position and length of these lines depends entirely upon the slopes of the reflecting planes in the crystal and the angular rotation of the crystal during exposure. It is possible with a single exposure to determine the distance between certain planes parallel to the axis of rotation by the spacing of characteristic lines and from the same plate the slopes of other planes with respect to the first and therefore to be able to determine the shape and volume of the elementary parallopipeda of the crystal lattice.

Formulæ are derived giving the locus of points on these lines in terms of the measured rotation of the crystal and the indices of the reflecting planes. For any assumed space lattice it is possible to compute the locus of these lines and check with the photograph. This method should be useful in the case of crystals which, on account of small dimensions in certain directions, *e.g.*, thin lamellar crystals, do not reflect well enough to give line spectra from all the possible faces.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA.

SECRET PHOTO-ELECTRIC SIGNALLING.

BY ELMER DERSHEM.

A DESCRIPTION and demonstration of an apparatus for signalling consisting at the sending end of a source of light and a rotating disc having a different number of sectors at different distances from the center but having the ratio of open to closed segments the same for all radii. If the sector disc is moved across the beam of light the total intensity remains constant and hence an unequipped observer is unaware of any change as the interruptions are too frequent to be noted by the eye. At the receiving station this beam which has been interrupted 500 to 1,000 times per second falls upon a photo-electric cell connected to a multiple stage audion amplifier and the signals, made by moving the disc across the beam, may be heard as changes in pitch in the telephones. Tests have shown that a light intensity equal to one per cent. of that of the full moon is sufficient for this purpose without condensing lenses while with their use the necessary intensity is less, depending upon the size of the lens.

Department of Physics,
University of California.

An Harmonic Curve Writer.

By William J. Raymond.

The Harmonic Curve Writer, briefly described at a meeting of the Physical Society,[1] has now been completed. The scope of the machine has been considerably extended by means of attachments, so that the visual representation of types of vibration, other than those originally planned, has been made possible.

In the machine as now constructed there are four vertical shafts, each carrying a driving cone. By means of friction wheels, which transmit the motion from cone to cone, four relative frequencies of rotation are possible, which may have any desired values between 1 and 27. Each shaft actuates a slotted cross-head, and by means of a summation-wire passing over pulleys, the vibratory motion of the cross-heads, doubled in amplitude, is communicated to a tracing pen. Circular motion of the pin which drives a cross-head causes simple harmonic motion. Spiral motion of the pin, obtained by an independently driven cam, causes damped harmonic motion, the value of the decrement depending upon the relative frequencies of rotation of the cam and cone-shaft. Equiangular spiral cams have been made for logarithmically damped motion, and Archimedean spiral cams for rectilinearly damped motion. The initial amplitude of one of the four component motions, given to the tracing pen, may have any value within 7.5 cm.

The pen moves transversely across a band of paper 30 cm. wide, which is driven at constant speed from one roller to another. With this arrangement the curve writer may be used in the following ways: (1) It will draw sinusoids of amplitudes less than 7.5 cm. and wave-lengths between 0.25 cm. and 125 cm. (2) It will draw damped sinusoids with a wide range of decrements. (3) It will combine two to four sinusoids of commensurable or incommensurable wave-lengths, as, for example, the representation of the four principal harmonics of a bell-tone. (4) It will combine damped sinusoids, drawing curves which represent the vibrations of coupled, oscillatory systems, mechanical or electrical. (5) It will combine two decrescent, exponential curves, representing over-damped or ultra-periodic motion. (6) It will combine two ultraperiodic curves, or an ultraperiodic curve and one or two damped sinusoids. (7) It will combine damped and undamped sinusoids, to represent forced vibrations of mechanical systems, or the so-called starting-currents in certain electrical circuits.

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[1] Phys. Rev., Vol. XI., p. 479.

As an attachment to the machine, a rotating circular plate may replace the band of paper, to receive the record of the tracing pen. With a wide range of frequencies of rotation, in terms of the period of the pen, many of the curves drawn by the older harmonographs may be reproduced. The trace of the pen may pass through the center of the table, or along a line parallel to a diameter.

A second attachment consists of a square table, rolling to and fro on rails, in a direction perpendicular to the motion of the pen. One or two of the slotted cross-heads may be diverted from their usual function of operating the pen, to produce simple or complex harmonic motion of the table. When two cross-heads drive the pen and two the table, the Duplex Harmonograph of Vincent and Jude[1] is imitated. A number of their more striking curves have been drawn, as well as some of the simpler forms originating with Lissajous.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA.

THE VARIATION OF THE THERMAL CONDUCTIVITY OF SOFT CAST IRON WITH THE TEMPERATURE.

BY ELMER E. HALL, O. R. HULL AND O. S. IMHOFF.

USING for the specimen a cylindrical shell capped by a hemispherical shell, the authors have measured the conductivity of a given casting of soft iron, such as used for stoves, at various temperatures in the range 190° to 540° C. Special attention was paid to securing radial flow, to the construction of the thermocouples, and to measuring the energy input. The difference in temperature between the inside and outside of the specimen was usually three or four degrees. The conductivity increased from 0.07 at 195° to 0.20 at 542°.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA.

THE INDEX OF REFRACTION OF WATER, ETHYL ALCOHOL, AND CARBON BISULPHIDE AT VARIOUS TEMPERATURES.

BY ARTHUR R. PAYNE AND ELMER E. HALL.

THE index of refraction of water was measured by the spectrometer method from 16° to 98.4° C., observations at thirty-four different temperatures in this range being made. Special attention was paid to the region above 50°. Precautions were taken to secure uniform and constant temperature during a set of readings. Angles were read to seconds. The prism was a specially designed gold plated brass prism with optical glass sides. Before gold plating the brass prism it was found that enough impurity dissolved in the water in a few hours to change the index by two units in the fourth decimal place. The variation of the refractive index with temperature is given at various temperatures throughout the range studied. The results are compared with those of previous workers. The results of similar measurements are given for ethyl alcohol for the temperature range 15° to 75°, and for carbon bisulphide for the range 16° to 45° C.

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[1] Phil. Mag., 6 Ser., Vol. XXIX., pp. 490–514, Pl. VIII.

Department of Physics,
University of California.

Preliminary Report on Spectroscopic Investigations in the Extreme Ultra-Violet.

By J. J. Hopfield.

A method has been developed for making Schumann films suitable for work in the extreme ultra-violet. These films can be developed at room temperature for ten minutes and longer and still show no trace of fog.

A method has been devised for obtaining the emission spectrum of highly absorbing gases even below the limit of fluorite absorption. The method consists in keeping the receiver of the vacuum spectrograph filled with hydrogen or some non-absorbing gas, while the discharge tube of internal capillary end-on type is filled with the gas under investigation. The two gases are kept localized by flowing constantly in contrary currents and by being continually pumped out through a common exit.

Using the above method of investigation, a very strong triplet λ 1302.5, 1305.2 and 1306.4 of relative intensities 8, 6, and 5 respectively was discovered when the vacuum tube contained oxygen and is attributed to that gas. If this is the first member of a fundamental series it corresponds to a resonance potential of 9.45 volts, a somewhat higher value than that experimentally observed.

By regulating the flow of gases so as to allow a mixture of hydrogen and oxygen or of hydrogen and air in the discharge tube, most of the secondary spectrum of hydrogen in the Lyman region was suppressed while the fundamental line λ 1215.6 and other lines, including the oxygen triplet mentioned above, were brought into prominence.

Department of Physics,
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The Effect of Pressure on the Band Spectra of Nitrogen.

By Jean Huddleston.

The only work that has been done on the effect of pressure on nitrogen band spectra is that of Villari, Cailletet and Cazin, who, about 1875, observed separately that with increasing pressure, the band spectra gradually became continuous. The object of the present investigation was to get photographic record of the changes in the spectra, and to discover at what pressure the various groups gave way to a continuous background.

A brass cylinder was first used, with a hand pump to give the increased pressure. When the spark was excited between copper electrodes by a large transformer run on 110 volts, it was found that the NO_2 formed in the cylinder produced such a strong absorption spectrum that all nitrogen bands were completely destroyed. The substitution of platinum electrodes for copper lessened the formation of NO_2 materially, and when the transformer was replaced by a small induction coil run on 8 volts, the plates showed no absorption. The other apparatus was of glass, and could be evacuated and filled with chemically prepared nitrogen, in which atmosphere no NO_2 would be formed. A 4-prism glass spectrograph was used, the range of wave-lengths recorded being from about 3700 to 5700; but since the spectrum was always faint above 5200, the bands available for study were the negative pole bands and the second group of positive bands.

The spark in pure nitrogen was submitted to pressures of 1, 1½, 2, 3, 3½, 4 atmospheres. At one atmosphere the positive and negative bands were of about the same strength. As the pressure increased, there was a gradual formation of a continuous spectrum, which appeared first in the longer wave-lengths, and worked on down toward the violet. At four atmospheres this hazy background had filled the entire spectrum, with band structure barely visible from 4344 to a little below 3997. Positive and negative bands seemed to be affected similarly. The continuous spectrum extended as far into the longer wave-lengths as the plate registered, but showed a decided decrease in intensity around 5200–5500.

When the spark was produced in air, in the brass cylinder, by a small induction coil run on 8 volts, the negative bands were stronger than the positive at one atmosphere. Pressures of 1, 2, 3, 4, 5, 6 atmospheres were used. With increasing pressure, the continuous spectrum gradually blotted out all band structure, closing in from the red, as before: but this change came earlier. At three atmospheres the positive bands had all given way, and the negative bands following the heads 4278 and 4236 were the only ones discernible. At four atmospheres all bands were gone; five and six atmospheres showed merely an increasingly heavy continuous background, crossed by some nitrogen and metal lines, but no band heads.

When the spark was produced in the cylinder by a large transformer run on 110 volts, the general progressive changes were much the same, except that here the continuous background was slower in forming, not blotting out all band structure until six atmospheres. Again the negative bands were stronger than the positive, and those following the head 4278 were the last to go. As in all other cases, the background had a decidedly diminished intensity around 5200–5500. At one atmosphere the maximum intensity of the band spectrum was around 4278, while at six atmospheres the maximum of the continuous spectrum was around 4630.

The region of the first group of positive bands, which was not recorded on the plates, was observed visually, by means of a constant deviation spectroscope. There was no evidence of any band structure at one atmosphere, or at any higher pressure, that region giving an apparently continuous background in all cases. This is in accordance with the evidence on the plates, namely that the bands in the longer wave-lengths tend to give way sooner to a continuous spectrum. The first group apparently becomes continuous below one atmosphere.

Department of Physics,
University of California.

The Reaction of Iron with Nitric Acid.

By Joseph G. Brown.

In previous papers the theory was advanced that the rate of solution of iron in nitric acid of different density is associated with different chemical reactions. This theory was based upon the study of a voltaic cell having an iron anode and with nitric acid as an electrolyte. In general it was concluded, (1) that the rapid rate which takes place in dilute acid is associated with the formation of ferrous nitrate, (2) that the moderate rate which occurs in acid of intermediate density is associated with a ferrous reaction involving some other product than ferrous nitrate, and (3) that the very slow rate in concentrated acid is associated with a ferric reaction.

The chemical products of this reaction and of the electrolysis of nitric acid were investigated by Freer and Higley in 1899. Veley and Manley measured the electrolytic conductivity of nitric acid in 1898. Young and Hogg measured the reaction rates of the process in 1915.

A consideration of the results of these studies and of the properties of the iron and nitrogen oxides and hydroxides has led to a more detailed theory of the action of iron with nitric acid.

Iron forms three known oxides: ferrous, ferric and ferroso-ferric; and three hydroxides: ferrous, ferric and meta-ferric. The hydroxides are amphoteric; *i.e.*, they act either as weak bases or weak acids. This indicates that they ionize either with hydroxyl or hydrogen ions and probably with both. The ease with which iron compounds change valence and the tendency which they have to form compounds with mixed valence are well known.

Nitrogen forms five oxides: pentoxide, peroxide, trioxide, nitric and nitrous, and in combination with hydrogen, amine oxide and ammonium oxide. These oxides unite with water to form the series: nitric acid, nitrous acid, hyponitrous acid, hydroxyl amine and ammonium hydroxide. Dilute nitric acid is one of the strongest acids known, while ammonium hydroxide and hydroxyl amine are bases. The intermediate compounds are weak acids.

A study of the properties of water solutions of nitric acid, of their action in electrolysis, and their reaction with metals, indicates that as the density increases the acid characteristics become weaker, and the basic characteristics become stronger, thus showing the amphoteric nature of nitric acid.

The reduction products which result when iron is dissolved in dilute acid are shown to be the consequence of the action of hydrogen ions in decreasing the valence of nitrogen. In consequence of this reduction electrons are removed from the iron by No^+ and N^+ ions. In concentrated acid, however, the hydrogen ion concentration is small, hence the reducing action upon the nitrogen does not occur. This results in electrons being removed from the iron by the No_2^+ ions, which, on account of their oxidizing character, increase the valence of the iron, thus forming ferric ions before any reaction takes place.

The observations are best accounted for by assuming that in concentrated acid the reaction is basic; *i.e.*, that the iron ions unite first with hydroxyl ions.

The products of the electrolysis of nitric acid solutions and the formation of the passive condition when an iron anode is used in the electrolysis, are in strict agreement with these conclusions.

It is believed that the principles involved in this theory are of general application. Chromium forms a series of oxides similar to iron but with an additional acid forming oxide. When chromic acid is used as an electrolyte in the cell with an iron anode, the results are similar to those obtained with concentrated nitric acid. No matter how dilute the chromic acid solution may be, there is no indication that the acid reaction takes place. This might be expected since chromium compounds show a more strongly amphoteric character and chromic acid is much weaker and more oxydizing than nitric acid.

STANFORD UNIVERSITY.

A BRAUN TUBE, UNDAMPED WAVE, PRECISION METHOD OF DETERMINING DIELECTRIC CONSTANTS OF GASES.

BY L. T. JONES AND H. G. TASKER.

AN undamped wave method of measuring small capacities has already been announced by Hyslop and Carman.[1] It has the disadvantage, however, of being applicable only to determining the dielectric constants of liquids, the sensitivity not being sufficiently great to measure smaller increments of capacity with the needful accuracy.

The new method combines all of the advantages of their arrangement but departs therefrom in the use of a Braun tube oscillograph in making frequency comparisons, by which means frequency ratios (of measuring and reference circuits) in any small whole numbers up to 15 may be determined with an accuracy of .05 cycle.

The method further departs from all others heretofore used for the measurement of changes in capacity, in that the desired value is determined without readjusting the circuits in any manner. This is accomplished by observing the frequency change in cycles as indicated by the changing Lisajous figures on the Braun tube and interpreting this quantity mathematically.

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[1] PHYS. REV., XV., p. 243, 1920.

It is found that the change in capacity (ΔC) produced by introducing gas to the test condenser may be expressed thus:

$$\Delta C = \frac{-1\left(\frac{2\Delta f}{f_0} + \frac{\Delta f^2}{f_0^2}\right) C_0}{1 + \frac{2\Delta f}{f_0} + \frac{\Delta f^2}{f_0^2}},$$

where C_0 = initial capacity of measuring circuit,
f_0 = initial frequency of measuring circuit,
and Δf = change in frequency of measuring circuit.

This equation is rigidly accurate but if f_0 exceeds 500,000 and Δf does not exceed 50, as will probably be the conditions of the work, then an error of less than .03 per cent. will be introduced by writing the equation thus:

$$\Delta C = \frac{-2\Delta f C_0}{f_0},$$

whereupon we have

$$K - 1 = \frac{-2\Delta f}{f_0} \times \frac{C_0}{C_v},$$

where C_v is the capacity of the test condenser with vacuum as dielectric.

It is obvious, then, that this method measures not K but the excess of K over unity, and that the accuracy of this measurement depends upon the relative rather than the absolute accuracy with which C_0, C_v and f_0 may be measured. The capacity sensitivity should be about 10^{-16} farads or 10^{-4} cm.

The authors propose to employ the method in one or more of the following investigations:

1. To obtain new and more accurate values of K for several gases.
2. To investigate variations of K with pressure of the gas.
3. To discover, if possible, a dependence of K on some molecular property.
4. To investigate variations of K with temperature.

In addition it may be used in any connection requiring the measurement of a very small capacity.

Department of Physics,
University of California.

THE EFFECT OF ULTRA-VIOLET LIGHT ON THE VISCOSITY OF RUBBER COLLOIDS.

By Richard Hamer.

During some research work on rubber colloids for the Honorary Advisory Council for Industrial and Scientific Research, Ottawa, Canada, the writer subjected a solution of Para rubber in pure benzol, to exposures of different lengths of time, and different limiting wave-lengths of ultra-violet radiation.

The periods taken for 34.2 c.c. to pass through a fine capillary tube at a constant temperature of 30.0° C. before and after the treatment, gave the viscosity relative to the initial reading taken as the standard.

The experimental results indicated that the radiation had a marked effect in decreasing the viscosity. The effect was cumulative with the time, and somewhat progressive after the radiation had ceased.

Radiation of shorter wave-length seemed more effective than that of longer wave-length.

The writer suggests that the cause lies in a depolymerizing effect of the radiation on the rubber particles.

The investigation is being continued for the purpose of following up some of the interesting points suggested.

UNIVERSITY OF CALIFORNIA.

THE SIMPLEST MERCURY VAPOR PUMP.

BY L. T. JONES.

SINCE the mercury vapor vacuum pump was first produced by Gaede it has rapidly taken its place as the most satisfactory of all pumps for the production of extremely high vacua. The quest may well be continued till not only the simplest form has been devised but also until one is designed that will pump against a very considerable back pressure.

The pump here described is like its predecessors in that the supporting pump must be capable of producing a vacuum of about .001 mm. Exhaustion will then be completed as far as adsorption of air by the vessel will permit.

A bulb containing about 25 c.c. of mercury is heated by a bunsen burner and the vapor rises through a nozzle to a chamber surrounded by a water jacket. The air intake is situated a little below the end of the nozzle but above the lower end of the water jacket. The air inlet tube is taken out through the side of the water jacket. After condensation the mercury drops down outside the nozzle and is readmitted to the heating chamber by a device first described by Knipp.[1] A forepump removes the air from the top.

The pump is made entirely of Pyrex glass and indeed cannot be made of any higher coefficient glass. A moderately skilled glass blower can make one in considerably less than an hour. The one demonstrated was made by the author in thirty minutes.

A more detailed description will be given at a later date.

DEPARTMENT OF PHYSICS,
UNIVERSITY OF CALIFORNIA.

A NEW ROTARY MERCURY PUMP.

BY L. T. JONES.

THIS pump is a liquid piston pump with mercury level remaining constant. A cylindrical housing closed by an iron plate and mercury seal and containing the rotating drum is divided by a baffle plate into two compartments, the low vacuum and the high vacuum. The rotation of the iron drum forces the air from the high vacuum side along the axis of the drum to the low vacuum side where it is removed by an auxiliary pump. This auxiliary pump should be capable of producing a vacuum of about 1 mm.

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[1] PHYS. REV., 9, p. 311, 1917.

Both the inlet and outlet connections are at the top of the main housing and are mercury sealed in a very simple manner. The shaft, entering below the mercury level on the low vacuum side, is provided with a mercury seal. The drum is of cast iron and freedom from breakage enables one to do away with elaborate glassware to separate the two chambers when the exhaustion has reached a certain stage.

The general performance is of the same order as that of the Gaede pump. The chief advantages lie in its sturdiness, the size of the intake and its smaller cost of construction. The experimental work is not yet completed.

Department of Physics,
University of California.

Atomic Radii.

By Maurice L. Huggins.

A comparison of "atomic volumes" is of much less value than a similar comparison of "atomic radii." W. L. Bragg has recently[1] shown how, from our present knowledge of crystal structure, it is possible to compute values of atomic radii for many of the elements, these values being fairly consistent in different crystals. His methods, however, are only applicable to those cases in which the electron bonds are on the center lines between adjacent atoms, if we identify (as we must in order to obtain even approximate constancy for a given element) the atomic radius with the distance from atomic center to valence electron group. It is necessary, then, before computing these distances, to know the arrangement of valence electrons in the crystal. In cases where we know the arrangement of the atoms, the orientation of the electrons can be arrived at by the proper use of the Lewis theory of valence and atomic structure. A few examples illustrating the method in simple cases were given.

The various factors affecting the distance between atomic center and valence electron groups were briefly discussed, with examples.

Department of Chemistry,
University of California.

Structure of the L-Series of Tungsten and Platinum.

By Frank C. Hoyt.

For the purpose of investigating the L-series of x-ray spectra a vacuum spectrograph has been constructed capable of furnishing very high dispersion.

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[1] Phil. Mag. (6), 40, 169, 1920.

This was made from a piece of 10-inch iron pipe 10 feet long with a large calcite crystal mounted at one end, the whole being sealed by iron face plates bolted on the ends. In one of these a slit about 1 mm. wide, covered with paper, was provided for the entrance of the rays from the x-ray tube outside. Both tube and crystal were movable about a common center, so that the reflected beam could always be sent straight down the pipe. With a good oil pump a vacuum of about ½ cm. of mercury could be maintained. A series of lead diaphrams provided excellent screening from stray radiation. At one setting of the crystal a spectrum covering about .2 Å. could be photographed, with a dispersion of about .018 Å./cm.

This arrangement is not adapted to direct wave-length measurements, but the values for lines near known ones can be found by interpolation with a certainty of about .2 per cent. The very weak lines in the β-groups of Pt and W have been measured in this way, and the results are given in the table below. The absorption spectrum of tungsten was photographed on the same plate as the emission spectrum by interposing a screen of sodium tungstate. The values thus obtained for the wave-lengths of the discontinuities have no very great accuracy, but with the high dispersion employed their positions relative to the weak lines can be determined, and this is of some theoretical importance.

Tungsten.

Interpolated Value.	Reported Value.	Notation.
1.236 Å.	1.2395 (Siegbahn[1])	
1.221	1.2205 (Siegbahn[1])	Webster, β_6; Coster, β_7
1.213	1.213 (Overn[2])	
1.209	1.209 (Overn[2])	
1.201	1.2031 (Siegbahn[1])	Siegbahn, β_5.
1.212	1.2140 (Duane and Patterson[3])	La_1 absorption limit
	Platinum.	
1.079 1.077	1.078 (Coster[4])	Webster, β_6; Coster, β_7
1.070	1.0701 (Coster[4])	Siegbahn, β_5.
1.059		
1.053		

Duane and Patterson[5] have reported that the critical absorption wave-length La_1 in tungsten is about .8 per cent. *longer* than that of the emission line β_5. This does not seem to be altogether in accord with present theories of emission, as β_5 has been found to have a critical potential equal to the quantum potential of the absorption limit La_1[2,3]. Here the discrepancy is not real, however, for the line which is just on the short wave-length side of La_1 is not the line assigned to L_1 in tungsten,[1] which was the line at 1.2118 Å. For some reason this line at 1.2118 Å. does not seem to have been observed by Duane and Patterson, although on photographic plates it is fully as strong as the line at 1.2031 Å. which they have taken as β_5, following Siegbahn.[2] That the line at 1.2118 Å is the one that corresponds to β_5 in the other heavy elements has already been pointed out.[3] This line is quite near to the absorption limit La_1 as measured by Duane and Patterson and the high dispersion picture showed it almost coincident with the lines at 1.209 and 1.213 Å, which constitute the doublet into which the 1.2118 Å line separates. Whether it was between them or on which side could not be determined with certainty, as the emission lines were very weak on this plate.

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[1] Phil. Mag., 38, 639, 1919.

[2] PHYS. REV., 14, 139, 1919.

[3] Ibid., 16, 526, 1920.

[4] Z. Physik, 4, 178, 1921.

[5] PHYS. REV., 16, 526, 1920.

Duane and Patterson have found La_2 about .7 per cent. longer than γ_2 and this was confirmed. It appears to be nearly coincident with γ_7 (1.072 Å.). The third absorption limit was too weak to show on this plate. This would indicate that the line γ_2 must belong in L_3. Experimentally the point is still uncertain. A series of photographs at 18, 16, and 15 kv. in platinum showed no noticeable change in intensity ratio between γ_2 and γ_3. On the other hand Dauvillier[4] has reported that γ_2 belongs in L_2 and γ_3 in L_3 in uranium.

STANFORD UNIVERSITY.

RECENT OBSERVATIONS OF ABSORPTION SPECTRA.

BY ARTHUR S. KING.

EXPERIMENTS with the electric furnace in obtaining absorption spectra by the use of a plug at the center of the tube have been directed mainly toward making the method more effective in producing in absorption the more difficult types of lines. With a new furnace, plug temperatures above 3200° C. have been used.

The observations have been largely of the iron spectrum. Iron lines to the red of λ5500 are difficult to reverse, having been previously obtained as absorption lines in the laboratory only in the explosion spectrum. In the emission spectrum of the furnace, they are relatively difficult of production, belonging largely in Class III. This orange and red section of the iron spectrum was obtained in absorption as far as λ6700 by the use of high temperatures, practically all of the furnace lines appearing.

In the ultra-violet, the lines are easily reversed, but the difficulty is to obtain a sufficiently strong continuous ground. This was produced by a plug temperature of 3000° C., the furnace lines being photographed in absorption as far as λ2448.

Between these limits, an extensive study has been made of the iron spectrum in emission and absorption at different temperatures, which will later be reported in detail.

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[1] Webster, Proc. Nat. Acad. Sci., 6, 26, 1920.

[2] Hoyt, Proc. Nat. Acad. Sci., 6, 639, 1920.

[3] Loc. cit.

[4] Compt. Rend., 176, 1350, 1921.

The furnace has also been used to obtain in absorption the subordinate series lines of the alkalis, which had not previously been observed as absorption lines. High temperature, together with the exclusion of oxygen, proved sufficient to bring out in absorption the subordinate series of sodium, potassium, caesium and rubidium. The extreme sharpness and freedom from dissymmetry of these absorption lines have enabled wave-length measurements to be made of greater accuracy than those previously made from arc spectra.

A further use of the absorption furnace was the mounting of a small tube inside a solenoid made to carry heavy currents and giving a strong magnetic field. In one of these solenoids, the furnace could be turned at different angles to the magnetic field. The trials made showed this to be a useful arrangement for the study of absorption lines produced under conditions which approximate in some respects those prevailing in sun-spots.

MOUNT WILSON OBSERVATORY.

PRELIMINARY MEASUREMENTS OF THE CRITICAL POTENTIALS OF THE M LINES IN THE X-RAY SPECTRUM OF LEAD.

BY P. A. ROSS.

THE critical potentials of five lines in the M series of lead have been measured. They are:

Line λ A°	Crit. Pot. in kv.	Edge of Gen. Radiation.
α 5.2751[1]	2.5	4.9 Å
β 5.0648[1]	2.7	4.7
γ 4.6637[1]	3.1	4.0
δ 4.073[2]		
ε 3.94[3]	4.0	3.1

A water jacketed brass x-ray tube with a Coolidge cathode was kept exhausted by a Langmuir pump. The direct current was supplied by the high tension system of D. L. Webster described elsewhere by him.[4] A stationary gypsum crystal was used with an optical path from target to plate of 24 cm.

This work is still in progress.

STANFORD UNIVERSITY.

A POSSIBLE ORIGIN OF THE DEFECT OF THE COMBINATION PRINCIPLE IN X-RAYS.

BY ARTHUR H. COMPTON.

THE original Kossel relation, which represents an application of Ritz's combination principle to x-ray spectra, and which follows directly from Bohr's theory, states that

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[1] Stenström. [2] Karcher. [3] Ross.

[4] Proc. Nat. Acad. Sci., vol. 6, p. 269, May, 1920.

$$K_{a_1}+L_{a_1}=\frac{1}{h}(W_{M_1}-W_K)=K_\beta\ ,$$

where K_a, L_a, K_β represent the frequencies of these lines respectively, W_{M_1} is the energy of an electron in the M_1 orbit and W_K its energy in the K orbit. It has been found experimentally, on the other hand, that

$$K_{a_1}+L_{a_1}=\frac{1}{h}(W_{M_1}-W_K)\neq K_\beta,$$

and that the difference

$$\Delta = K_{a_1}+L_{a_1}-K_\beta$$

is of the order of $+\frac{1}{2}$ per cent. of K_β. This difference has constituted a formidable difficulty for the Bohr-Sommerfeld theory of the origin of spectrum lines.

Perhaps the most obvious explanation of this discrepancy, proposed independently by Kossel[1] and Duane,[2] is that the K_β line may not be due to an electron falling from the same M orbit as the L_{a_1} line. According to Rubinowicz's principle of selection, indeed, no electrons should fall from the M_1 orbit to the K ring, but only from the $M_{2,\ 3\text{ or }4}$ orbits. On this view, therefore, K_β should equal

$$\frac{1}{h}(W_{M_{2,\ 3\text{ or }4}}-W_K).$$

A critical examination of the wave-lengths by Sommerfeld[3] and particularly by Birge[4] indicates, however, that this explanation probably does not account completely for the observed difference between K_β and $K_{a_1}+L_{a_1}$. In fact the difference Δ should on Kossel's view be approximately proportional to the 4th power of the atomic number, whereas Sommerfeld finds the experimental values of Δ to be nearly a linear function of N.

An alternative, or perhaps supplementary, explanation of this difference Δ is that as the electron passes from the M to the K orbit, traversing the L orbit, it sets the electrons of the L orbit into oscillation, thus being able itself to radiate less energy than $(W_K - W_M)$ when it reaches the K orbit. We know from a study of the tracks of β-rays and cathode rays through air that energy is lost in thus traversing groups of electrons. If the electron which gives rise to the radiation moves gradually from the M to the K orbit, the electrons of the L ring will assume their new positions gradually, and the work done on these electrons will be the same as that done by successive transfers from M to L, producing the L_a line, and from L to K, producing the K_a line. In this case the combination principle should therefore hold. If, however, the radiating electron were transferred instantaneously from the M to the K

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[1] Kossel, Zeitschr. f. Phys., 1, 126, 1920.

[2] Duane and Stenström, Nat. Ac. Sci. Proc., 6, 484, 1920.

[3] Sommerfeld, Zeitschr. f. Phys., 1, 142, 1920.

[4] Birge, Phys. Rev., 16, 371, 1920.

orbit, the electrons of the L ring would be set into oscillation with an energy equal to their difference in energy for p and for $p - 1$ electrons in the inner ring. Such energy of oscillation must be taken from the falling electron, and can be shown to represent a loss in frequency of this electron's radiation by approximately

$$\Delta = \frac{1}{h}\Delta W = \frac{2qR}{2^2}(N - p - \varphi_q), \tag{1}$$

where p and q are the numbers of electrons in the K and L rings respectively, R is the Rydberg constant, N is the atomic number, and φ_q is Moseley's correction for the repulsion between the electrons in the L ring. The energy spent by the radiating electron in setting up oscillations in the ring of electrons through which it passes will therefore lie between zero and the value represented by expression (1). The fact that this view makes Δ a linear function of N is in accord with Sommerfeld's interpretation of the experiments, and the numerical values also are of the right order.

WASHINGTON UNIVERSITY,
SAINT LOUIS, MO.

Second Series *November, 1921* *Vol. XVIII, No. 5*

THE

PHYSICAL REVIEW.

THE INFRA-RED ABSORPTION SPECTRA OF ALKALI HYDROXIDES.

By G. E. Grantham.

Synopsis.

Infra-red Absorption of Aqueous Solutions of Alkali Hydroxides, 1–3 μ.—The absorption of various concentrations of solutions of *NaOH, KOH and LiOH* and of one solution each of *NH₄OH and CsOH* was determined with reference to that of water, using two like cells with quartz windows, and a suitable spectrometer with thermopile and sensitive galvanometer. All curves show a broad, intense absorption band with a maximum at about 2.29 μ except in the case of the NH_4OH solution for which the maximum is at 2.20 μ. This agreement suggests that the band is due to dissociated hydroxide ions; yet in the case of other hydroxide solutions previous observers have not obtained an absorption band at this place. The absorption was found to be proportional to the concentration but not to vary systematically with the atomic weight of the base of the hydroxide. In the case of KOH the relative absorption results were checked by actual absorption measurements.

The Coefficient of Absorption of Water, 1.3 to 2.0 μ was measured by comparing the transmissions of cells of different thicknesses. Maximum absorption was observed at 1.48 and 1.98 μ. *The effect of dissolving an alkali hydroxide* is to decrease the absorption of the water in the band at 1.48 μ by an amount proportional to the concentration of the solution and to the atomic weight of the base.

Introduction.

An investigation of the ultra-violet and visible absorption spectra of aqueous alkaline solutions of phenoltetrachlorophthalein by Howe and Gibson[1] showed absorption bands at frequency numbers 3590, 2760 and 1740. These bands are approximately harmonic and it seemed desirable to search for the next band of the harmonic series which should have a frequency number of about 900, or a wave-length of approximately 1.1 μ. This search proved fruitless, but in the region 1.7 μ it was found that a given thickness of a ten gram molecule per liter aqueous solution of NaOH transmitted less energy than did an equal thickness of phenoltetrachlorophthalein in a ten gram molecule per liter aqueous solution of NaOH. Further trial showed that NaOH has a strong absorption in this region and that the result just described was due to a di-sodium salt being formed in the cell containing the phenoltetrachlorophthalein, which formation used up *n* gram molecules per liter of the NaOH, thus, in effect, making this cell contain a less thickness of NaOH. This suggested a study of the absorption spectra of alkaline hydroxides in the region 1.0 μ to 3.0 μ. The measurements were made by means of a spectrometer in conjunction with a thermocouple and galvanometer.

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[1] Phys. Rev., S. II., Vol. X, 1917.

METHOD.

Owing to the strong absorption of aqueous solutions in this region, a differential method was adopted. The deflection d_1 was taken when the radiation was passing through a given thickness of water, and an instant later the deflection d_2 was taken when the radiation was passing through an equal thickness of the aqueous hydroxide solution. This made it possible to have the two galvanometer deflections of the same order of magnitude, and to work with the galvanometer in a more sensitive condition. The values d_1 and d_2 were taken at least three times for each wave-length, or until consistent results were obtained. Readings were taken at each 0.1 μ except in the region of a peak on the transmission curve where they were made more frequently. A sample of the data obtained is shown in Table I. These values were selected at random from a complete set of data for KOH and represent about one third of the values taken for one curve. The per cent. relative transmission was calculated from the equation: Transmission $= (d_1/d_2) \times 100$.

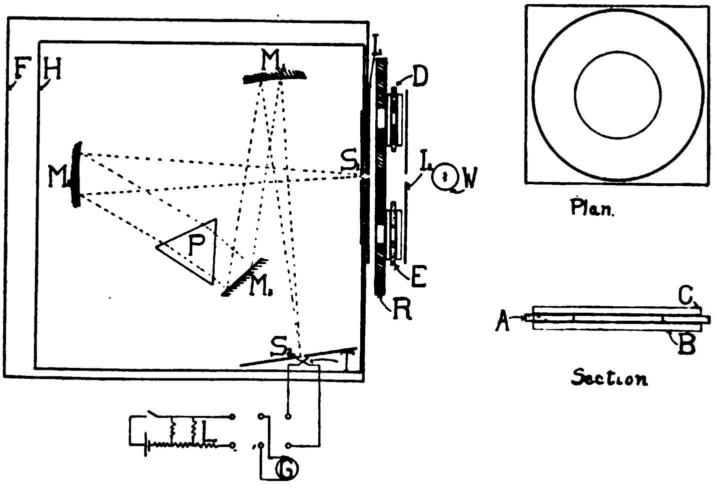

Fig. 1.

APPARATUS.

The apparatus used is shown in Fig. 1. The galvanometer and spectrometer have been quite fully described by the writer in a previous article.[1] The spectrometer was of the Hilger constant deviation type equipped with a rock salt prism *P* and a Coblentz[2] bismuth-silver thermopile *T*. When adjusted the instrument gave the following values for the three absorption bands of water and the carbon dioxide emission band respectively; 1.48 μ, 1.98 μ, 3.0 μ and 4.36 μ. These points were checked from time to time during this work. The spectrometer was covered by a glass case *H*, which was in turn covered by a sheet iron case *F*. The inside walls of *H* were covered with black cloth to prevent reflection. A 2.5 watt electric lamp was kept burning continuously immediately under the spectrometer to prevent dew formation, and P_2O_5 was used to keep the air surrounding the prism dry. During the experiments the slits S_1 and S_2 were 0.25 mm. wide.

A Coblentz[3] astatic galvanometer *G* was used. The sensibility of the galvanometer was adjusted to approximately 3.3×10^{-11} ampere per mm. at three meters distance. The sensibility was measured by means of the testing box *L*, which was furnished with the galvanometer. No deflection greater than 35 cm. was used.

The source of radiation *W* was a 400-watt nitrogen filled lamp which was supplied with current from a 120-volt storage battery.

The cells which contained the solutions were made as follows: A piece of plane glass was ground until its thickness was uniform and was then cut into two parts, each of which was four centimeters square. Through the center of each of these parts a hole 2 cm. in diameter was ground, making two glass washers. To assemble a cell one of these washers *A* was placed upon a quartz plate *B*, the small volume formed by the hole and the quartz plate filled with the solution, and another quartz plate *C* placed over the glass washer. These three parts were then clamped together and placed upon the slide *R* at *D*. The other cell was made similarly, using the other glass washer filled with water and mounted on the slide at *E*. The four quartz plates used were of equal thickness. Sheet iron screens I_1 and I_2, with holes 1.5 cm. in diameter, permitted no radiation to enter the slit S_1, except that which had come through the water, or solution in a cell. The pairs of washers used for different regions in the spectrum were 3.77 mm., 1.29 mm., 0.84 mm. and 0.15 mm. thick. Unless otherwise stated in this paper the thickness of the cell used was 0.84 mm.

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[1] Phys. Rev., S. II., Vol. XVI., 1920.

[2] Bulletin of the Bureau of Standards, Vol. 9, 1912.

[3] Bulletin of the Bureau of Standards, Vol. 4, 1908. Bulletin of the Bureau of Standards, Vol. 9, 1912. Scientific Papers of the Bureau of Standards, No. 282.

Table I.

Ten gram molecule per liter aqueous solution of KOH.
Thickness 0.84 cm.

μ	d_1 mm.	d_2 mm.	Transmission, Per Cent.	Average, Per Cent.
1.0	85	85	100	100
	85	85		
	85	85		
1.3	199	201	101	100.3
	195	195		
	202	202		
1.4	135	165	122.3	121.6
	135	163	121	
	135	164	121.5	
1.5	81	109	135	135.3
	80	109	136	
	81	109	135	
1.7	212	103	48.6	48.9
	210	103	49.0	
	210	103	49.0	
2.0	141	50	35.5	35.1
	139	48	34.6	
	139	49	35.2	
2.1	137	32	23.3	23.3
	132	32	24.3	
	131	31	23.7	
2.2	168	25	14.9	15.5
	165	26	15.8	
	165	26	15.8	
2.3	252	38	15.1	14.9
	252	37	14.7	
	250	37	14.8	
2.5	59	21	35.6	35.6
	59	21		
	59	21		
2.7	25	18	72	71.7
	24	17	71	
	22	16	72.7	
	24	17	71	
2.9	18	15	83.4	83.4
	18	15		
	18	15		

Results.

The absorption for different concentrations of NaOH, LiOH and KOH was observed as described above. The absorption for a 0.25 gram molecule per liter aqueous solution of CsOH and for an approximately 15 gram molecule per liter solution of NH_4OH was also observed. The results of these investigations are shown graphically in Figs. 2–7. Figs.

2, 3 and 4 show the absorption curves for different concentrations of the aqueous solutions of the alkali hydroxides. These curves are similar, each showing evidence of strong absorption in the region between 1.5 μ and 2.6 μ with a maximum of absorption at about 2.29 μ. There is practically no difference in the absorption of a given thickness of water such as was used in this investigation, and that of an equal thickness of solution of X-OH in the region from 1.0 μ to 1.3 μ. From 1.3 μ to 1.55 μ the absorption of a given thickness of water is much greater than

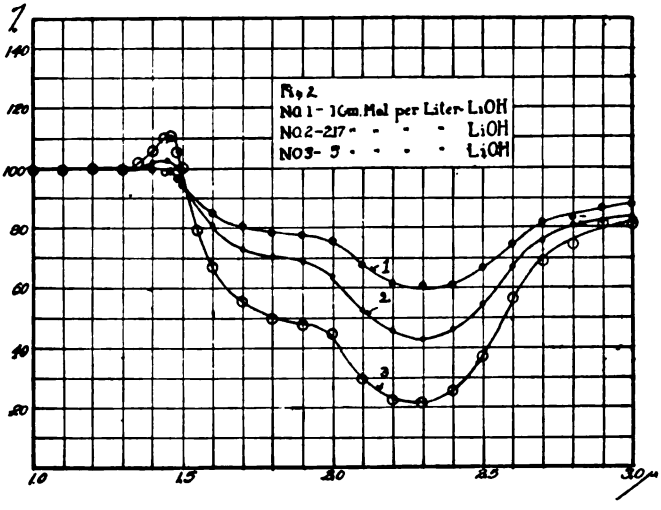

Fig. 2.

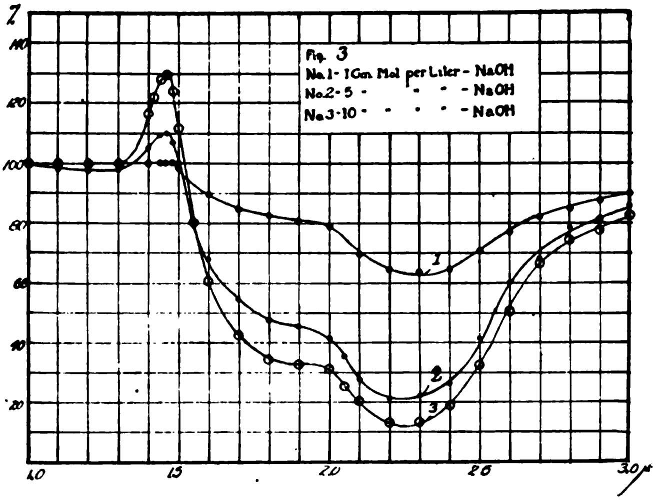

Fig. 3.

the absorption of the same thickness of a solution of X-OH. This range (1.3 μ to 1.55 μ) includes a sharp absorption band due to water at 1.49 μ. It also includes a region of weak absorption for X-OH. There is in the X-OH cell a less number of water molecules than in the water cell due to the replacement of water molecules by the X-OH molecules. If the X-OH absorption was weak in this region, a transmission value higher than 100 per cent. would be expected by this method. This explanation, however, does not account for the high values of transmission obtained. The density of a ten gram molecule per liter aqueous solution of KOH was found from chemical tables to be 1.414 grams per c.c. In this solution there were 561.00 grams of KOH per liter. The thickness of the solution used in the cell was 0.84 mm. The actual thickness of the water in this cell was then

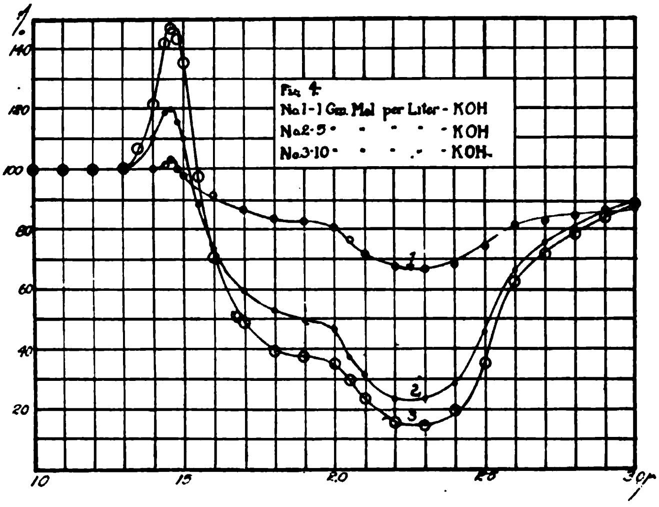

Fig. 4.

$$\frac{1414 - 561}{1000} \times 0.84 = 0.716 \text{ mm.}$$

If the coefficient of absorption of water was known, the per cent. of transmission for equal thicknesses of solution and of water, assuming that the KOH had no effect either as an absorbing agent itself or in modifying that of the water, could be calculated. The coefficient of absorption of water was measured in the following manner: Two cells of different thickness, filled with water, were mounted on the slide and the energy transmitted by each, for wave-lengths in the region 1.3 μ to 2.0 μ, was measured. The values of the coefficient of absorption were then calculated from the relation:

$$\beta = \frac{1}{c(t_2 - t_1)} \log I_1/I_2$$

where c is the concentration, taken as unity, and I_1 and I_2 the energy transmitted at a given wave-length through cells of thickness t_1 and t_2 respectively. The different values of β were obtained using cells of thickness 0.84 mm. and 1.183 mm. The measurements were then repeated using cells of thickness 0.84 mm. and 1.290 mm. and the mean value for β used. Using these values of β and 0.84 mm. and 0.716 mm. for t_2 and t_1 respectively, the values of I_1/I_2 were calculated for this region of the spectrum. These results are shown graphically in Fig. 5, curve No. 2. Curve No. 1 is the observed per cent. transmission curve for a ten gram molecule per liter aqueous solution of KOH, while No. 2 shows the corresponding values for the same solution calculated in the manner just described. Curve No. 1 shows a maximum ordinate at 1.46 μ of value 147 per cent., while, if this increase in transmission was due to the different thickness of water, the maximum value of the ordinate should be 112 per cent. This indicates that the presence of KOH has an effect upon the absorption of water, which effect is to make the transmission greater. A similar calculation with similar results was made for NaOH. The heights of these peaks as seen from Figs. 2, 3 and 4 are roughly proportional to the concentration for a given hydroxide, and to the atomic weight of the base for different hydroxides. The peak of the transmission band is at 1.46 μ in each case, while the maximum absorption of water in this region is shown by Fig. 7, curve No. 2 to be at 1.49 μ. This apparent shift of 0.03 μ toward the shorter wave-lengths is explained by the overlapping of the X-OH absorption band on the transmission band. If it

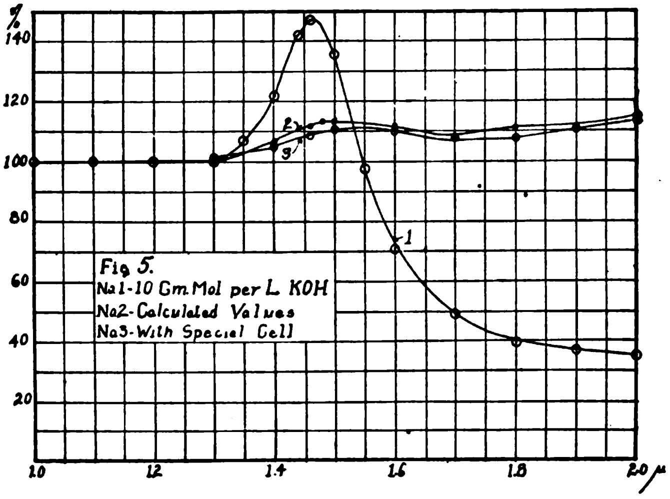

Fig. 5.

were not for this overlapping, the maximum of the transmission bands, shown in Figs. 2, 3 and 4 might coincide with the maximum absorption of water.

In order to check the method just described for testing the replacement of water molecules by KOH the following suggestion was acted upon.

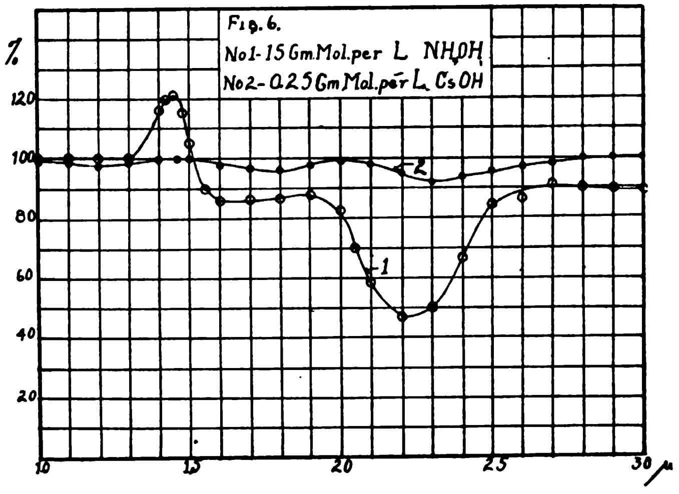

Fig. 6.

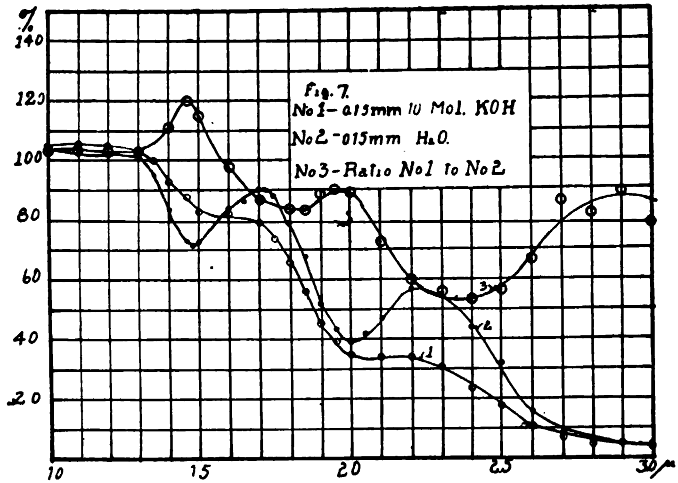

Fig. 7.

By using one cell 0.84 mm. thick and the other of the same thickness as the actual layer of water in a cell 0.84 mm. thick, filled with a ten gram molecule per liter aqueous solution of KOH, namely, 0.716 mm., the per cent. transmission could be obtained directly. In an attempt to grind a glass washer of this thickness and keep it uniform the grinding was carried too far, and a cell 0.705 mm. thick was obtained. The results of the use of the cells 0.84 mm. and 0.705 mm. thick are shown in Fig. 5, curve No. 3. These results check, as well as could be expected, with those calculated above.

On the transmission curves shown in Figs. 2, 3 and 4 there is a hump at wave-length 2.0 μ. This is also the location of a strong absorption band of water, as shown by Fig. 7, curve No. 2. This hump may be due to the same effect as the peak at 1.46 μ. It lies, however, in the region of strong absorption of X-OH, and for this reason a similar conclusion cannot be drawn from calculations. Owing to this increase in transmission at 2.0 μ the minimum of the transmission curves, which lies at 2.29 μ, may be shifted toward the longer wave-lengths, and it is therefore probable that the maximum of absorption of the X-OH solutions is at a slightly shorter wave-length than 2.29 μ. Since the absorption curves for the hydroxides of the alkali metals tested are similar, it is evident that the absorption is due to the OH radical. This conclusion is further substantiated by the fact that, at a given wave-length, the magnitude of the absorption for the same strength of different solutions shows no systematic variation with atomic weight of the base. The position of the minimum at 2.29 μ does not shift with the change of atomic weight of the base.

Fig. 7, curve No. 1 shows the actual transmission of a 0.15 mm. thickness of a ten gram molecule per liter aqueous solution of KOH, and No. 2 the transmission of the same thickness of water. Curve No. 3 shows the result of plotting the ratio of the ordinates of curves No. 1 and No. 2. This curve is similar to those of Figs. 2, 3 and 4. The relatively strong transmission of the empty cell made it necessary to use the galvanometer at low sensibility, and consequently the accuracy of measurement was not as great for the curves of this figure as for those of Figs. 2, 3 and 4. A study of Fig. 7, curves No. 1 and No. 2 shows the region of the strongest absorption of the KOH to be from 1.8 μ to 2.5 μ.

Fig. 6, curve No. 1 shows the transmission of an approximately 15 gram molecule per liter solution of NH_4OH. In this curve there is a shift of the minimum to 2.2 μ, and the hump in the curve at 2.0 μ is much more pronounced than in the case of the hydroxides of the alkali metals. Curve No. 2 shows the transmission of a 0.25 gram molecule per liter aqueous solution of CsOH. The solution was very weak, due to lack of chemicals. No attempt was made to study the absorption of RbOH for the same reason.

Ransohoff[1] has studied the effect of the OH group on absorption using six alcohols. He found bands at 1.71 μ and 3.0 μ, neither of which falls in the region of the strong absorption found in this investigation. Coblentz[2] has studied the absorption of $Mg(OH)_2$ and other hydroxides of elements not in the alkali metal group, and his results show no absorption in the region 2.29 μ. This would indicate that the band is present only in strongly alkaline hydroxides.

From Fig. 7, curve No. 1 it is seen that the effect of the absorption band of water at 1.49 μ is much less marked in the absorption curve of the KOH solution, than the effect of the absorption band of the water at 2.0 μ. It is probable that this absorption band of water would disappear, if the concentration of the X-OH were sufficiently high. Coblentz[3] has shown that the strong absorption band of solid iodine at 7.3 μ disappears when the iodine is placed in solution. This is caused by the action of the solvent upon the absorption of the solute, while the effect here described is the effect of the solute upon the absorption of the solvent.

In papers by Guy, Schaeffer and Jones[4] and by Schaeffer, Paulus and Jones,[5] curves appear that show similar effects upon the absorption bands of water at 1.0 μ and 1.24 μ, when various metallic salts are dissolved in water. The maximum absorption of the solutions does not, in many of the cases shown, occur at the same wave-length as that of the water. The ratio of transmission of solution to the transmission of a layer of water, equal in thickness to that contained in the solution at the positions of maximum absorption for each, is as large as 1.21 for 2.14 normal solution of $MgSO_4$ and 1.16 for 1.02 normal solution of $Al_2(SO_4)_3$.

Livens[6] in a theoretical paper discusses the effect of solute upon the absorption of a solvent, and points out that a shift in the position of the absorption bands should accompany a change in the intensity of the maximum absorption.

CONCLUSIONS.

The following conclusions may be drawn from these investigations:

1. There is a broad, intense absorption band in the spectra of the alkaline hydroxides in the region 1.5 μ to 2.7 μ.

2. The absorption is roughly proportional to the concentration of the solution for a given hydroxide.

3. There is no systematic variation of the absorption with the atomic weight of the base.

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[1] Berlin Inaugural Dissertation, January, 1896.

[2] Scientific Papers of the Bureau of Standards, No. 45, 1907.

[3] PHYS. REV., XVI., I., 1903.

[4] Phys. Zeits., Vol. 14, p. 278, 1913; Amer. Chem. Jour., 49, p. 265, 1913.

[5] Phys. Zeits., Vol. 15, p. 447, 1914; Chem. News, 110, p. 223, 1914.

[6] Phys. Zeits., Vol. 14, p. 660, 1913.

4. The absorption is probably due to the dissociated hydroxide ions, as the absorption is not present in the alcohols, or other compounds which have the OH radical.

5. The presence of an alkaline hydroxide in solution with water decreases the actual absorption of the water present. This effect of the solute upon the absorption of the solvent increases with the concentration. This shows evidence of a change of the physical properties of the water molecule, when it becomes the solvent.

This investigation was carried on in the Physics Laboratory of Cornell University. The writer wishes to thank Professor R. C. Gibbs, under whose direction this work was done, for his interest and advice throughout this investigation and the writing of this report. He wishes also to thank Professor W. R. Orndorff of the Chemistry Department for advice regarding the chemistry involved, Dr. S. D. Jackson and Mr. R. T. K. Cornwell of the Chemistry Department for making up the standard solutions and his wife, Margaret Grantham, for assistance in taking the data and computing the results.

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AN ABSOLUTE SCALE OF X-RAY WAVE-LENGTHS.

By Oswald B. Overn.

Synopsis.

Absolute Scale of X-Ray Wave-Lengths Based on Discontinuities Due to Analyzing Crystal.—In a previous study of the continuous spectrum from a tungsten target, an absorption line was observed at 0.7928×10^{-8} cm. This result has been confirmed with a molybdenum target, light lines being obtained at 0.7926 and 0.4288×10^{-8} cm. To get these lines with a rock salt crystal a thickness of less than 10^{-2} cm. was necessary. *In explanation of the discontinuities* it was suggested by A. W. Hull that when any wave-length is reflected by two sets of crystal planes, the energy is divided so that the intensity of the part which reaches the photographic plate is less than for wave-lengths on either side. For each pair of sets of planes the critical wave-length is fixed by the geometry of the crystal alone. Taking the grating constant for rock salt as 2.814×10^{-8} cm., the wave-lengths computed for the planes 100/210 and 100/310 are 0.7959 and 0.4316×10^{-8} cm. respectively. Using these as *reference points*, the center of the scale of wave-lengths may be accurately determined and a scale of wave-lengths may be established independent of everything except the grating constant.

Bromine and Silver X-Ray Absorption Limits come near the above reference points and were thus *accurately located* at 0.9186 and 0.4842×10^{-8} cm., in close agreement with the values given by Blake and Duane.

K Series of Molybdenum.—The wave-lengths of five lines were determined by reference to 0.7959×10^{-8} cm. more accurately than in previous measurements.

In a former paper[1] the author has described some work done at the University of Iowa on the tungsten spectrum and has shown a photograph of a light line which appeared very clearly in the region of the bromine absorption band at a wave-length of $.7928 \times 10^{-8}$ cm.

Later de Broglie[2] has published a paper in which he states that he has verified all the other results of the author on the tungsten spectrum but that he has not found this line. Since de Broglie's work has evidently been done with great care, this line might appear to have been a false effect.

The experiment has therefore been tried again with a

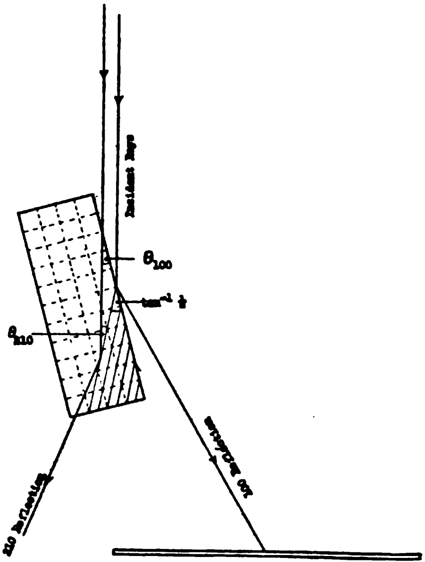

Fig. 1.

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[1] Phys. Rev., N. S., Vol. XVI., Dec., 1920, p. 526.

[2] Phil. Mag., Nov., 1919.

molybdenum tube at the University of Chicago. A light line has again been found to appear unmistakably at a wave-length of .7926 × 10^{-8} cm. as compared with the wave-length .7928 × 10^{-8} cm. previously found.

Apparatus and Method.

The source of the rays was a Coolidge tube with a molybdenum target excited by means of a transformer, the tube rectifying its own current. The continuous spectrum was produced by a thin crystal of rock salt rotated slowly by clockwork. The exposure was twenty hours per degree of rotation with a constant input of 150 watts. The distance from plate to crystal axis was kept in the neighborhood of 30 cm. and the slit was .030 cm. in width. The wave-lengths were measured by Dershem's method[1] thus eliminating the necessity of correcting for depth of penetration into the crystal. The spectrometer although differing widely in detail, was constructed on the same principle as that used by Dershem and will therefore require no description here.

Results.

It was found that these lines could not be produced clearly using a thick crystal. Indeed, one crystal which failed to produce an effect when it had a thickness of .013 cm. gave a clear photograph of the line .7926 × 10^{-8} cm. when ground down to a thickness of .006 cm. It would seem that the crystal must have a thickness less than about .01 cm. in order to produce a clear image of this line. The crystal which produced the best photograph had an average thickness of .0059 cm. Under conditions as outlined above, this line will appear very readily.

The line of wave-length .4288 × 10^{-8} appears very feebly as would be expected from the fact that there are fewer atoms in the planes reflecting the radiation away from the plate at this point and it is therefore very difficult to measure it accurately. It has been found on only one plate since lack of time prevented my finding it a second time. For this reason the measurement is probably somewhat uncertain.

Theory.

I am indebted to Dr. A. W. Hull for suggesting the following very interesting interpretation of the meaning of these light lines.

The depth of penetration of the reflected rays into the crystal and hence the intensity of the photograph is limited by absorption. For the small angles under consideration a large part of this absorption is scattering. But the scattering due to any set of planes is small except when it is at the angle of perfect reflection.

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[1] Phys. Rev., N. S., Vol. XI., June, 1918, p. 461.

Let us suppose, for example, that the homogeneous spectrum is being reflected to the photographic plate by a set of planes A. As the crystal is rotated it may come to the angle of perfect reflection for another set of planes B with the result that the crystal suddenly becomes more opaque to the rays which should be reflected from A; *i.e.*, some of the radiation which we should expect to be reflected by the set A is reflected in another direction by the set B and does not reach the photographic plate. The intensity of the photograph will therefore be diminished at the wave-length where this occurs and a light line will result.

The angle at which this will occur depends upon the relative positions of the two sets of planes. To illustrate, let us suppose that the continuous spectrum is being produced by the 100 planes and that the crystal is rotated until it reaches a position where the same wave-length, λ, is reflected simultaneously by the 100 and 210 planes.

Then

$$\lambda = 2d_{100} \sin \theta_{100} = 2d_{210} \sin \theta_{210},$$

or

$$\sin \theta_{210} = \frac{d_{100}}{d_{210}} \sin \theta_{100}.$$

But since

$$\frac{d_{100}}{d_{210}} = \sqrt{5}.$$

We have

$$\sin \theta_{210} = \sqrt{5} \sin \theta_{100}. \tag{1}$$

Another relation is evident from Fig. 1, namely,

$$\theta_{100} + \theta_{210} = \tan^{-1} \tfrac{1}{2}. \tag{2}$$

Equations (1) and (2) fully determine θ_{100} and θ_{210}. Solving, we get,

$$\theta_{100} = 8^\circ\ 7'\ 48'',$$
$$\theta_{210} = 18^\circ\ 26'\ 06''.$$

And

$$\lambda = 2d \sin \theta = .7959 \times 10^{-8} \text{ cm}.$$

Other combinations of atomic planes should produce other lines as indicated in Table I.

Table I.

Grating constant for rock salt = d = 2.814 × 10^{-8} cm.

Planes.	Wave-Lengths in 10^{-8} cm.	Observed Values.	Correction.
100 and 210	.7959	.7926 and .7928	+.0033 and +.0031
100 and 310	.4316	.4288	+.0028
100 and 320	1.103		
100 and 110	1.779		

The intensity of these lines would vary in accordance with the relative number of atoms in the planes producing them. The absorption coefficients of the crystals used would also have some effect in changing the relative intensity. However, it seems reasonable to suppose that the first three lines mentioned in the table should be visible with a rock salt crystal.

Several attempts were made to find the first three lines. As shown in the third column, these attempts have met with success in the first two cases only. The line of wave-length 1.103 × 10^{-8} was obscured on the photographs (if it did exist) by the fact that it is very close to the γ_1 line of tungsten which appeared on all the molybdenum photographs of this region.

An Absolute Scale of X-Ray Wave-Lengths.

The wave-lengths .7959 and .4316, as determined above, depend only upon constants of the crystal and are totally independent of the apparatus. They therefore furnish a method of finding the center correction more accurate than any other yet devised.

Furthermore, any wave-lengths of lines appearing on the same plat in the immediate region of one of these light lines may be corrected bye adding the difference between the calculated and observed values of these light lines as shown in the fourth column. Wave-lengths so found should be more accurate than those found by other methods provided the measurements are accurate. They would be subject to twice the error in the dividing engine used in measuring the plates and to the error in the grating constant but to *no other error*. An *absolute* scale of wave-lengths is thus established.

The Bromine Absorption Band.

The bromine absorption band always appears on photographs with a sharp critical absorption edge and so furnishes a convenient reference line. It lies very close to the line .7959 and hence the above-mentioned correction can be applied to it.

Table II.

Bromine Critical Absorption. Wave-Length × 10^8.

	Obs. Value.	Correction.	Corrected Value.	Blake and Duane.[1]
Plate A,..........	.9151	+.0033	.9184	.9179
Plates 210, 213.....	.9157	+.0031	.9188	

The average value of the critical absorption wave-length found by this method is seen from Table II. to be .9186 × 10^{-8} cm. which agrees within one tenth of one per cent. with the value found by Blake and Duane.[1]

THE SILVER ABSORPTION BAND.

The silver critical absorption wave-length appears on the same plate as the line .4288 × 10^{-8} and lies near it in the spectrum. Hence it can be determined as follows:

Observed Ag. abs. wave-length	.4814 × 10^{-8} cm.
Correction	+.0028
Corrected value	.4842 × 10^{-8} cm.

Here the corrected value is within two tenths of one per cent. of the value given by Blake and Duane, namely, .4850 × 10^{-8} cm.

THE K SERIES OF MOLYBDENUM.

The K series lines of molybdenum all appear on the same plates with the line .7959 × 10^{-8} and can be corrected as follows:

	Plate A.		Plates 210 and 213.		
Designation.	**Observed Value.**	**Corrected Value.**	**Observed Value.**	**Corrected Value.**	**Average Corrected.**
Light 210/100	.7926	.7959	.7928	.7959	.7959
Mo α_2	.7099	.7132	.7099	.7130	.7131
Mo α_1	.7055	.7088	.7056	.7087	.7087
Mo β	.6294	.6327	.6291	.6322	.6324
Mo	.6235	.6268	—	—	.6268
Mo γ	.6181	.6214	—	—	.6214

These corrected wave-lengths should be more reliable than any other values heretofore found.

THE K SERIES OF TUNGSTEN.

For comparison, one photograph has been made of the second order K series lines of tungsten. Only α_2 and α_1 appeared on the plate. These lines are quite close to the silver absorption edge and so can be calibrated from its wave-length after this has been corrected by means of these light lines. The distance between crystal and plate for this photograph was 48.136 cm. and the center of the scale was found by calling the wave-length of the silver absorption edge .4842 × 10^{-8} as determined above. This gives the correct values at once. The measurements are as follows:

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[1] PHYS. REV., N. S., Vol. X., 1917, p. 700.

K Series of Tungsten.

Designation.	**Distance from Center.**	**Wave-Length in 10^{-8} cm.**	**Siegbahn.**	**Duane and Patterson.**
Ag. Abs.	8.3760 cm.	.4842		
W K α_2	7.3735 cm.	.2136	.21352	.21348
W K α_1	7.1863 cm.	.2083	.20885	.20867

The results of Siegbahn[2] and Duane and Patterson[1] are listed for comparison, those of Duane and Patterson being changed slightly to make them conform to the grating constant here used.

The extreme faintness of the line $.4316 \times 10^{-8}$ by which the silver absorption edge is calibrated introduces a certain degree of uncertainty into these results. However, the agreement is as good as could be expected under the circumstances.

More accurate determinations of the wave-lengths of these light lines would seem to be of great importance to the study of x-ray spectroscopy. Further work along these lines is now being done at the Ryerson Physical Laboratory.

In conclusion, the author wishes to express his thanks to Dr. A. W. Hull for his suggestions, to Professor R. A. Millikan for his kindly interest, and to Mr. Erik Andersen for valuable assistance rendered in the experimental work.

THERMAL CONDUCTIVITY OF SOME WEARING MATERIALS.

By EMILY S. ROOD.

SYNOPSIS.

Thermal Conductivity of Woolen, Cotton, Linen and Silk Materials.—Measurements of the conductivity of samples of knitted and woven materials have been made by the disk method of Lees, corrected for variation of emissivity with temperature. From one to eight or more layers of one of the materials were held between two copper disks with a pressure of 6 gm./cm.^{2}, one disk being heated electrically by a coil between it and a third disk, and all three radiating to a constant temperature enclosure. The temperature of the samples was 30 to 40°. The values obtained are greater for dense than for loosely woven or knitted samples, ranging from 76 for unspun silk, 94 to 120 for wool, 101 to 122 for silk, 131 for flannelette, 158 to 167 for linen, to 168 to 184 for cotton, all times 10^{-6}. The results come out greater for several layers than for a few, and greater for moist than for dry samples. When the materials are arranged according to conductivity for equal weight instead of equal thickness, the order depends largely on the looseness of texture, beginning with unspun silk, loosely woven wool and knitted artificial silk, and ending with closely woven silk cotton and linen.

THE common textbooks on heat and the standard physical tables show both a lack of data, and great discrepancies in the results given for the thermal conductivity of materials such as wool, cotton, silk. Poynting and Thomson quote Péclet's value for wool .00012 and for cotton .00011. Edser gives the value .00003 for flannel, and Draper .00011 for wool and neither give values for cotton. The Smithsonian Tables (sixth edition) give .00012 for flannel, while three other well-known tables give the value .000035 with Forbes as authority. Lees obtained in 1892 for flannel the value .00023, and Lees and Chorlton in 1896 the value .00012, with no explanation of the difference. Similarly the values given for cotton vary from .00003 to .0005. It was hence suggested by Professor Laird that it would be worth while to undertake a new set of measurements of thermal conductivities of some materials used in clothing.

The method chosen was the disc method described in detail by Lees in the Philosophical Transactions of the Royal Society, A, 191, 1898. In the present experiment three copper discs, 4 cm. in diameter and approximately 3 mm. thick, were used. Copper and constantan wires were soldered at opposite ends of a diameter of each disc. Between the first and second discs was a flat heating coil, and between the second and third discs the material to be tested made into disc form. The whole was shellacked, tied with a silk thread, and suspended in a constant temperature enclosure made from a Lavoisier ice calorimeter, with oil between the walls. A thermocouple was soldered to the inner surface of the calorimeter. The other junctions of the thermocouples were in mercury cups outside the calorimeter, the temperature of which was read on a calibrated .1° thermometer. The various thermocouples were calibrated in position; a heating wire wound around in the oil filled space between the walls of the calorimeter, and separated from the inner wall by some pieces of wood, served to raise the temperature as required.

The formula for conductivity used by Lees for this disc method, with some change in notation, is:

$$k = H\frac{(\pi r^2+2\pi r t_3)v_3+\frac{1}{2}(v_2+v_3)\pi r t_s}{(\pi r^2+2\pi r t_3)v_3+\pi r t_s(v_2+v_3)+2\pi r t_2 v_2+(\pi r^2+2\pi r t_1)v_1}\times\frac{t_s}{(v_2-v_3)\pi r^2},$$

where H = total heat supplied,

r = radius of discs,

t_1, t_2, t_3, t_s = thickness of the three copper discs and of the substance respectively,

v_1, v_2, v_3 = temperature excess of the three copper discs over the surroundings.

In computing the thickness of the discs half the thickness of the heating coil is added to the first disc and half to the second. A correction for the heat conducted along the wires of the thermocouples is also made by adding $\sqrt{qkph}/2\pi rh$, where q is the cross section, p the perimeter, k the conductivity, and h the emissivity of the wires.

In the above formula it is assumed that for the intervals used the heat emitted per square centimeter by each disc is proportional to the corresponding temperature excess, or that the emissivity is a constant. In the course of the experiments it was found that this assumption is unjustified for the temperature range used. Accordingly separate observations were made on the rate of cooling of the discs and a curve plotted with emissivity and temperature as coordinates, this method was checked later by a measurement of the steady temperature attained by the two copper discs alone for different values of the heating current. This curve gave a correction factor to be used in each case with the above formula. The emissivity at 10° and 15° temperature excess was found to be 0.000256, and 0.000276 respectively; the difference is greater than that found by McFarlane.[1] The effect of the correction is to reduce the value of the conductivity over that given by the Lees formula from four to seven per cent., or up to ten per cent. in extreme cases, since the numerator should be reduced by a factor h_3/h_1 nearly, where h_1 and h_3 are the emissivities at the temperatures of the corresponding discs, and only one term of the denominator is reduced. In the conditions used by Lees the difference in temperature between the two sides of the substance being tested was smaller than here, but even in some of his experiments the correction would probably amount to over two per cent., and it is possible that some of the apparant decrease in conductivity with rise in temperature observed by him was due to failure to apply such a correction.

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[1] Smithsonian Tables, 6th edition, p. 252.

The power was supplied from a storage cell, and was measured by a volt-ammeter method. The voltmeter and ammeter were tested by potentiometer methods. A correction was applied for the heat lost in the connecting wires. No readings were used when the temperature of a disc changed more than .1° in 15 minutes, or when the difference in temperature on the two sides of the material being tested changed more than .05° in 15 minutes. The whole difference here was from 3° to 9°, but was usually between 3° and 4°, and the temperature of the material being tested was between 30° and 40°. Lees and Chorlton[1] worked out the problem of the variation of temperature in a brass disc similarly placed, and from their solution it would appear that the temperature throughout the copper discs used in these experiments could be considered uniform. As an experimental check the one thermocouple was soldered to the top of the upper disc for a time instead of in a small hole at the side and this alteration made no perceptible change in the results.

Twelve different wearing materials, and unspun silk were tested. Five were from common knitted underwear, namely, artificial silk, linen mesh, medium weight cotton, and two kinds of woolen wear, wool no. 1 a loosely knit light weight sample, wool no. 2 a more tightly knit one. The woven materials were cotton cloth, flannelette, a closely woven wool suiting, white dress linen, and three kinds of silk dress material. Silk no. 1 was a soft dress silk, pure except for the dye, silk no. 2 tussah silk, (wild silk), silk no. 3 white parachute silk (cultivated Japanese silk). Silks nos. 2 and 3 and the unspun silk (French cocoons combed) were kindly furnished by Cheney Bros. as pure silk. The knitted materials and the cotton cloth had all been washed many times, being cut from worn articles of clothing. The others were in new condition.

Several thicknesses were generally used in making the discs. The edges were sewed together with ravellings to make a smoother surface, and were coated with shellac, melted or in solution. In a number of instances no shellac was used with no difference in the result. In order to check the values found and to see that no error was introduced due to a constant difference of temperature between the substance and the copper, experiments were made with discs of different thickness for most of the various materials. The values agreed for small thicknesses, but as the thickness increased the values increased as illustrated by a set of values obtained for the wool suiting, 125, 114, 119, 137, and 154 $\times 10^{-6}$ with discs of one, two, three, four and six layers respectively. It is evident that the formula is not intended for use when the difference in temperature on the two sides of the substance being tested is large, since it assumes that the temperature decreases uniformly in it and that the heat conducted through may be taken as the mean of that entering and leaving the disc. A formula was worked out by assuming a simple exponential distribution of temperature in the given disc, which gave relatively smaller values for the greater thicknesses, but not sufficiently smaller to bring them into agreement with the first values, and it probably did not take into consideration all the factors involved. It is true also that the correction for the variation of emissivity with temperature is large in these cases, and makes the results more doubtful.

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[1] Phil. Mag., 41, p. 497, 1896.

The discs were kept in a drying chamber with calcium chloride, and calcium chloride was placed in the calorimeter. With one of the knitted woolen samples, that by chance was not dried before using, the value .000152 was obtained instead of .000120 as found later. It was hence suggested that the effect of moisture should be investigated. For this purpose the discs were left in a moist atmosphere, and a little water was put in the bottom of the calorimeter. When the discs were heated they naturally dried somewhat, so that in the steady state there was a limit to the moisture possible. Wool with 4 per cent. moisture gave a value .000099 as against .000093 when dry, and with 8 per cent. moisture a value .000104; and for a sample of cotton the value increased from .000185 to .000191 on an average for 1.5 per cent. moisture. These increases are less than those given by Lees and Chorlton (*l.c.*), and less than that suggested by the discrepancy noted with the earlier sample. Somewhat lower values on an average were obtained for cotton, wool and silk in months when the laboratory was heated, than in the summer, but the amount of moisture must have been small at any time as fresh calcium chloride was constantly used.

In carrying out the experiments the largest source of error apart from the effects of moisture is in the measurement of the thickness of the discs. It is thought that this error would not be more than 2 per cent. The materials were generally under a pressure of 6 gm./cm.^{2}, determined by the weight of the discs. Small constant errors in the calibration factor for the thermocouples, in the constants of the discs, and in the correction for the power used might amount to between 1 per cent. and 2 per cent. These errors would not affect the relative values. The "probable error" computed from the measurements on any one material is less than 1 per cent., except for the woven cotton where with a small number of measurements it is 2 per cent.

Table I. gives values of the conductivity obtained for knitted cotton at different times and with different samples and different thicknesses. They show fair agreement. Table II. shows in column 1 the materials used. Column 2 gives the thickness of a single layer of the material, column 3 its weight in grams per 100 sq. cm., column 4 the average value of the conductivity found not including the values for thick discs, column 5 this conductivity multiplied by the density. This latter is the flow of heat per sq. cm. per sec. for a unit difference of temperature between surfaces such that a gram of material would lie between, this may be called the conductivity per equal weight. Column 6 gives values obtained by Lees and Chorlton[1] by a slightly different method for some similar materials. The woolen materials show plainly the effect of loose texture in increasing warmth where wind is not a factor, both in the values of absolute conductivity and more strikingly in the conductivity for equal weight. The silk materials show this even more so, there being an approximately linear relation between conductivity and density, so that it should be possible to estimate the conductivity of any given sample fairly closely if the density were known. Extrapolating

TABLE I.

Cotton (Knit).

Date.	Number of Pieces.	Thickness.	Conductivity.
June 2, 1919	4	.32 cm.	.000191
June 6, 1919	8	.632 cm.	.000211
June 9, 1919	8	.632 cm.	.000212
June 9, 1919	6	.46 cm.	.000190
June 10, 1919	6	.46 cm.	.000194
July 25, 1919	4	.32 cm.	.000186
Feb. 18, 1921	4	.316 cm.	.000180
Feb. 19, 1921	4	.32 cm.	.000184
May 2, 1921	4	.32 cm.	.000180
May 5, 1921	4	.35 cm.	.000177
July 12, 1921	4	.344 cm.	.000182
July 15, 1921	4	.32 cm.	.000182
July 20, 1921	4	.32 cm.	.000184

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[1] Phil. Mag., 5, 41, p. 495, 1896.

TABLE II.

Material.	Thickness per Piece.	Weight per 100 sq. cm.	Conductivity.	Conductivity × Density.	Conductivity, Lees and Chorlton.
Wool no. 1	.068 cm.	1.20 gm.	.000094	1.66×10^{-5}	
Wool no. 2	.126 "	2.65 "	.000120	2.53	.00012 (New flannel)
Wool suiting	.094 "	3.6 "	.000118	4.51	.00013 (Old flannel)
Unspun silk		.12[1] "	.000076	0.91	.0000613[2]
Silk no. 1	.0194 "	.578 "	.000101	3.00	.000095
Silk no. 2	.0138 "	.605 "	.000111	4.87	
Silk no. 3	.0138 "	.64 "	.000122	5.66	
Silk (artificial)	.045 "	.75 "	.000117	1.95	
Flannelette	.097 "	1.55 "	.000131	2.09	.00015
Cotton (woven)	.030 "	1.0 "	.000168	5.60	.00018 (Calico)
Cotton (knit)	.079 "	1.94 "	.000184	4.52	
Linen (mesh)	.098 "	2.6 "	.000158	4.19	
Linen (woven)	.033 "	1.95 "	.000167	9.87	.00021

[1] Density in gm./cm.^{3} under a pressure of 6 gm./cm.^{2}

[2] Rubner, Landolt and Bornstein Tables, 1905, p. 509.

however to pure silk (density 1.56) gives only about .0003 instead of .0008 as found by Rubner.[1] The advantage of wool over cotton for warmth shows up in column 5, for in considering the favorable position of flannelette, it should be recalled that the sample was new. The linen appears to have a distinctly lower conductivity than previous measurements show; one set of readings was made on an unbleached sample, of slightly less density, and a still lower value was found. From the results it may be inferred that at equal density the substances would come in the order silk, wool, artificial silk, linen, cotton, as to conductivity, and that of materials as actually found unspun silk is the warmest and woven linen the coolest, the linen changing place with cotton because of its actual greater density in manufacture.

In conclusion, the author expresses her appreciation of the aid received from Professor Laird in carrying out the experiment, and especially of the help in working out the correction of the errors involved.

MOUNT HOLYOKE COLLEGE.

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[1] Landolt and Bornstein Tables, 1905, p. 509.

VAPOR PRESSURE OF METALLIC CALCIUM.

By Norman B. Pilling.

Synopsis.

Vapor Pressure of Calcium, 500–700° C., from Rate of Evaporation in Vacuum.—A drawn wire of calcium was coiled into an open helix and placed in a glass tube which was then evacuated and heated to a definite temperature for a definite time. From the loss of weight due to evaporation the vapor pressure was computed by using a formula derived from the kinetic theory but involving a correction factor for the reflection of molecules by the walls of the hot tube. This factor was assumed to be a constant of the apparatus and was determined by making measurements for zinc and cadmium whose vapor pressures are known. The calcium used had 1.6 per cent. Mg and 1.25 per cent. $CaCl_2$. The values obtained for 500 to 700° C. are well represented by the formula: $\log p = 9.73 - (10{,}170/T)$, the corresponding equations for zinc and cadmium being $9.41 - (7070/T)$ and $9.02 - (5940/T)$ respectively.

Vapor Pressure of Liquid Calcium, Computed using Richards's relation between the vapor pressures of liquid and solid phases, is given by the equation $\log p = 9.27 - 9670/T$. From this equation the *boiling point* comes out 1240° C.

Experimental Method.

The method of measurement involved the determination of the rate of evaporation of a certain weight of metallic calcium of known surface area at constant temperature and in vacuo. From this rate it is possible to compute by means of the kinetic theory of gases the equilibrium vapor pressure. In this way the vapor pressures of tungsten, molybdenum and platinum have been computed.[1]

The following expression has been derived[2] for the vapor pressure of a substance in terms of its rate of evaporation in vacuum:

$$m = \sqrt{\frac{M}{2\pi RT}}\cdot p, \qquad (1)$$

in which

M = molecular weight of vapor (assume metallic vapors to be monatomic).

R = gas constant = 8.32×10^7 ergs/degree.

T = absolute temperature.

p = pressure, dynes/cm.^{2}

m = rate of evaporation, gm./(cm.^{2} sec.).

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[1] I. Langmuir, Vapor Pressure of Metallic Tungsten, Phys. Rev., II., Vol. 2, p. 329, 1913. I. Langmuir and C. M. J. Mackay, Vapor Pressure of the Metals Platinum and Molybdenum, Phys. Rev., II., vol. 4, p. 377, 1914.

[2] I. Langmuir, Vapor Pressure of Metallic Tungsten, Phys. Rev., II., Vol. 2, p. 329, 1913.

This expression is valid when the following postulates hold

1. That at low vapor pressures (few millimeters) the mutual interference between vapor molecules is so slight as to make the rate of evaporation a function of temperature only and independent of the external pressure.

2. That the rate of condensation is proportional to the vapor pressure.

3. That the coefficient of reflection with respect to both metal surface and enveloping walls is zero.

The above equation then will give the rate of evaporation in a vacuous space whose enveloping walls are cold or remotely distant. As will be shown later, in the experimental arrangement adopted the enveloping walls have been made neither remote nor cold, and hence another factor, the reflection of calcium molecules from this enveloping surface back to the metallic surface with consequent condensation has to be taken into account.

Let s = coefficient of reflection from the enveloping walls. Then

$$m = \sqrt{\frac{M}{2\pi RT}} \cdot p(1 - s). \tag{2}$$

Solving for p, and changing p from dynes/cm.^{2} to millimeters of mercury (1 mm. Hg = 1333 dynes/cm.^{2})

$$\log p = \log m + \tfrac{1}{2} \log T + \tfrac{1}{2} \log \frac{2\pi R}{M} - \log (1 - s) - 3.125. \tag{3}$$

Details of the experimental method used are shown in the apparatus diagram (Fig. 1). A 19/32″ bore pyrex glass tube closed at the bottom extended to the middle of a vertical electrically heated tube resistance furnace. This ignition tube was connected to a standard set of mercury diffusion pumps through a greased stopcock; a liquid air trap was placed between the stopcock and the furnace to prevent vapors from the lubricant of the cock and mercury from the pumps diffusing back into the ignition tube. The calcium in the form of a drawn wire helix was cleaned, weighed and immediately dropped into the ignition tube through a suitable opening at the top. In its initial position (Fig. 1) the helix

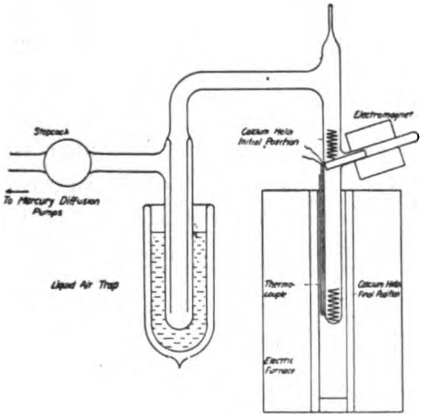
Stopcock
To Mercury Diffusion Pumps
Liquid Air Trap
Calcium Helix Initial Position
Electromagnet
Thermo-couple
Calcium Helix Final Position
Electric Furnace

Fig. 1.

rested on a steel pin which projected across the heating tube from an inclined lateral tube above the top of the furnace. As soon as the specimen was in place the top of the heating tube was sealed off and the entire system was evacuated. A platinum-platinrhodium thermocouple within a slender quartz protection tube was lashed to the outside of the glass tube with asbestos tape. When the furnace had reached a steady temperature and time had been given for the removal of absorbed moisture, etc., from the inside of the tube, the steel pin was withdrawn with an external electromagnet and the calcium helix dropped into the furnace. At the end of the run, the furnace was lowered away from the ignition tube, and the calcium cooled in vacuo. When cold, the tube was cracked open and the loss in weight of the calcium helix determined.

At temperatures above 600° C. the pyrex glass collapsed. A somewhat larger tube was then used, into which an unglazed usalite porcelain liner, 19/32″ bore, was slipped, which acted as a support for the softened glass.

In all cases except cadmium the evaporated metal condensed as a mirror at the mouth of the furnace. The cadmium condensed just below the furnace mouth.

DETERMINATION OF COEFFICIENT OF REFLECTION.

In evaluating the coefficient of reflection, the rates of evaporation of several metals with known vapor pressures were measured under various conditions, from which the coefficient was computed by substitution in equation (2). A search of the vapor pressure literature showed the data to be very incomplete and unsatisfactory regarding values of the vapor pressures of the solid metals. Almost all of the data are based on extrapolated curves from the liquid state, oftentimes with the weight of very few measurements.

The vapor pressure of solid zinc and solid cadmium has been measured over a considerable temperature range by a dynamic method by Egerton.[1] Accordingly equations were deduced from his data which were used as standards of reference in measuring the reflection coefficient with zinc and cadmium. These equations follow.

$$\text{Zinc:} \qquad \log p = -\frac{7070}{T} + 9.41,$$

$$\text{Cadmium:} \quad \log p = -\frac{5940}{T} + 9.02.$$

The zinc which was used in this determination was 0.124 cm. diameter Horse Head brand zinc wire furnished drawn to size by the New Jersey

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[1] A. C. Egerton, Vapor Pressure of Zinc, Cadmium and Mercury, Phil. Mag., Vol. 33, p. 33, 1917.

Composition of Zinc and Cadmium Wire.

	Zn	Cd	Fe	Pb	As
Zinc (no. 077).....	99.94(*d*)	nil	.01	.05	——
Cadmium (no. 075)	nil	99.99(*d*)	——	nil	.006

d = by difference.

TABLE I.

Coefficient of Reflection, Zinc and Cadmium Vapor.

Metal.	No.	Temperature, ° C.	Ignition Tube Reflect. Surf.	Vapor Pressure, mm.	s	log $(1 - s)$
Zinc........	3558	301	Pyrex glass.........	.00133	.99369	−2.200
	3556	404	Pyrex glass.........	.0813	.99230	−2.113
	3557	405	Pyrex glass.........	.0840	.99236	−2.117
	3582	402	Usalite porcelain	.0759	.99198	−2.096
Cadmium...	3559	302	Pyrex glass.........	.0488	.99213	−2.104

Zinc Company. The cadmium was a rod rolled and drawn to wire 0.103 cm. diameter in the laboratory.

Upon examination of this table it will be seen that within the limits of experimental error the coefficient of reflection is a very large but constant quantity, practically independent of temperature, nature of reflecting surface (glass or porcelain) and vapor (metal). The experimental error was largest with no. 3558; the mean value of the four other measurements of log $(1 - s)$ is -2.107, which was taken as a characteristic constant of the apparatus and used in the reduction of data on calcium.

VAPOR PRESSURE OF CALCIUM.

The calcium metal with which the following determinations were made was a laboratory product prepared electrolytically by Mr. P. H. Brace.[1] This was forged, rolled and drawn cold to wire 0.106 cm. diameter, and stored for use in Lectroseal oil. Analysis of a large piece of the undrawn material follows.

Analysis of Calcium. No. 092.

Ca..	97.16
Mg..	1.62
Fe..	0.009
$CaCl_2$..	1.25
	100.04

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[1] See P. H. Brace, Transactions, American Electro-chemical Society, 1920, p. 69.

In preparing the wire for use, a measured length was coiled into an open helix 8 mm. diameter. The oil was removed by soaking in xylene, and the crust of calcium salt on the surface removed by pickling in dilute hydrochloric acid in alcohol. When the surface was bright, the acid was rinsed off in alcohol, the alcohol removed by momentary immersion in ether, which attacked the metal vigorously, and then dried quickly in air. The wire diameter was then measured, the helix weighed, placed in the apparatus and sealed off as soon as possible.

TABLE II.

Rate of Evaporation of Calcium.

No.	Temperature ° C.	Specimen.		Weight of Calcium Evaporated, gm.	Time, min.	Rate of Evaporation, gm./(cm². sec).
		Weight, gm.	Surface Area, cm².			
3448	503	0.3694	10.8	0.0024	64	0.058×10^{-6}
3452	553	0.9312	25.8	0.0267	76	0.227
3451	605	0.4414	11.7	0.0366	58	0.900
3449	608	0.4460	12.7	0.0413	50	1.08
3456	652	0.5696	13.5	0.0843	18	5.78
3457	700	0.5689	13.1	0.1621	12.5	15.9

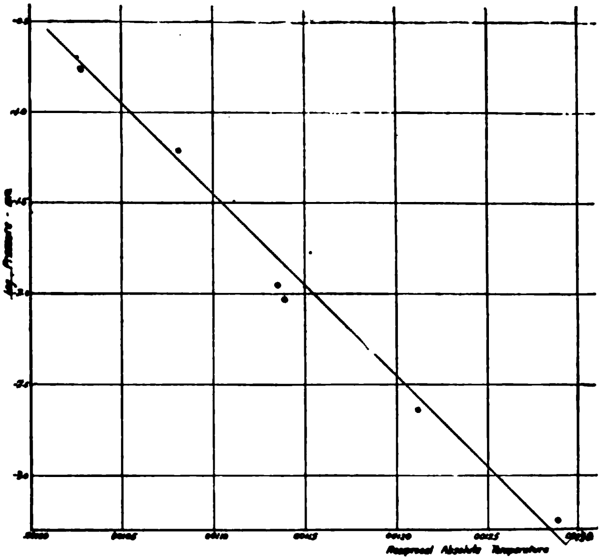

Fig. 2.

Experimental data have been obtained for the temperature range 500–700° C.

TABLE III.

Vapor Pressure of Calcium.

No.	Temperature, °C.	Pressure, mm. Hg.
3448	503	.000561
3452	553	.00227
3451	605	.00931
3449	608	.0112
3456	652	.0614
3457	700	.173

The experimentally determined values of vapor pressure are plotted in Fig. 2, the coördinates being log pressure and reciprocal absolute temperature. The relation is linear, and the best straight line is

$$\log p = -\frac{10{,}170}{T} + 9.73, \tag{4}$$

representing the vapor pressure of solid calcium. Richards[1] making use of the Clausius-Clapeyron equation and Richards's rule, has derived a relation between the vapor pressure of the solid and liquid phases which has experimental verification for several substances. If the vapor pressure curves are expressed

$$\log p = -\frac{A}{T} + B \text{ (solid)}$$

and

$$\log p = -\frac{A'}{T} + B' \text{ (liquid)},$$

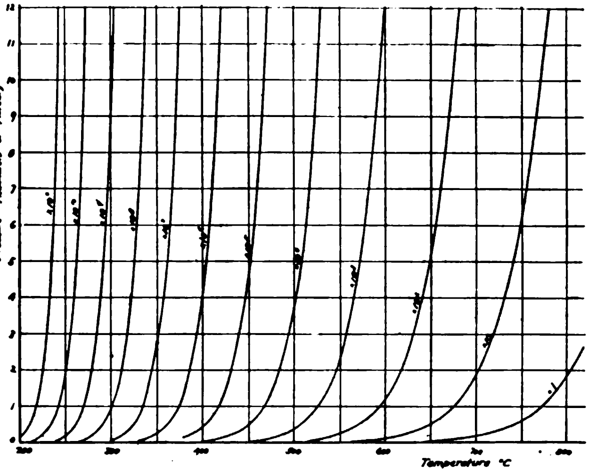

Fig. 3.
Vapor Pressure of Solid Calcium.

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[1] J. W. Richards, The Vapor Tensions of the Metals, Journal Franklin Institute, Vol. 187, p. 581, 1919.

the relation between the four constants is as follows:

$$A' = A - 0.46\ Tm.p.$$
$$B' = B - 0.46.$$

Taking the melting point of calcium to be 805° C.[1]

$$\log p = -\frac{9670}{T} + 9.27, \qquad (5)$$

the vapor pressure curve for liquid calcium.

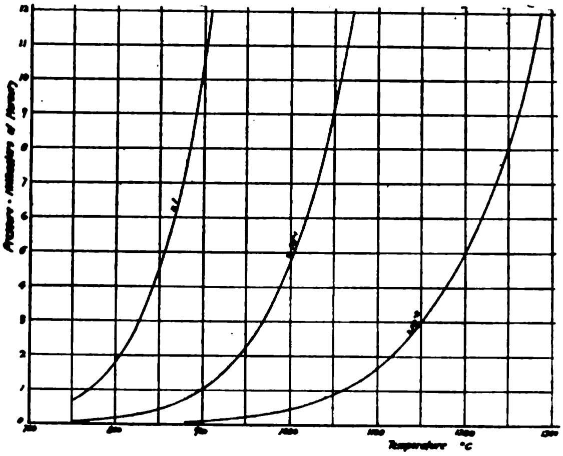

Fig. 4.

Vapor Pressure of Liquid Calcium.

Charts representing equations (4) and (5), the vapor pressures of solid and liquid calcium respectively, are given in Figs. 3 and 4.

From the vapor pressure curves the following constants may be computed.

Normal boiling point, 760 mm.	1240° C
Vapor pressure at melting point, 805° C.	2.0 mm.
Heat of vaporization	
Trouton's rule $\frac{23T}{\text{At.Wt.}}$	867 cal./gm
Clapeyron eq. $\frac{4.57A'}{\text{At.Wt.}}$	1100 cal./gm

Westinghouse Research Laboratory,
East Pittsburgh, Pa.,
May 24, 1921.

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[1] Soc. Franc. de Physique, p. 258.

LIGHT EMISSION FROM A MOVING SOURCE IN CONNECTION WITH THE RELATIVITY THEORY.

By N. Rashevsky.

Synopsis.

Light Emission from a Moving Source; a Generalized Explanation of the Michelson-Morley Experiment.—The paper is based upon unpublished work left by the late *Nikolas Pashsky* of Kieff. It is pointed out that the experiment does not prove the constancy of the velocity of light but merely that if light is sent out in all directions from the center of a spherical mirror, all rays after reflection will return simultaneously to the center no matter what the velocity of the system through space. This result requires merely that with respect to axes moving with the source, the velocity of light in a direction making an angle φ with the direction of motion is given by $\sigma_\varphi = \sigma/\{1 + \sqrt{[\psi \cos \varphi/(\sigma + \psi)]}\}$, when σ and ψ are arbitrary functions of the velocity of translation v, with the only conditions that for $v = 0$, $\psi = 0$ and $\sigma = c$. There are therefore an ***infinite number of possible hypotheses*** which will explain the experiment, ***in addition to the relativity theory***, which assumes that $\sigma_\varphi =$ constant. The only way to decide the matter would be to measure the velocity of light by a method in which the ray of light would not return to its starting point.

THE following paper is based upon the unpublished works of Nikolas Pashsky, late assistant in physics at the University of Kieff.[1] His manuscripts were entrusted to me by his widow to prepare for publication, but I was forced to leave them in Kieff.

Wishing to delay no longer in presenting his ideas to the scientific world I am presenting my own explanation of some of his ideas.

The present paper represents only a very small portion of the large mass of work which he had nearly ready for publication.

1. The classical theory of the electromagnetic field, among other fundamental statements, on which it is based, contains the following postulate:

(*a*) The velocity of propagation of any electromagnetic disturbance in free space is independent of the motion of the source, and is, relative to a system at rest, equal to $c = 3 \times 10^{10}$ cm./sec.

Einstein, denying *the reality of absolute rest* and *absolute motion* and requiring that the velocity of light should have the same value c, relative to any chosen system, came to the familiar conclusions, which all together form the relativity theory.

The principal experimental fact, on which are based his speculations, is the Michelson experiment, to which however the relativity theory

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[1] Died June 26, 1918, aged 32 years.

must ascribe more than it gives in reality. According to this theory the velocity of light in a moving system is in all directions equal to c, *i.e.*, all the points of a sphere, from the center of which at the moment t an electromagnetic disturbance is sent out, will receive the impulse simultaneously, while the Michelson experiment shows only that all those disturbances, reflected by the points of the sphere, will *come back to the center simultaneously*, and this is not the same.

H. Lorentz, wishing to give an explanation for this true result of the Michelson experiment, made the contraction hypothesis. Einstein, going beyond the limits of the experiment and requiring the simultaneity of the arrival of impulses on the sphere, was *forced* to make the assumption as to the change of time-units in the moving system.

The purpose of this article is to show that without going further than the experiment leads us, and consequently abandoning the principle of relativity, we can, without making the contraction assumption, find an infinite number of hypotheses satisfying the Michelson experiment and that all the theories based on those hypotheses are equivalent one to another and to a *certain extent* equivalent to the relativity theory. But for this purpose *we must abandon the postulate I of the constancy of velocity of light.*

Such an attempt was made by W. Ritz, who made the assumption that the velocity of light emitted by a moving source is equal, relative to a system at rest, to $c + v$ and $c - v$ respectively in the direction of movement of the source and opposite it, where v is the velocity of translation. We shall now make a more general assumption.

(*b*) The velocity of light emitted by a moving source is equal, relative to a system of rest, to $c + f_1(v)$ in the direction of motion and $c + f_2(v)$ in the opposite direction, where $f_1(v)$ and $f_2(v)$ are arbitrary functions of v, with the only limiting condition that

$$f_1(0) = f_2(0) = 0. \tag{1}$$

Relative to axes moving with the source the velocities will be respectively

$$\begin{aligned} c + v + f_1(v) &= U_1(v) = \sigma + F_1(v), \\ c - v + f_2(v) &= U_2(v) = \sigma + F_2(v), \end{aligned} \tag{2}$$

where σ is a function of v with the condition $\sigma(0) = c$; and $F_1(v)$ and $F_2(v)$ are other arbitrary functions of v with the same limiting conditions

$$F_1(0) = F_2(0) = 0.$$

Here it will be of use to define what we understand by the words "velocity of light."

All the methods employed up to the present to measure this velocity are based on the principle of Fizeaux or Foucault in both of which the light travels along a path of a known length, and comes back again in the opposite direction. The time necessary to traverse this path is measured, and the velocity is obtained from the equation

$$T = \frac{2l}{\sigma},$$

where l is the length of the path. It is obvious that this does not give us in any case the "true" velocity of light, for the velocities in opposite directions may be different. The velocity obtained in this way we shall denote as "measured velocity" of light, in distinction to the "true velocity" which could be obtained by a method using the propagation of light in one direction. But in all our physical experiments we know only the "measured velocity," a fact which is of importance. Now we shall make the assumption that σ is the "measured velocity" of light in our moving system. But, as our hypothesis must satisfy the conditions of the Michelson experiment, we must require that the time necessary for light to pass from a moving point A to another point B, situated at the distance l from A and moving with it, and return to A, must be independent of whether the system is moving in the direction AB or BA and equal to σ.

Hence

$$\frac{l}{\sigma + F_1(v)} + \frac{l}{\sigma + F_2(v)} = \frac{2l}{\sigma}, \tag{3}$$

or, after some transformations,

$$-\frac{\sigma}{2}[F_1(v) + F_2(v)] = F_1(v)F_2(v). \tag{4}$$

If we put

$$F_1(v) + F_2(v) = 2\psi(v), \tag{5}$$

where $\psi(v)$ is an arbitrary function with the same limiting condition

$$\psi(0) = 0,$$

we have

$$\begin{aligned} -\sigma\psi(v) &= F_1(v)F_2(v), \\ 2\psi(v) &= F_1(v) + F_2(v); \end{aligned} \tag{6}$$

we see that $F_1(v)$ and $F_2(v)$ are roots of the quadratic equation

$$-F^2 + 2\psi(v)F + \sigma\psi(v) = 0.$$

Hence

$$\begin{aligned} F_1(v) &= \psi(v) + \sqrt{\psi^2(v) + \sigma\psi(v)}, \\ F_2(v) &= \psi(v) - \sqrt{\psi^2(v) + \sigma\psi(v)}, \end{aligned} \tag{7}$$

and for the velocities relative to a moving system we find respectively

$$\sigma + \psi(v) + \sqrt{\psi^2(v) + \sigma\psi(v)}$$

and

$$\sigma + \psi(v) - \sqrt{\psi^2(v) + \sigma\psi(v)}. \tag{8}$$

But from (3) we see that we can change the rôles of the functions $F_1(v)$ and $F_2(v)$ and so finally we get

$$\begin{aligned} &\sigma + \psi(v) \pm \sqrt{\psi^2(v) + \sigma\psi(v)}, \\ &\sigma + \psi(v) \mp \sqrt{\psi^2(v) + \sigma\psi(v)}. \end{aligned} \tag{9}$$

So far we have neglected the velocity of light in a direction making a definite angle with the direction of translation. We may show that we obtain a result consistent with the Michelson experiment if we assume that the hodograph of velocities relative to moving axes is an ellipsoid of revolution along the major axis, this axis being parallel to the direction of motion, and the source being situated in one of the foci of the ellipsoid. The velocities along the axis of revolution are given by (9).

Then we have for the major semi-axis

$$a = \sigma + \psi(v), \tag{10}$$

the linear excentricity is

$$e = \sqrt{\psi^2(v) + \sigma\psi(v)}, \tag{10$_a$}$$

the parameter which gives the velocity in a direction perpendicular to the direction of motion is

$$p = \sigma. \tag{10$_b$}$$

The numerical excentricity is

$$\frac{e}{a} = \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}}. \tag{10$_c$}$$

Hence the velocity of light in a direction making an angle φ with the line of motion is

$$\sigma_\varphi = \frac{\sigma}{1 \pm \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}} \cos\varphi}. \tag{11}$$

The positive sign in the denominator corresponds to the assumption that the source lies in the front focus; and the negative that it lies in the rear one.

It is easy to show that our assumption explains the Michelson experiment, if we make the assumption that every point of the reflecting sphere behaves like an independent source, emitting light in accordance with (11).

Really the time necessary for the light to pass from the center to a point of the sphere, situated in a direction defined by the angle φ is, if l = radius of the sphere,

$$T_1 = \frac{l}{\sigma_\phi} = \frac{l\left(1 \pm \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}}\cos\varphi\right)}{\sigma}.$$

The time necessary to return by the same path from the sphere to the center is

$$T_2 = \frac{l}{\sigma_{\pi+\phi}} = \frac{l\left(1 \mp \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}}\cos\varphi\right)}{\sigma}.$$

The change of sign before the radical is due to the fact that

$$\cos(\pi + \varphi) = -\cos\varphi.$$

The entire time necessary for the light to go and return is

$$T = T_1 + T_2 = \frac{2l}{\sigma}, \tag{12}$$

i.e., independent of the direction of propagation.

Formula (12) shows also that if we measure the velocity of light by ordinary methods, sending the beams twice by the same path and dividing the double length of the path by the time that is necessary for the light to go and return, we will always obtain the value σ. If σ is independent of the velocity of translation of the source, and σ = const. $= c$, then we will never detect in this manner the absolute movement of the system. If σ is a function of v, this would be possible.

As $\psi(v)$ is a quite arbitrary function, only limited by the condition

$$\psi(0) = 0,$$

we have thus found an infinite number of solutions of the "Michelson problem," all equivalent one to another.

2. To show now the mutual relation between our theory and the relativity theory, we may proceed as follows:

Let us consider a sphere of radius l, moving with a uniform velocity v.

Let the center of the sphere be the origin of moving coördinate axes and let the x-axis lie in the direction of motion. Let us consider now the phenomena of light propagation from the point of view of the classical theory.

The velocity of light referred to the moving axes will be, as easily shown, respectively $c - v$ and $c + v$ in the direction of motion, and $\sqrt{c^2 - v^2}$ in a direction perpendicular to this. The time necessary for the light to travel from the center of the sphere and back will be in the latter case

$$\frac{2l}{\sqrt{c^2 - v^2}}, \tag{13}$$

and in the former

$$\frac{l}{c+v}+\frac{l}{c-v}=\frac{2lc}{c^2-v^2}, \tag{14}$$

i.e., if this time is greater, and if we desire to maintain the simultaneity of return of light to the center, we *must* assume that l is contracted along the line of motion in the ratio

$$\frac{\sqrt{c^2-v^2}}{c}=\sqrt{1-\frac{v^2}{c^2}} \quad \text{(assumption 1)},$$

and then it is easily shown that the time of return of light to the center will be independent of the direction of propagation.

This would account for the Michelson experiment and, as Lorentz showed, for all the other known experimental phenomena.

But Einstein, as remarked above, went further and postulated, that the "measured velocity" of light in a moving system should be c; as in our case it is equal to

$$\sqrt{c^2-v^2},$$

there remains nothing but to make the assumption that in the moving system time runs slower in a ratio

$$\frac{\sqrt{c^2-v^2}}{c}=\sqrt{1-\frac{v^2}{c^2}} \quad \text{(assumption 2)},$$

for then the time required for light to go and return will be not

$$\frac{2l'}{\sqrt{c^2-v^2}} \quad \text{but} \quad \frac{2l'}{c}.$$

But even if we make this assumption we will find that the absolute movement of the system could be detected, for, though the time of return of the light to the center and its measured velocity are independent of the direction of propagation, the time of arrival of light at the sphere and the "true velocity" will depend upon this direction, and to remove this difficulty, we must assume that a watch whose x-coördinate is equal to x', shows the time $t-(u/c^2)x'$, if t is the time at the center (assumption 3).

In this way we obtain the complete set of assumptions which form the basis of relativity theory.

Let us now return to our hypothesis. Formula (11) without any contraction hypothesis satisfies the conditions of the Michelson experiment and shows that the measured velocity of light is independent of the direction of propagation. Thus our hypothesis is equivalent to the contraction one and furnishes an equally good explanation of all the known experimental facts, our infinite set of assumptions giving the same result as the contraction hypothesis. All the experiments made up to the present had to do with the propagation of light in a closed path, this being true also for experiments of electromagnetic character, where we always have to do with "mutual actions" of two charges. No experiment has been made that shows the necessity of assumptions 2 and 3. They are beyond the limits of experiment. And in the same way, as between the modified Lorentz theory and the relativity theory of Einstein the only decisive experiment would be to measure the velocity of light by a method involving its propagation in one direction only; our assumptions are *to this extent* equivalent to the relativity theory.

If we should postulate the simultaneity of arrival of the light impulses at the surface of the sphere in our theory, we should be obliged to assume that the watch on the surface of the sphere shows a time

$$t' = t \mp \frac{k}{\sigma} x, \tag{15}$$

where x is its moving coördinate and

$$k = \sqrt{\frac{\psi(v)}{\sigma + \psi(v)}}. \tag{16}$$

In reality the time necessary for the light to pass from the center to a point of the sphere is equal to

$$\frac{l(1 \pm k \cos \varphi)}{\sigma},$$

and if we introduce t', it becomes

$$\frac{l(1 \pm k \cos \varphi)}{\sigma} \mp \frac{k}{\sigma} l \cos \varphi = \frac{l}{\sigma},$$

i.e., the velocity of light measured by a "one direction method" will be σ.

Of course, if we desire that even from a source at rest the velocity of light in the moving system should be σ, we must change in an appropriate manner the unit of time in our moving system.

But, without this assumption, since all the apparatus with which we make our measurements has the same velocity of translation as the earth, we shall find (applying the principle used in the deduction of 12) for the "measured velocity" of light even from extra-terrestrial sources the value σ.

If by astronomical methods, given by de Sitter and others, we obtain a constant value of the velocity of light, even from extra-terrestrial sources,[1] it does not disprove our theory; it shows only that the value of $\psi(v)$ for a given v is below a certain limit, for example

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[1] By "one-direction method."

$$\psi(v) < \frac{v^2}{c^2}, \text{ etc.}$$

It will be the problem of experimental physics to find out the precise form of ($\psi(v)$).

By applying the Huygens principle and taking instead of spherical elementary waves, ellipsoidal ones, all the optics of moving systems can be built up. This was done by the late N. Pashsky. Those results I hope to publish later.

In conclusion I wish to express my sincerest thanks to Dr. P. H. Dike for the revision of the English translation of the Russian original manuscript, and for many critical remarks.

PHYSICAL LABORATORY,
ROBERT COLLEGE,
CONSTANTINOPLE,
February, 1921.

A NOTE ON THE PLANE WAVE IN AN ISOTROPIC DIELECTRIC.

By Satyendra Ray.

Synopsis.

Electric and Magnetic Forces in the Direction of Propagation of a Plane Wave in an Isotropic Dielectric need not be zero if the ether has a constant electric density P_e and a constant magnetic density P_m, for it is shown that these forces are equal, respectively, to $P_e\lambda/k$ and $P_m\lambda/\mu$ where λ, k and μ denote wave-length, inductivity and permeability respectively.

An Application of Dimensions.—The equations of a plane wave, referred to an axis of Z parallel to the direction of propagation, are,

$$K\frac{dE_x}{dt} = -\frac{dM_y}{dz}; \quad K\frac{dE_y}{dt} = \frac{dM_x}{dz}; \quad K\frac{dE_z}{dt} = 0; \qquad (1)$$
$$\mu\frac{dM_x}{dt} = \frac{dE_y}{dz}; \quad \mu\frac{dM_y}{dt} = -\frac{dE_x}{dz}; \quad \mu\frac{dM_z}{dt} = 0.$$

The solution of the third and the sixth equations above gives

$$E_z = \text{constant}; \quad M_z = \text{constant},$$

instead of $E_z = 0$, $M_z = 0$, as usually stated in text-books.

In a paper on the equivalent shell of a circular current[1] a radius of the electric particle infinitely smaller than 10^{-13} cm. has been found. If it is taken to be the diameter of the electric particles of which the ether is composed an expression for the constant forces in the direction of propagation can be obtained in terms of the constants of the ether and the plane wave by the method of dimensions.

The electric force will be a function of the electric density ρ_e and inductivity K of the medium and of the wave-length Λ of the disturbance if it is periodic.

$$E_z = c\frac{\rho_e\Lambda}{K}$$

is such a function, where c is a numerical constant, and

$$[E_z] = \left[\frac{\rho_e\Lambda}{K}\right]$$

dimensionally. Similarly, assuming magnetic density ρ_m,

$$M_z = c\frac{\rho_m\Lambda}{\mu}.$$

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[1] Phil. Mag., Jan., 1920.

To find the constant c. For any electromagnetic disturbance,

$$\frac{\mu K}{c^2}\cdot\frac{d^2E_x}{dt^2}=\frac{d^2E_x}{dx^2}+\frac{d^2E_x}{dy^2}+\frac{d^2E_x}{dz^2}-\frac{d}{dx}\left(\frac{dE_x}{dx}+\frac{dE_y}{dy}+\frac{dE_z}{dz}\right);$$
$$\frac{\mu K}{c^2}\cdot\frac{d^2E_y}{dt^2}=\frac{d^2E_y}{dx^2}+\frac{d^2E_y}{dy^2}+\frac{d^2E_y}{dz^2}-\frac{d}{dy}\left(\frac{dE_x}{dx}+\frac{dE_y}{dy}+\frac{dE_z}{dz}\right);\quad(2)$$
$$\frac{\mu K}{c^2}\cdot\frac{d^2E_z}{dt^2}=\frac{d^2E_z}{dx^2}+\frac{d^2E_z}{dy^2}+\frac{d^2E_z}{dz^2}-\frac{d}{dz}\left(\frac{dE_x}{dx}+\frac{dE_y}{dy}+\frac{dE_z}{dz}\right).$$

with three similar equations in M_x, M_y, M_z; also,

$$\left(\frac{dE_x}{dx}+\frac{dE_y}{dy}+\frac{dE_z}{dz}\right)=\frac{4\pi\rho_e}{K}$$
$$\left(\frac{dM_x}{dx}+\frac{dM_y}{dy}+\frac{dM_z}{dz}\right)=\frac{4\pi\rho_m}{\mu}.$$

The equations (2) reduce to the usual form

$$\frac{\mu K}{c^2}\cdot\frac{d^2E_x}{dt^2}=\nabla^2E_x,\text{ etc.,}$$

and

$$\frac{\mu K}{c^2}\cdot\frac{d^2M_x}{dt^2}=\nabla^2M_x,\text{ etc.,}$$

only on condition that ρ_e and ρ_m are absolute constants, independent of both time as well as position.

For a plane wave of period τ we write

$$E_x=\lambda Ae^{\iota\frac{2\pi}{\tau}\left(t-\frac{lx+my+nz}{V}\right)};$$
$$E_y=\mu Ae^{\iota\frac{2\pi}{\tau}\left(t-\frac{lx+my+nz}{V}\right)};$$
$$E_z=\nu Ae^{\iota\frac{2\pi}{\tau}\left(t-\frac{lx+my+nz}{V}\right)},$$

where l, m, n are the direction cosines of the normal to the wave front, and λ, μ, ν of the electric vector.

Differentiating the equations with respect to x, y, z, respectively and adding, we have

$$\frac{4\pi\rho_e}{K}=-i\frac{2\pi}{\Lambda}Ae^{\iota\frac{2\pi}{\tau}\left(t-\frac{lx+my+nz}{V}\right)}+(l\lambda+m\mu+n\nu)$$
$$=\frac{2\pi}{\Lambda}A\sin\frac{2\pi}{\tau}\left(t-\frac{lx+my+nz}{V}\right)\cos\varphi,$$

where φ is the angle between the electric vector and the direction of propagation.

Or

$$A\sin\frac{2\pi}{\tau}\left(t-\frac{lx+my+nz}{V}\right)\cos\varphi=\frac{2\rho_e\Lambda}{K}.$$

It will be noticed the left-hand expression is the electric force in the direction of propagation and is equal to the constant $2\rho_e\Lambda/K$. A similar expression in ρ_m, Λ, μ would be obtained for the steady magnetic force in the direction of propagation. Maxwell distinctly indicates the existence of such steady forces in addition to the periodic effects.[1]

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[1] Vide Vol. II., § 783.

NOTE ON RADIATION PRESSURE.

BY R. N. GHOSH AND N. K. SUR.

SYNOPSIS.

Derivation of the Formula for Radiation Pressure.—Larmor's formula was derived on the assumption that the energy density of the radiation is proportional to $1/\lambda^2$. The same formula is now derived *for a pulse with an energy distribution in accordance with Wien's law.* If the reflector is moving toward the source with a speed v and if we assume that the amplitude for each component part of the incident pulse is unchanged by reflection, then because of the change of wave-length on reflection (Doppler effect) there is a change in energy density per second which is equal to the corresponding radiation pressure times v. Hence $p_\lambda = 2e_\lambda(c+v)/(c-v)$ and the total pressure is $2I(c+v)/c(c-v)$, when I is the intensity of the total radiation.

INTRODUCTION.

TO Larmor is due a method of argument to show that any wave motion in which the energy density is inversely proportional to the square of wave-length will exert a pressure which is equal to the energy density in the beam. His method of argument has been applied by Edser[1] to Electromagnetic waves. He on the principles of moving electric field laid down by Heaviside, arrives at the conclusion that the amplitude of the wave must undergo a change given by the formula

$$a/a' = c - v/c + v, \tag{1}$$

(1) where a and a' are the amplitudes before and after reflection. And the pressure is given by

$$p = 2E_1\{(c+v)/(c-v)\}. \tag{2}$$

(2) Where c and v are the velocities of light and reflector respectively, and E_1 is the energy density in the incident beam.

But Prof. Larmor[2] finds that he cannot accept the result contained in (1) on the mode of identification of the magnetic force with the velocity of displacement of the medium (ether).

Wiener has shown that the surface of a stationary reflector is a node, and if we assume that this condition holds good when the reflector is moving with a constant velocity, then the pressure of radiation can be demonstrated, on the basis of argument given by Larmor, when the amplitude is a complicated function of wave-length. The particular case when the energy density of the wave motion obeys Wien's or

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[1] Edser, Phil. Mag., Oct., 1914. (See also Hargreaves Phil. Mag., June, 1920.)

[2] Larmor, Phil. Mag., Nov., 1914.

Planck's law of distribution of energy has been discussed in this paper, and the pressure has been found to agree with Larmor's result.

II. Sound Waves.

That sound waves exert a pressure follows readily from Larmor's theorem. The late Lord Rayleigh[1] discussed the question of pressure of Sound waves in several papers. He came to the conclusion that Sound waves of finite amplitude exert no pressure, if they are to preserve their form unchanged. Recently it has been shown by Elder Bartel[2] that such waves exert a pressure, which is not zero by taking into account the disturbance in the medium by producing the moving reflector.

III. Light Waves.

The late Lord Rayleigh[3] showed that a type of pulse given by $y = e^{-c^2x^2}$ was suitable for white light representation, and which at that time agreed with the energy distribution law of Weber. Recently Dr. Houston[4] has discussed in two papers a type of pulse which account for the various properties of white light. His pulse is given by

$$y = \frac{A \cos (5/2 \tan^{-1} x/h)}{(x^2 + h^2)^{5/4}}. \tag{3}$$

Expression (3) can be transformed into

$$y = \frac{4A}{3\sqrt{\pi}} \int_0^\infty \alpha^{3/2} e^{-h\alpha} \cos \alpha x d\alpha,$$

where $\alpha = 2\pi/\lambda$. It shows the component waves present in the pulse. They have a physical significance when the pulse passes through a dispersive medium. The amplitude of a component wave λ is given by

$$A_\lambda = \frac{4A}{3\sqrt{\pi}} e^{-h\alpha} \cdot \alpha^{3/2}.$$

So that the amplitude is a function of the wave-length.

The energy density $E_\lambda d\lambda$ comprised between λ and $\lambda + d\lambda$ is given by

$$E_\lambda d\lambda = \frac{c_2 A^2}{\lambda^5} e^{-4\pi h} d\lambda$$

where c_2 is a constant. Now if we identify h as the reciprocal of absolute temperature corresponding to the radiation we have

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[1] Lord Rayleigh, 1905, pp. 364, 1902, pp. 338, Phil. Mag.

[2] Edler Barter, Proceedings of the Asiatic Society of Bengal, Nov., 1919. See also Warren Weaver, Phys. Rev., May, 1920.

[3] Scientific Papers, Vol. III., p. 268.

[4] Houston, Proceedings of the Royal Society, 1913, p. 339; 1914, p. 288.

$$E_\lambda = \frac{c_2A^2}{\lambda^5}e^{-4\pi/\lambda T} = \frac{c_2A^2}{\lambda^5}e^{-c_1/\lambda T},$$

which is Wien's Law.

Now let us apply Larmor's argument to the pulse given by (3) incident normally on a perfect reflecting surface. The reflector is moving with a velocity v in opposite direction to that of the incident pulse. By Doppler's principle the component wave of length λ is changed into λ' so that

$$\frac{\lambda}{\lambda'} = \frac{c+v}{c-v}. \tag{4}$$

The amplitude A_λ before reflection is proportional to

$$Ae^{-c_1/\lambda T}/\lambda^{3/2},$$

and after reflection it must remain the same, so that the surface of the reflector may be a node. Therefore A is changed in such a way so as to fulfill this condition given by

$$\frac{A'}{A} = \left(\frac{c-v}{c+v}\right)^{3/2}$$

in accordance with Wien's displacement law.

The change of A is connected in some manner with the disturbance in the ether produced by the moying mirror not known at present. Dr. Green[1] in a very recent paper on the "Fluid analogue of the ether" has discussed the properties of the ether in the light of the ordinary fluid. By pushing the analogy a little further, we may conceive that similar disturbance may occur in the ether, as in the ordinary fluid when a plane lamina moves through it with a constant velocity. If pulses travelling with the minimum velocity are incident upon the lamina, they are reflected back, but near the lamina, the disturbance will be the sum of the disturbances produced by the pulses and the motion of the lamina, the surface of the lamina always being a node. Another point to be noticed is the pressure experienced by the lamina. It will again be sum of the pressures due to the pulses, and the additional pressure depends upon the velocity of the lamina. This is precisely the pressure which we are going to calculate. But the analogy cannot be pushed further for numerical calculation.

The change in energy density corresponding to the change of A into A' is given by

$$\frac{E_\lambda'}{E_\lambda} = \left(\frac{c+v}{c-v}\right)^2,$$

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[1] Dr. Green, Phil. Mag., June, 1920.

i.e., the energy in the reflected train corresponding to the wave-length λ is therefore increased in the ratio of $[(c+v)/(c-v)]^2$.

An incident pulse of length $(c+v)$ is changed by reflection into a pulse of length of $(c-v)$, on account of motion of the reflecting surface. Hence the increase in energy density per second is

$$E_\lambda\left(\frac{c+v}{c-v}\right)^2(c-v) - E_\lambda(c+v) = E_\lambda(c+v)\left[\frac{c+v}{c-v} - 1\right].$$

This energy is supplied by the work done in advancing the reflecting surface a distance v per second against a pressure p so that

$$pv = E_\lambda(c+v)\left[\frac{c+v}{c-v} - 1\right];$$

$$\therefore\quad p = 2E_\lambda\left(\frac{c+v}{c-v}\right),$$

which is in accordance with the results of Larmor and Edser.

Equation (5) shows that p varies with E_λ, since c and v are constants. It also shows that the pressure on the reflector is maximum for the wave-length for which the energy density is maximum, and will be less than this maximum value for different values of the wave-length as shown in the following table:

T = 2000° abs. carbon glow lamp,

C/I = 2.8 × 10^{-3} dynes per sq. cm.

$\lambda \times 10^{-4}$ Cm.	Pressure × 10^{-6} Dynes per Sq. Cm.
8.3	.32
4.8	.64
2.7	.90
1.8	.71
1.2	.32

The total pressure due to whole pulse, consisting of different wave-lengths is given by

$$\begin{aligned} P &= \int_0^\infty 2E\lambda\left(\frac{c+v}{c-v}\right)d\lambda \\ &= \frac{2I}{c}\left(\frac{c+v}{c-v}\right), \end{aligned} \tag{6}$$

where I is the intensity of total radiation of the incident beam. In the case of stationary reflector, the pressure is given by twice the energy density in the beam, in agreement with Larmor. Equation (6) shows that the pressure is infinite when the reflector is moving with a velocity equal to that of light; in that case the reflector piles up the radiation as it advances with the velocity of radiation—which of course no one would deny. In the case when it moves away from the radiation with an equal velocity the pressure is zero, since it just keeps ahead and there is no reflection.[1]

The total energy density in front of the reflector corresponding to the wave-length λ is given by E

$$E = E_\lambda + E_\lambda\left(\frac{c+v}{c-v}\right)^2;$$

$\therefore \quad p = kE$, where

$$k = \frac{c^2 - v^2}{c^2 + v^2},$$

which reduces to unity if we neglect second order terms in v/c.

EWING CHRISTIAN COLLEGE,
ALLAHABAD,
Dec., 1920.

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[1] Larmor, loc. cit.

THE IONIZATION POTENTIAL OF SELENIUM VAPOR.

By A. D. Udden.

Synopsis.

Ionization potential of selenium vapor was determined from current potential curves obtained with a four electrode, pyrex glass tube. Nickel was used for the electrodes. After the cleaning out process the selenium was spilled into the bottom of the tube and vaporized by means of an oil bath kept at 168 to 282° C. Both total negative current measurements were made and positive current measurements according to the Lenard method, and corrections were applied for the potential fall along the electrically heated salted platinum cathode and for the initial energy of the electrons. The ionization potential comes out 12.7 ± 0.4 volts. Putting this value in the quantum equation we get 980 Å as the short wave-length limit of the selenium spectrum.

THE ionization and resonance potentials of several elements belonging to group VI. of the periodic table have already been determined.[1] In continuation of the study of the elements of this group, this paper presents the results of a determination of the ionization potential of selenium vapor.

The significance of critical potential determinations has frequently been pointed out. When an electron possesses a certain critical speed it is capable of ionizing a given vapor atom by collision. That potential which can impart this speed to an electron is known as the ionization potential of the given vapor. According to Bohr's theory, if the ionization potential V of a vapor has been determined, the limiting frequency ν of the spectrum of this vapor may then be calculated from Planck's formula, $\nu = Ve/h$.

The ionization potential of selenium vapor was obtained by the usual method of accelerating the electrons from a hot filament to metal electrodes to which suitable potentials were applied. Current potential curves were plotted and the ionizing potential was indicated by an abrupt change of curvature in each of the graphs. Corrections were applied for the drop in potential along the filament and for initial velocity of emission of the electrons. The initial velocity correction was made by applying back potentials on the grid just sufficient to stop the electron current.

A diagram of the apparatus used in the present investigation is shown in Fig. 1. The pyrex glass tube was made in two sections, ground to a

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[1] Scientific Papers of the Bureau of Standards, No. 400.

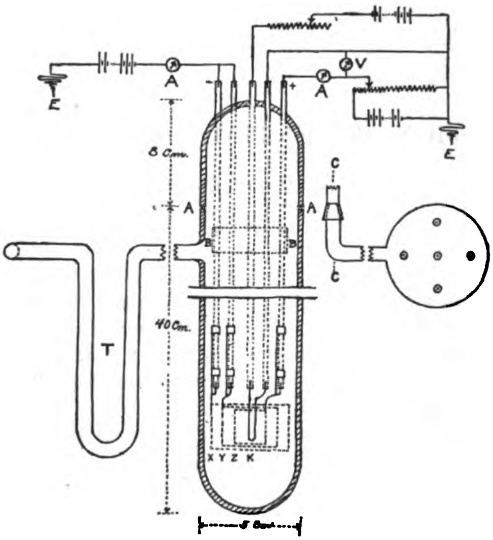

Fig. 1.

smooth fit at A–A, and these were sealed together with DeKhotinsky cement. The nickel plate X and the grids Y and Z were cylindrical in shape and placed coaxially with reference to the cathode K. The electrodes were held in place by means of clasps fitting on slender pyrex tubes sealed into the upper section of the large tube. A small mica cup at B–B contained the selenium which was to be vaporized. The cathode consisted of a thin strip of platinum coated with several applications of barium and strontium oxides. When measurements were being made a freezing mixture of carbon dioxide snow surrounded the trap T.

With proper heat treatment of the tube and the electrodes a vacuum of about .003 millimeter pressure was obtained, after which the selenium was spilled into the bottom of the tube by rotating the latter about a horizontal axis C–C. The selenium was then vaporized by means of an oil bath surrounding the lower part of the tube. Some difficulty was experienced in finding a metal suitable for use as electrodes in the vapor. Copper was attacked by the vapor and a polarizing layer appeared to form on the surface of iron. A white powder formed on the nickel electrodes but this did not seriously interfere with the flow of electron currents.

The ionization potential was determined by both total current and positive current measurements. In the case of total current measurements, the plate and two grids were connected together so as to form a single electrode to which positive accelerating potentials were applied.

For the positive current or Lenard method the electrodes were connected as shown in the diagram. In the latter case a negative potential of about 15 volts was applied to the outer electrode.

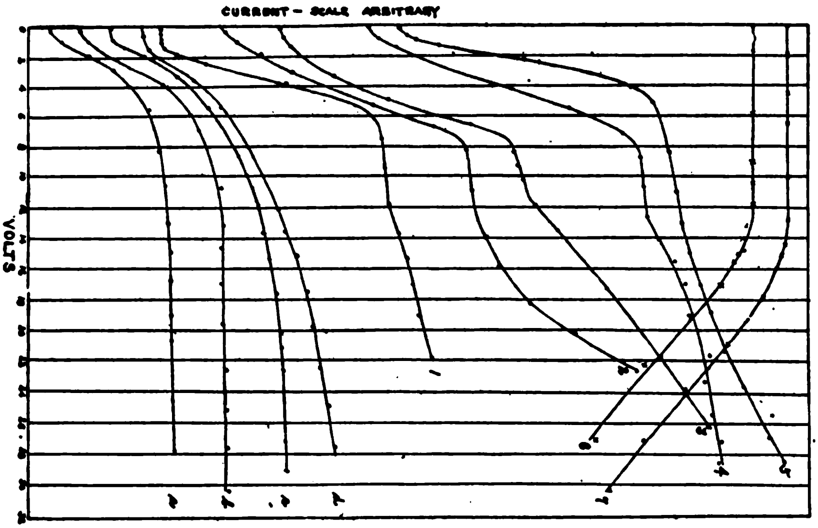

Fig. 2.

Runs were first made without the selenium vapor, keeping the conditions otherwise the same as when the vapor was present in the tube. Curves such as *a*, *b*, *c* and *d* (Fig. 2) were then obtained. It will be noticed that these curves flat off quite regularly and do not show any sudden increase in the electron current. Curves 1, 2, 3, 4, 5, 6 and 7 were made with vaporized selenium present in the tube, the temperature of the vapor varying from 168 to 282 degrees Centigrade for the different runs. Graphs 1 to 5 are total current measurements while 6 and 7 were obtained by the Lenard method. The graphs as plotted include the corrections for potential drop along the filament, the corrections varying from 1.0 to 2.4 volts. The initial velocity corrections are shown in the accompanying table. In some cases the temperature of the filament was so low that no appreciable initial velocity was obtained.

No. of Curve.	Break, Volts.	Initial Velocity, Volts.	Ionization Potential, Volts.	Deviation from Mean.
1	12.0	—	12.0	−.7
2	11.8	1.5	13.3	+.6
3	11.6	.5	12.1	−.6
4	12.7	.1	12.8	+.1
5	13.0	—	13.0	+.3
6	12.5	.1	12.6	−.1
7	13.0	—	13.0	+.3

The average ionization potential of selenium vapor calculated from these measurements is 12.7 ± .4 volts. This is quite close to the value obtained for sulphur by Mohler and Foote.[1] The ionization and resonance potentials for elements belonging to group VI. so far determined are given below.

Element.	Ionization Potential.	Resonance Potential.
Oxygen	155. ± .5	7.91 ± .1
Sulphur	12.2 ± .5	4.78 ± .5
Selenium	12.7 ± .4	——

If the potential 12.7 volts is substituted for V in the expression, $\nu = Ve/h$ the limiting frequency of the spectrum of selenium vapor is seen to be about 980 Ångstrom units. Inasmuch as measurements do not appear to have been made in this region of the selenium spectrum this result can not be confirmed. No arc was at any time visible in the selenium vapor up to thirty volts, the highest potential employed.

It was also desired to measure the resonance potential of selenium vapor. For this purpose, following the work of Mohler and Foote, a second grid was brought into use according to their method. At present it has not been possible to obtain any indication of a resonance potential. With further modifications of the dimensions and distances separating the electrodes it is possible that a resonance potential may be determined. This might provide a more satisfactory correction for the initial velocity of emission of the electrons.

The writer is under obligations to Dr. C. B. Bazzoni for suggestions and advice received during the course of the work upon this experiment

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ON TALBOT'S BANDS AND THE THEORY OF THE LUMMER-GEHRCKE INTERFEROMETER.

BY NIHAL KARAN SETHI.

SYNOPSIS.

New Simple Method of Observing Talbot's Bands.—If a thin strip of *cover glass is immersed in a dispersive medium*, such as a suitable mixture of CS_2 and benzene, with a refractive index such that the group velocity in it is less than in the plate, and if a vertical slit illuminated with white light is observed through the glass strip held in a vertical plane very oblique to the plane of the incident light, two out of the four spectra formed by the edges of the plate exhibit Talbot's bands. The *theory* of the formation of the bands in this way is closely similar to that of the Lummer-Gehrcke plate, and as in the latter, the maxima are narrow and the minima crossed by secondary maxima when the number of internal reflections is small. The appearance, too, is not affected by opening the slit wide.

Interference Bands Analogous to Those given by a Fabry-Perot etalon were also observed with the same simple arrangement. The appearance and behavior of these bands are *described and explained.*

ATTENTION has been drawn by Prof. R. W. Wood[1] to the relation between the theory of Michelson's echelon grating and of the well-known bands in the spectrum discovered by Talbot which are observed on covering half the aperture of the beam in a spectroscope by a thin plate of mica or glass. Wood has pointed out that Talbot's plate is in effect an echelon grating of two elements and has published photographs of Talbot's bands obtained by using a mica echelon of two or more elements as the retarding plate. In the present note, the writer wishes to describe a new and extremely simple method of observing Talbot's bands, which is of interest as it suggests an analogy between these bands and the theory of the Lummer-Gehrcke interferometer.

A thin strip of glass conveniently obtained by cutting from a microscope cover-slip is immersed in a mixture of carbon disulphide and benzene in adjustable proportions. The liquid is contained in a flat cell and means are provided to enable the observer to turn the slip of glass round a vertical axis while its edges are always maintained in a vertical position. On turning the strip so that white light from a distant slit is incident very obliquely on it, an eye placed close to the strip is able to see a pair of spectra on either side of the source of light and another pair on either side of its reflected image. When the inclination of the glass strip and the refractive index of the liquid mixture are suitably adjusted, one of either pair of the spectra is seen to exhibit Talbot's bands, while the remaining two show some curious curved fringes. We shall first deal with the former case.

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[1] Phil. Mag., Nov., 1909, p. 758.

Fig. 1 shows the main features of the arrangement. The edge *A* of the glass strip acts as a prism and a ray *OP* is deviated in the direction *QR* in which the usual spectrum is observed. But owing to the very oblique incidence of light, a number of internal reflections take place and we have a corresponding number of parallel emergent rays $Q'R'$, $Q''R''$, etc., the mutual interference of which is undoubtedly the cause of the Talbot's bands observed in this spectrum. The rays *ST*, $S'T'$, $S''T''$, etc., give another spectrum similarly crossed by Talbot's bands. These multiple reflections can be easily observed by removing the eye to a distance or by means of a small magnifying lens.

Figs. 2 and 3 (in the plate) are photographs of Talbot's bands observed in this manner. It should be observed that in addition to the very bright principal maxima, the photographs also show feebler secondary maxima in the spectrum—one in Fig. 2 and two in Fig. 3.

That these bands represent a more general case of Talbot's bands is abundantly clear when we remember that the well-known Talbot's bands are caused by the redistribution by a prism of the diffraction pattern due to two adjacent narrow parallel beams which are caused to differ in phase by the introduction of a thin retarding plate in the path of one of them. In the present case we have not two but several such beams differing in phase by regularly increasing amounts, although no thin plates are expressly introduced for this purpose. The prism which re-distributes the diffraction pattern is also included in the glass strip, so that we have a considerable simplification of apparatus.

It is a noteworthy fact that in this experiment, the fringes are observable in the spectrum both when the refractive index μ of glass is smaller than μ' of the liquid mixture, and to some extent also when $\mu > \mu'$. The essential condition is that the group-velocity of light in glass is higher than in the liquid surrounding it. And when by the addition of more benzene, the refractive index of the latter is reduced so much that the group-velocity becomes equal to that in glass for any particular color,[1] these bands disappear in the part of the spectrum lying on the red side of this position. The reason why this is so becomes clear on recalling the condition for the visibility of Talbot's bands, namely, that the light pulses or groups passing through different parts of the dispersing

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[1] For a method of ascertaining this see the paper on Powell's Bands by the writer in PHYS. REV., Dec., 1920.

PHYSICAL REVIEW, VOL. XVIII., SECOND SERIES.
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PLATE I.
To face page 390.

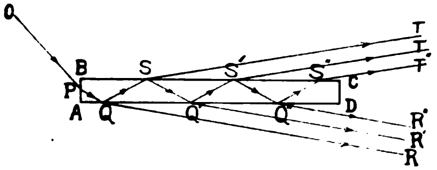
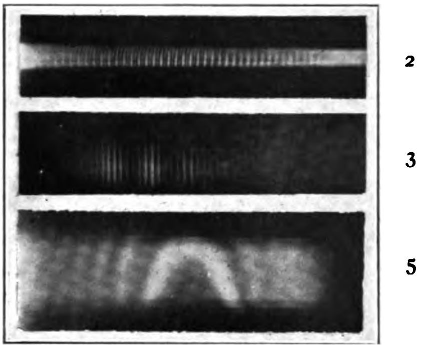


Fig. 1.



Figs. 2, 3, and 5,

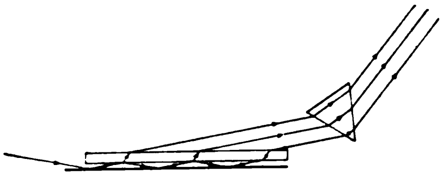


Fig. 4.

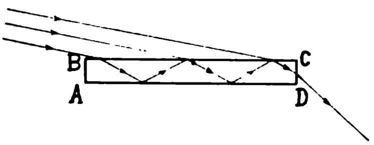


Fig. 6.

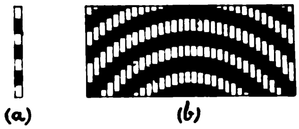


Fig. 7.

NIHAL KARAN SETHI.

prism should be caused to arrive together at the focal plane of the observing telescope. In this case the liquid surrounding the plate has the higher dispersive power and hence the condition cannot be satisfied unless the group-velocity in it is less than within the plate.

In calculating the distribution of light in the resulting spectrum we may follow the treatment of the usual case of Talbot's bands by Rayleigh,[1] and assume in his notation that the successive beams extend from $x = -h$ to $x = -h + d$; $x = -h + 3d$ to $x = -h + 4d$; $x = -h + 6d$ to $x = -h + 7d$, etc., since it can be easily shown that the widths of the beams are all equal being limited by the thickness of the glass strip and that they are separated from each other by intervals equal to the width of any one of them. Remembering that the amplitude of each beam is b^2 times that of its predecessor where b is the coefficient of internal reflection, we shall obtain on integration and addition the value of the intensity I to be given by

$$I = \frac{1}{\xi^2} \sin^2 \frac{\pi \xi d}{\lambda f} \frac{1}{(1 - b^2)^2 + 4b^2 \sin^2 \frac{\pi}{\lambda}\left(R - \frac{2d\xi}{f}\right)}$$

for the complete set of beams. When the slit is illuminated by white light and the diffraction pattern is spread out into a spectrum we shall have $R = R_0 + \bar{\omega}\xi$. If now $\bar{\omega} = 2d/f$,

$$I = \frac{1}{\xi^2} \sin^2 \frac{\pi \xi d}{\lambda f} \cdot \frac{1}{(1 - b^2)^2 + 4b^2 \sin^2 \frac{\pi R_0}{\lambda}}.$$

The condition for the darkest minima (for they cannot be absolutely dark) is thus seen to be $\bar{\omega} = 2d/f$.

From what has been said above it is clear that in forming these Talbot's bands, the thin glass plate acts in a manner essentially similar to that in which the well-known Lummer-Gehrcke plate acts. This is made still clearer when monochromatic light is employed. The diffraction pattern now observed shows the same sharp and narrow maxima separated by broad dark bands which are themselves crossed by feeble secondary maxima when the number of internal reflections is not very large. As the incidence inside the plate becomes more and more oblique, the bands also gradually increase in width. And lastly the bands remain quite clear even when the slit is widened, so much so that Talbot's bands produced in this way can be easily observed without any slit whatsoever.

That the Lummer-Gehrcke plate does not ordinarily show Talbot's bands when illuminated by white light is due not only to the very large retardations which it produces and which will require the dispersing apparatus to have an exceedingly large resolving power,[1] but also to the more important fact that its dispersion takes place in the wrong direction. This becomes the clearer when we consider the small correction, first pointed out by Baeyer,[2] in calculating the separation of two close wave-lengths λ and $\lambda + d\lambda$ observed with the Lummer–Gehrcke plate. The angular distance between the maxima of these wave-lengths can be easily shown to be

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[1] Ency. Brit., 11th edition, Vol. VIII., p. 250 and Sc. Papers, Vol. III., p. 123.

$$d\theta = -\frac{1}{\sin\gamma\cos\theta}\left\{\frac{n\cos\gamma}{2t} - \frac{d\mu}{d\lambda}\right\} d\lambda,$$

where t is the thickness of the plate, γ and θ are the angles of internal incidence in and of emergence from the plate and n represents the order of the maxima and is equal to $(2\mu t\cos\gamma)/\lambda$. This expression for the separation consists of two terms, one due to the change in the retardation consequent on a change in λ and the other due to the dispersion of the plate. Putting

$$\mu = 1.5, \qquad \frac{d\mu}{d\lambda} = -\frac{2B}{\lambda^3}, \qquad B = .5 \times 10^{-10}, \qquad \lambda = 5 \times 10^{-5}$$

and taking the emergence to be almost grazing, so that

$$\sin\gamma = \frac{1}{\mu} = \frac{2}{3} \quad \text{and} \quad \cos^2\gamma = \frac{5}{9},$$

we get

$$\frac{n\cos\gamma}{2t} = \frac{\mu\cos^2\gamma}{\lambda} = 1.5 \times 10^4,$$

and

$$\frac{d\mu}{d\lambda} = -\frac{2B}{\lambda^3} = -.08 \times 10^4.$$

But if the plate is surrounded by a medium whose refractive index μ does not differ much from that of the plate itself and whose dispersion is *greater*, these two terms become comparable and since the sign of the second is reversed, it is possible under favorable conditions to have the effect of the first term completely destroyed by the second; in other words, to have the maxima remaining fixed while the wave-length is altered. In the case of a mixture of carbon disulphide and benzene adjusted till μ' differs from μ by .01, the value of $\cos^2\gamma = 1 - 1/(1.01)^2 = .02$ nearly; so that the first term in the above expression becomes

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[1] See T. E. Doubt, PHYS. REV., Oct., 1917, p. 322.

[2] Verh. der Deutsch. Phys. Ges., 10, 1908, p. 733.

$.06 \times 10^4$ nearly. And the resultant dispersion $d/d\lambda(\mu - \mu')$ is about three fourths of the dispersion of glass, so that the second term also is equal to about $.06 \times 10^4$.

It may also be mentioned that Talbot's bands can also be observed with the interference plate recommended by Barnes.[1] The arrangement is shown in Fig. 4.

The thin plate of air is formed between two interferometer plates one of which is heavily silvered in front. A separate prism is used to produce the spectrum. In the position of the prism shown in the figure, the Talbot's bands with the subsidiary maxima described in the previous case are obtained, but when the prism is turned round so as to receive the retarded beams on the thicker side, no bands are produced.

Although not connected with the subject of Talbot's bands the writer must take this opportunity of describing a phenomenon which one cannot fail to notice while observing these bands with the arrangement of Fig. 1. It has already been mentioned that in addition to two spectra showing Talbot's bands and formed by the edge *A* of the plate, two other spectra are also observed. They are found to be due to the edges *C* and *D* of the plate and in general within certain limits of adjustment of the refractive index of the liquid, each of them exhibits in some part of it a series of curved dark bands (Fig. 5 in the plate) which drift up or down and give place to new ones in the centre somewhat in the same way as the rings in a Fabry-Perot etalon while the center itself slowly shifts across the spectrum when the glass strip is slowly turned round. The curvature of these arcs is found to depend on the inclination of the edges *C* and *D* to the slit which is the source of light. When they are carefully adjusted to be strictly parallel, we have a series of vertical bands on either side of a rather broad central patch and these become narrower and fainter as the distance from the central patch increases until they disappear altogether. Rotation of the glass strip makes them close in or spread out exactly like the curved bands described above. But the slightest inclination of the edge *C* to the slit makes them curved and the sign of the curvature may be altered by an inclination in the opposite direction. When the inclination is very considerable, they appear as horizontal arcs of circles of very large radii.

The disposition of the rays which cause this phenomenon is shown for one of the spectra in Fig. 6. A number of parallel rays incident on the face *BC* come out of *CD* after multiple internal reflections as a system of coincident beams, the components of which differ in phase by regularly increasing amounts. That these are the cause of the curved bands is seen from the fact that they disappear when the beams are cut off by a screen. Such a system will no doubt produce the usual Fabry-Perot rings, but on account of the very small retardations produced by the plate, the whole of the slit will be covered by the central ring only. A spectrum in which each monochromatic element consists of such a pattern cannot be expected to show any bands. But there is one circumstance which considerably alters the situation. The retardation in phase which is easily seen to be $(2\mu t \cos \gamma)/\lambda$ does not continually increase from one end of the spectrum to the other, but becomes stationary at a certain point and then begins to decrease. The condition for its becoming stationary is evidently

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[1] Astrophys. Journal, 19, 1904, p. 200.

$$\frac{d}{d\lambda}\left(\frac{2\mu t \cos \gamma}{\lambda}\right) = 0.$$

This combined with the condition $\mu \sin \gamma = \mu' \sin i$ will give

$$\tan^2 \gamma = \frac{\frac{1}{\mu}\frac{d\mu}{d\lambda} - \frac{1}{\lambda}}{\frac{1}{\mu'}\frac{d\mu'}{d\lambda} - \frac{1}{\mu}\frac{d\mu}{d\lambda}}.$$

Putting roughly

$$\mu = \mu' = 1.5, \quad \frac{d\mu}{d\lambda} = -1 \times 10^3, \quad \frac{d\mu'}{d\lambda} = -1.5 \times 10^3 \text{ and } \lambda = 5 \times 10^{-5} \text{ cm.},$$

we obtain $\gamma = 81^\circ$ nearly, which not only proves the possibility of this condition being fulfilled but is moreover of the right order of magnitude.

In the neighborhood of the wave-length for which this condition is satisfied, the phase change is very slow and the condition of maximum or minimum intensity persists for a number of neighboring wave-lengths. The spectrum in this neighborhood will therefore appear to be crossed by vertical dark bands which will gradually become narrower on either side of this point.

Another fact confirmed by observation is that this point of stationary retardation is shifted towards the red end of the spectrum when the angle of incidence is increased, because the denominator in the formula for $\tan \gamma$ decreases faster than the numerator when λ is increased.

The above discussion refers to the case when the slit is strictly parallel to the edge of the glass strip. When, however, any one of them is slightly turned about the line of sight, there is another factor which gives rise to the curved bands. The light from all the points of the slit is not now incident on the strip at the same angle, so that the interfering beams due to different parts of the slit have different retardations and therefore the image of the slit in monochromatic light becomes crossed by dark bands; and the more is the tilt increased, the greater is the number of these bands. An easy numerical calculation shows that a very small change in the angle of incidence is enough to change the retardation by π. Since the retardation

$$\rho = \frac{2\mu t \cos \gamma}{\lambda},$$

$$\frac{d\rho}{d\gamma} = -\frac{2\mu t \sin \gamma}{\lambda}.$$

Putting

$\mu = 1.5$, $t = .025$ cm., $\lambda = 5 \times 10^{-5}$ cm. and $\sin \gamma = 1$ nearly, $\frac{d\rho}{d\gamma} = 1.5 \times 10^3$ therefore, the change in γ required for a change of π in the value of ρ is

$$\frac{\pi}{1.5 \times 10^3}.$$

The result, then, of this inclination of the slit is that the image of the slit appears broken up as in Fig. 7, *a*. The disposition of the dark and bright portions will vary with the wave-length of the light used. With a monochromator as the source of illumination, one can watch the slow shift of these upwards or downwards as the wave-length is continually altered, till the constancy of retardation with change of λ is reached and then they begin to march in the reverse direction. Illuminated with white light, these various images must be distributed in the spectrum much as in Fig. 7, *b* and will give rise to the curved bands referred to above. Inclining the slit in the opposite direction produces the curves with the concavity turned upwards.

The experimental work was done in the laboratory of the Indian Association for the Cultivation of Science, Calcutta, and the writer's best thanks are due to Prof. C. V. Raman for his unfailing interest in the work.

210 BOWBAZAAR STREET,
CALCUTTA, INDIA,
15th March, 1921.

THE REFLEXION OF X-RAYS BY CRYSTALS.

By H. A. Wilson.

Synopsis.

Theory of Reflection of X-rays from a Point Source, by a Perfect Crystal.—The derivation of the expression for the total reflected energy as obtained by other physicists, is given in simpler form. It is shown that the reflected rays are parallel in the plane of incidence but diverge in the perpendicular plane. Making the simplifying assumption that no ray is reflected more than once from any plane of atoms and that the absorption by the crystal is negligible, expressions for the intensity at any point in the reflected beam are derived. It is shown that interference fringes are to be expected whose distance apart under ordinary circumstances may be of the order of 4 × 10^{-4} cm. The error due to the above mentioned simplifying assumptions can be corrected by multiplying by the appropriate exponential factor.

THE theory of the reflexion of x-rays by crystals has been discussed by W. H. Bragg,[1] C. G. Darwin,[2] A. H. Compton[3] and W. L. Bragg.[4] The last three of these investigators obtained equivalent results for the total energy reflected.

The main object of the present paper is to give the theory of the reflexion of homogeneous x-rays, coming from a point source, by a perfect crystal, in a simpler form than has hitherto been done. The results obtained agree with those of the previous investigators but some points of detail with regard to the character of the reflected beam are brought out clearly by the new method of calculation.

The very interesting results recently described by Bergen Davis[5] seem to show that in some cases reflexion takes place as though the crystal were nearly perfect.

If an electron with charge e and mass m is exposed to a beam of plane polarized x-rays in which the amplitude of the electric intensity at the electron is A, then the amplitude A' in the scattered radiation in the plane perpendicular to the electric intensity is given by

$$A' = Ae^2/rmc^2,$$

where c denotes the velocity of light and r is the distance from the electron.

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[1] W. H. Bragg, Phil. Mag., May, 1914.

[2] C. G. Darwin, Phil. Mag., Feb. and April, 1914.

[3] A. H. Compton, Phys. Rev., Jan., 1917.

[4] W. L. Bragg, R. W. James and C. H. Bosanquet, Phil. Mag., March, 1921.

[5] Bergen Davis and W. M. Stempel, Phys. Rev., May, 1921.

If instead of an electron we have an atom containing N electrons then, following W. L. Bragg, we may write

$$A' = AFe^2/rmc^2,$$

where F is a function of the angle between the incident and scattered radiation which should approach N as the angle approaches π.

Consider first the reflexion by a single layer of atoms containing n atoms per unit area. Let the incident radiation come from a point source S (Fig. 1), and let it be plane polarized with the electric intensity perpendicular to the plane of incidence. Consider the reflected amplitude at any point I. The reflected rays at I will have gone along the path SPI such that the angles SPA and IPB are equal. The area of the first Fresnel zone on the surface AB of the crystal can be easily shown to be $\pi\lambda r_1 r_2/(r_1 + r_2) \sin\theta$ where $r_1 = SP$, $r_2 = PI$, θ is the angle SPA and λ the wave-length of the incident radiation.

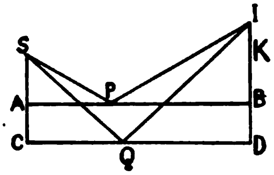

Fig. 1.

Let the amplitude in the incident spherical waves at P be a/r_1, then the amplitude at I of the reflected radiation is

$$\frac{1}{2}\frac{2}{\pi}\frac{n\pi\lambda}{\sin\theta}\frac{r_1 r_2}{r_1 + r_2}\frac{a}{r_1}\frac{F}{r_2}\frac{e^2}{mc^2} = \frac{n\lambda aFe^2}{(r_1 + r_2)\sin\theta\cdot mc^2}.$$

This is inversely as $r_1 + r_2$ as it should be, for the reflected rays appear to come from the image of S formed by the plane AB.

Suppose now that we have a series of similar layers of atoms equally spaced below AB. The vibration at I will be the resultant of the vibrations due to all the layers. Let d denote the distance between successive layers. Successive reflexions up and down will cause a single incident ray to emerge as a series of parallel rays. The calculation will be made now on the simple assumption that successive reflexions up and down can be neglected so that each plane reflects only once. It will also be assumed that the reflected rays from each plane have the same intensity although of course owing to absorption and reflexions the intensity must diminish with the depth of the planes below the surface of the crystal. The effect of these assumptions on the results obtained will be considered later.

Consider the vibration at I due to a layer CD at a distance pd below AB so that p is an integer. Let $SA = h_1$, $IB = h_2$, $AP = k_1$, $PB = k_2$ then the path SQI of rays reflected by the layer CD is longer than the path SPI by

$$\sqrt{k^2 + (h + 2pd)^2} - \sqrt{k^2 + h^2},$$

where $k = k_1 + k_2$ and $h = h_1 + h_2$ for the angles SQC and IQD are equal.

Now suppose the angle SPA or θ satisfies the equation $\lambda = 2d \sin \theta = 2dh/r$ then the difference between the paths SQI and SPI will be nearly but not exactly equal to $2pdh/r$. Let this difference be equal to $(2pdh/r) + N\lambda$ where N is a number which is small unless p is large. Then we have

$$\sqrt{k^2 + (h + 2pd)^2} - \sqrt{k^2 + h^2} = \frac{2\,pdh}{r} + N\lambda,$$

so that

$$4p^2d^2\left(1 - \frac{h^2}{r^2}\right) = 2rN\lambda + N^2\lambda^2 + 2pN\lambda^2.$$

The terms $N^2\lambda^2$ and $2pN\lambda^2$ are quite negligible compared with $2rN\lambda$ so that since $h/r = \sin \theta$,

$$4p^2d^2 \cos^2 \theta = 2rN\lambda,$$

or

$$p = \frac{1}{d \cos \theta}\sqrt{\frac{rN\lambda}{2}}.$$

In calculating the resultant vibration at I path differences equal to a multiple of λ may be neglected so that the vibration at I due to the layers for which p is between p and $p + \delta p$ may be taken equal to

$$\frac{an\lambda Fe^2}{rmc^2 \sin \theta} \cos \frac{2\pi}{\lambda}(ct - N\lambda)\delta p,$$

since the change in N when p is increased by unity is clearly very small. Hence the resultant vibration at I is

$$\frac{an\lambda Fe^2}{rmc^2 \sin \theta}\int_0^\infty \cos \frac{2\pi}{\lambda}(ct - N\lambda)\delta p$$

$$= \frac{an\lambda Fe^2}{rmc^2 \sin \theta}\int_0^\infty \left[\cos 2\pi N \cos \frac{2\pi}{\lambda} ct + \sin 2\pi N \sin \frac{2\pi}{\lambda} ct\right]\delta p.$$

Substituting for p in terms of N and putting $N = x^2/4$ we get for the amplitude at I

$$\frac{an\lambda Fe^2}{rmc^2 \sin \theta}\,\frac{1}{d \cos \theta}\sqrt{\frac{r\lambda}{2}\,\frac{1}{2}}\left\{\left[\int_0^\infty \cos \frac{\pi x^2}{2}\,dx\right]^2 + \left[\int_0^\infty \sin \frac{\pi x^2}{2}\,dx\right]^2\right\}^{1/2}$$

$$= \frac{an\lambda Fe^2\sqrt{r\lambda}}{4rmc^2 \sin \theta \cdot d \cos \theta} = \frac{an\lambda^{1/2} Fe^2}{2mc^2 r^{1/2} \cos \theta}.$$

Thus it appears that the amplitude at I varies inversely as the square root of the length of the path from the point source. This shows that the reflected rays at I form a parallel beam in the plane of incidence. This is clear also from elementary considerations for it is only when θ is extremely close to the value given by $\lambda = 2d \sin \theta$ that there is appreciable reflexion from a series of planes so that the reflected rays must proceed as in Fig. 2. The reflected rays are parallel in the plane of incidence but diverge in the perpendicular plane so that the amplitude varies as $r^{-1/2}$ and the intensity as r^{-1}.

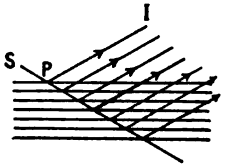

Fig. 2.

The intensity at I is due to rays which have passed through only a very small thickness of the crystal so that the result obtained for the amplitude at I will be approximately correct in spite of the simplifying assumptions made that each plane reflects only once and that the amount reflected from each plane is the same.

Consider now the amplitude at K (Fig. 1) on the perpendicular from I to the crystal surface. Denote IK by z. The amplitude at K, if the same simplifying assumptions are made, will be double that at I provided IK is big enough for K to be outside the diffraction fringes at the upper surface of the reflected beam. For at K in this case we get reflected beams from each layer of atoms for which p varies practically from $-\infty$ to $+\infty$ instead of from 0 to ∞ as at I. The distribution of intensity along IK will be the same as that in the shadow of a straight edge so that the reflected energy may be calculated by assuming the intensity to be constant along IK below I and zero above I and then allowing for absorption and the effect of successive reflexions. The radiation at K is reflected in the crystal at a depth $\frac{1}{2}IK$ below the surface so that it has passed through a thickness $IK/\sin\theta$.

It is easy to see that the intensity at K will fall off as z increases approximately exponentially even when all the possible reflexions up and down are taken into account. The effect of reflexions will be to cause the intensity at K to fall off less slowly as z increases than it falls off on the assumption that each plane reflects only once. We may therefore suppose that the intensity is proportional to $\epsilon^{-\mu z/\sin\theta}$ where μ is the apparent coefficient of absorption giving the diminution of intensity due to true absorption and to reflexions from the planes. Hence the total energy reflected per unit time in a length w perpendicular to the plane of incidence is

$$c \cos\theta \cdot w \left(\frac{an\lambda^{1/2}Fe^2}{mc^2r^{1/2}\cos\theta}\right)^2 \int_0^\infty \epsilon^{-\mu z/\sin\theta}\,dz = \frac{wa^2n^2\lambda F^2e^4 \sin\theta}{\mu m^2c^3 \cos\theta \cdot r},$$

since the energy flow is parallel to PI and so equal to $c \cos\theta$ times the square of the amplitude per unit area of the plane IK.

Consider the energy reflected between two planes meeting on the perpendicular from S to the surface of the crystal having an angle ϕ between them so that $w = \phi r \cos\theta$ and the reflected energy E_r is given by

$$E_r = \frac{a^2n^2\lambda F^2e^4\phi \sin\theta}{m^2c^3\mu},$$

which is independent of r, as it should be. If we put $\mu = 0$ then E_r becomes infinite. This is because the vertical width of the reflected beam becomes infinite. The intensity at any point remains finite.

Now consider the incident energy in the angle ϕ and between glancing angles θ_1 and θ_2 such that $\theta_1 > \theta > \theta_2$ and let $\theta_1 - \theta_2 = \psi$. The incident energy per unit time is then

$$E_i = c\frac{a^2}{r^2} r \cos\theta \cdot \phi \cdot r\psi,$$
$$= ca^2\psi\phi \cos\theta.$$

Hence

$$\frac{E_r}{E_i} = \frac{n^2\lambda F^2e^4 \sin\theta}{m^2c^4\mu\psi \cos\theta}.$$

Let N denote the number of atoms per unit volume so that $n = Nd$ which with $\lambda = 2d \sin\theta$ gives

$$\psi\frac{E_r}{E_i} = \frac{N^2\lambda^3F^2e^4}{2m^2c^4\mu \sin 2\theta}.$$

When this is multiplied by the polarization factor $\frac{1}{2}(1 + \cos^2 2\theta)$ and the Debye temperature factor $\epsilon^{-B\sin^2\theta}$ it agrees with the results obtained by Darwin, A. H. Compton and W. L. Bragg. Darwin and W. L. Bragg got it for an imperfect crystal only. In the case of a perfect crystal it appears that μ should be greater than the true absorption coefficient.

So far only the intensity at I and at K when K is not very near to I have been calculated. It is easy to calculate the intensity for any position of K if we make the same simplifying assumptions as before. The plane which reflects to K exactly at the angle θ is at a depth $z/2$ below the surface of the crystal and the length of the path from S to K of the ray reflected from this plane is the same for all values of z. The quantity p may therefore be taken to vary from 0 to ∞ for the part of the crystal below this plane and from 0 to $-z/2d$ for the part above it. The vibration at K is therefore, when the diminution of intensity due to absorption and reflexion is neglected

$$\frac{an\lambda Fe^2}{rmc^2 \sin\theta} \int_{-z/2d}^{\infty} \cos\frac{2\pi}{\lambda}(ct - N\lambda)\delta p,$$

so that the amplitude at K is equal to

$$\frac{an\lambda^{1/2} Fe^2}{\sqrt{2}\, mc^2\, r^{1/2} \cos\theta} \left\{ \left[\int_{-\beta}^{\infty} \cos\frac{\pi x^2}{2}\, dx \right]^2 + \left[\int_{-\beta}^{\infty} \sin\frac{\pi x^2}{2}\, dx \right]^2 \right\}^{1/2},$$

where

$$\beta = z \cos\theta \cdot \sqrt{\frac{2}{r\lambda}}$$

just as in the diffraction fringes near the shadow of a straight edge in parallel light. This agrees with the previous conclusion that the reflected rays form a parallel beam in the plane of incidence.

The greatest intensity occurs almost exactly when $\beta = \sqrt{3/2}$ so that $z \cos\theta = \sqrt{3r\lambda/4}$. If $r = 40$ cm. and $\lambda = 2 \times 10^{-8}$ cm. then this gives $z \cos\theta = 8 \times 10^{-4}$ cm. The second maximum occurs when $\beta = \sqrt{7/2}$ so that $z \cos\theta = 12 \times 10^{-4}$ cm. the distance between the first two maxima is therefore 4×10^{-4} cm., which is big enough to observe with a microscope. If z is greater than say 3×10^{-3} cm. the fluctuations in the amplitude become small and the amplitude may be taken to be twice that at I multiplied, of course, by the absorption factor.

Rice Institute,
Houston, Texas,
May 24, 1921.

THE EFFECT OF THE AGE AND THE CONCENTRATION OF A SOLUTION OF RHODAMINE-B UPON THE PHOTOELECTRIC CURRENT DEVELOPED.

By Ruth Yeaton Jenkins.

Synopsis.

Study of Rhodamine-B Photoactive Electrolytic Cell.—Two identical semi-transparent films of Pt, both sputtered on one piece of glass and baked at 500° C., were used as electrodes of a cell into which various solutions of rhodamine-B in alcohol were successively introduced. For a fresh solution, the photoelectric current developed by illuminating one electrode was found to increase with time to a steady value. The sensitiveness was decreased by exposure to room light, but recovered in the dark. With increasing concentration the sensitiveness increased to a maximum for 3 per cent. and then decreased. The current-time curves obtained with intermittent illumination vary in form with the concentration, showing saturation for low concentrations, reaching maxima for concentrations of 1 to 3 per cent. and increasing indefinitely for higher concentrations. These and other observations show that the phenomenon is quite complicated.

Relative Conductivity of Solutions of Rhodamine-B in Alcohol was measured for concentrations up to 8 per cent.

Effect of a Flowing Electrolyte upon the Potentials of the Electrodes for a Solution of Rhodamine-B in Alcohol.—The Pt electrode against which the electrolyte flowed became negative with respect to the other, the effect being proportional to the rate of flow.

If one takes a cell consisting of two platinum electrodes in a solution of certain fluorescent substances, such as eosin, fluorescein, rhodamine, etc., and keeps one electrode in the dark, illuminating the other through the solution, or, as has proved more satisfactory, illuminating the solution through a semi-transparent electrode sputtered on the inside of the cell wall, an electromotive force is developed, such that the photoelectric current tends to flow from the illuminated to the dark electrode, in the external circuit.

As a minor part of his investigation of cells of this type, A. Goldmann[1] tried the effect of the concentration of an alcoholic solution of Rhodamine-B upon the current obtained. He started with a saturated solution and repeatedly diluted it to half its previous strength, finding that, with a fixed strength of illumination, the current increased continuously with the concentration.[1]

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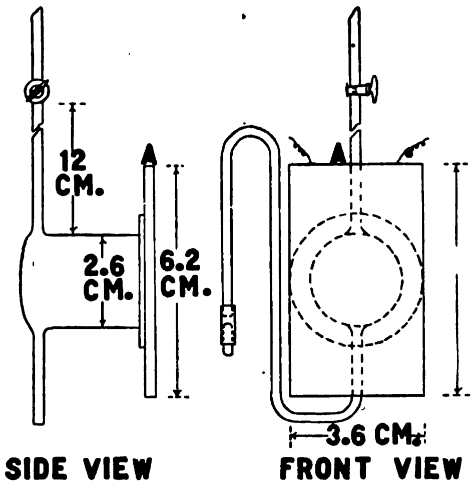
[1] A. Goldmann, Lichtelectrische Untersuchung an Farbstoffzellen, Annalen der Physik, Vol. 27, p. 449, 1908.

Concentration	1	1/2	1/4	1/8	1/16	1/32	1/64
Current	100	70	35	10	5	2	0

C. C. Murdock[2] experimenting with a 6 per cent. solution of the same dye, found that his results could be reproduced accurately only as long as he confined the illumination of the cell to relatively short periods of time. The limiting time of illumination for which results could be reproduced was not determined.

As Goldmann also spoke of a "fatigue" effect which caused a decrease in sensitiveness when the solution had been exposed to very intense light, or had been illuminated for a long time, it was thought advisable to repeat that portion of his investigation which dealt with concentration, since it seemed possible that, by the time the lower concentrations were reached, this "fatigue" had reduced the sensitiveness of his solution.

FIGURE 1



SIDE VIEW FRONT VIEW

Fig. 1 shows front and side elevations of the cell used. One end of a short piece of one inch glass tubing was closed and small tubes sealed in at the opposite ends of a diameter near this end. One of these tubes contained a stopcock and the other was bent as shown in the front view. They were used for the purpose of filling and emptying the cell. On the other end of the large tube there was blown a flange, the face of which was ground plain and closed with a plate, *A*. This was a piece of plate glass, 6.2 cm. long by 3.6 cm. wide, upon which there had been deposited by cathodic sputtering the semitransparent platinum films shown in Fig. 2. Each film was 6 mm. wide with a resistance of approximately 400 ohms, and the distance between the nearest edges of the two films

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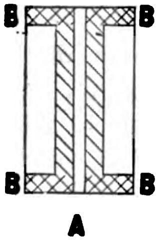
[1] Goldmann (p. 476) gave the following table, in which 1 represented a saturated solution and 100 the current obtained from it. Other concentrations and currents were given in terms of these arbitrary values.

[2] C. C. Murdock, A Study of the Photo-active Electrolytic Cell, Platinum—Rhodamine-B—Platinum, Phys. Rev., Vol. XVII., p. 626, 1921.

FIGURE 2



was 4 mm. Both films were sputtered at the same time and were therefore as nearly alike as possible. This and the small distance between them, insured a small "dark current." (The size of the dark current increases with the difference in density and with difference in temperature of the two electrodes.)[1]

It was found necessary to bake the films at about 500° C. for several hours. The heavy ends *B*, *B* were then copper-plated and the connecting wires soldered to them, thus insuring a better contact than could be obtained with clamps. The parts of the cell were then cemented together. Du Pont Household Cement, thinned with a little amyl acetate, was found very satisfactory for this purpose. Several preliminary coats were applied to the separate parts of the cell and allowed to dry thoroughly before the two pieces were finally put together. In this way it was found possible to apply the cement over the electrodes without injuring their conductivity.

One of the semitransparent electrodes was screened by black paper and the cell was placed in a light tight box fitted with a shutter. In this way the solution could be illuminated through the other film.

A Leeds and Northrup (No. 2285 High Sensitivity) galvanometer, internal resistance of 295 ohms, critical damping resistance of 150,000 ohms, sensitivity of 10,200 megohms, was so arranged that it was shunted by a little less than its critical damping resistance throughout the experiment. One terminal of the cell was connected directly to one end of the damping resistance; the other, to a movable contact on the damping resistance which permitted the sensitiveness to be varied.

The source of illumination was a 40-watt tungsten lamp, used at a constant voltage and at a constant distance from the cell. Since relative measurements only were desired, however, the transmission of the electrodes, the area exposed to the solution and the intensity of the light falling upon them were not determined. A water cell with glass sides 2.5 mm. thick, inside width of 11 mm. and large enough completely to cover the shutter opening, was put between the lamp and the photoelectric cell to stop any heat radiation from the lamp.[1]

In order to have the previous history of all solutions as nearly alike as possible, Goldmann's method of diluting a concentrated solution was not followed. Instead the 95 per cent. alcohol and the dye were mixed in the desired proportions at a definite time before each solution was to be used (about one and one half days). All solutions were kept in clear glass bottles, exposed to the general light of the room, but not to direct sunlight. Later experiments tend to show that these conditions were not sufficiently uniform for very exact work, inasmuch as there was no way to regulate the light in the room with varying weather conditions. Variations caused by such irregularities would not, however, change the general form of the results.

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[1] During the readings on the first solution the water cell was not used and the lamp was at a correspondingly greater distance from the photoelectric cell.

The solution under observation was placed in the cell and illuminated for intervals increasing by 18 or 27 second steps from 18 seconds to 9 minutes. For the measurement of time, a chronograph drum was arranged so that it sounded a signal once every revolution. The interval between these signals was very constant and approximately equal to 9 seconds. The time between periods of illumination was, at first, only long enough for the galvanometer to return to its undeflected position, about 3 min. for short exposures and 6 min. for longer ones. This interval was later made four times the length of the previous exposure. Galvanometer readings were taken once every 9 seconds for both the growth and the decay of the current developed. Since, however, the decay curves do not have any obvious bearing upon the points taken up in this paper they are not included.[1]

Observations on the most dilute solution were taken first and the cell rinsed with a little 98 per cent. alcohol before the next solution was drawn in. The concentration of each solution was determined at the end of the run. The curves[2] obtained by plotting galvanometer deflections as abscissae against time in revolutions of the chronograph drum as ordinates show several interesting features:

1 cm. deflection = 1.82×10^9 amp.
1 revolution = 9 seconds.

1. The photoelectric sensitiveness, instead of increasing steadily with increasing concentration as found by Goldmann, passes through a maximum value at a concentration of about 3 per cent. This is shown in Figs. 3 and 4.[3] To confirm this result and to fix more closely the position of the maximum a second set of solutions was made up and tested. In Fig. 4 the results obtained from the first set of solutions are indicated by ×; those from the second by ⊙.

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[1] Representative decay curves for each solution have been filed in the Library at Cornell University.

[2] No attempt was made to indicate the separate points plotted, as, with very few exceptions, the line drawn completely covered them.

[3] In making Fig. 4, the concentration by weight was plotted against the deflection at the end of the first 18 sec. exposure, assuming that the fatigue effects would then be negligible and that the currents would still be proportional to the true sensitiveness of the solutions.

2. Three different forms of curves were obtained. In the first, the photoelectric current increases to a constant value; in the second, it passes through a maximum; in the third, it continues to increase indefinitely. In Fig. 3, *A* and *B* are of the first type, *C* and *D* of the second, and *E* and *F* of the third. The length of time before the current maxi-

FIGURE 3

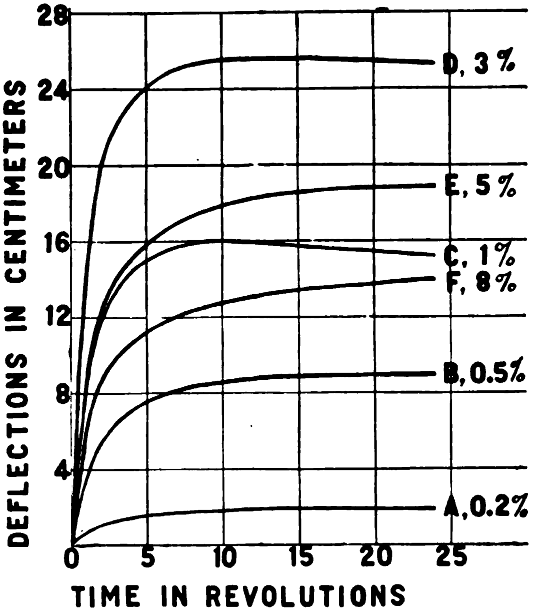
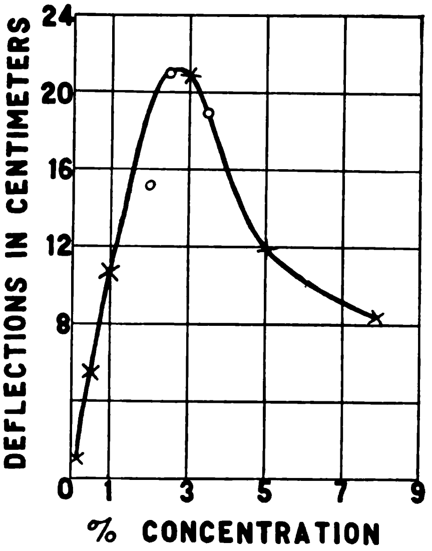


FIGURE 4



mum was reached in the curves of the first and second types varied slowly at first and then very rapidly with the change in concentration. This is shown in Fig. 5. These results are in agreement with those of C. C. Murdock who, working with a 6 per cent. solution, found the current still rising after several hours. Goldmann, on the contrary, says that in all cases the current comes to a steady value with illumination.

The current maximum exhibited by certain concentrations may be caused by a "fatigue" effect, working in opposition to the photoelectric sensitiveness of the solution. This "fatigue" might be thought of as increasing during illumination to a nearly constant value; which value is a function of the concentration of the solution. The time to reach this fairly constant value might also be a function of the concentration, increasing slowly at first and then rapidly with increased concentration. Such an effect would explain the differences in the rapidity with which the current fell, after reaching its maximum, for different solutions and the shift of the position of the current maximum.

FIG. 5

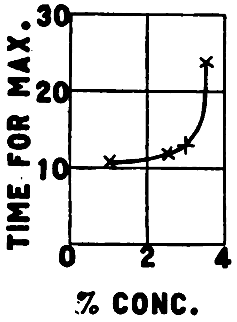
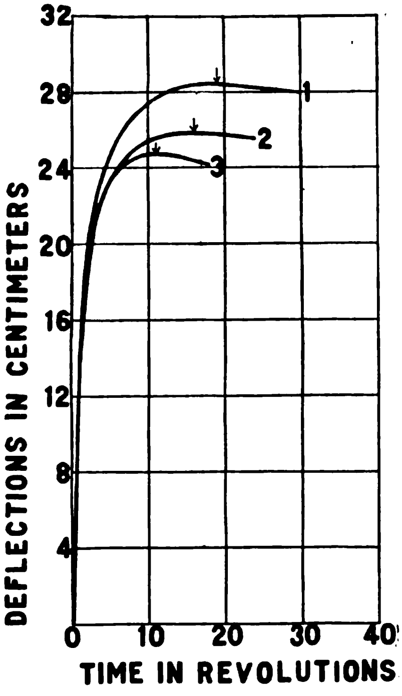


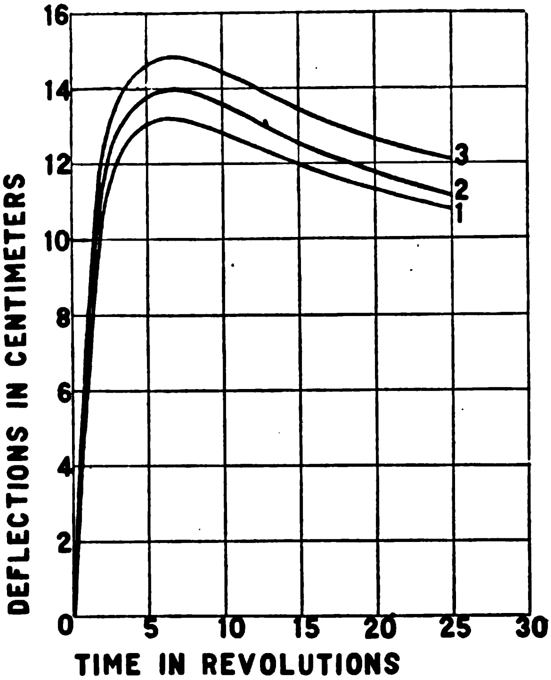
FIGURE 6



3. For a given solution, the current maximum was found to shift toward the left with the length of time the solution had stood "in the light." No such shift was observed for the four days the solution had been standing "in the dark." ("In the light" is defined as:—In a clear glass bottle, exposed to the general light of the room. "In the dark" is defined as:—In the cell on closed circuit and inside the light tight box.) (See Figs. 6 and 7.)

Parts of these same solutions gave the same type of curves after having been kept in bottles in a light tight box for six months and not exposed to the light until they were in the cell and the experimental illumination was begun. The maximum

FIGURE 7



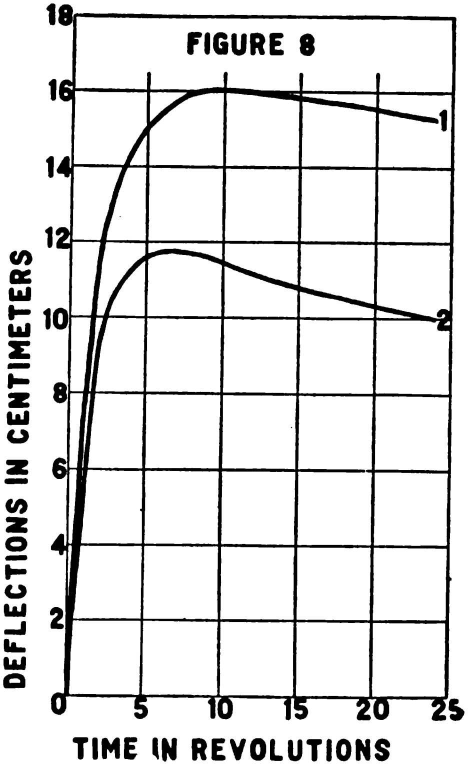


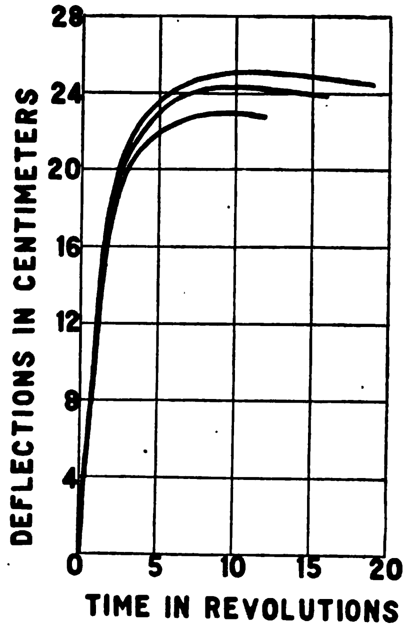
FIGURE 8

had, however, shifted quite far to the right and the sensitiveness of the solutions was only about half its former value.

4. In investigating this point, it was found that there was a marked decrease in the sensitiveness of a solution which had been left standing "in strong light" (Fig. 8) but that a solution which had recently been standing in strong daylight, increased in sensitiveness again when put "in the dark" (Fig. 7) and that this recovery was begun even during the first run made with the solution (Fig. 9). In this figure it is seen that the curves for exposures of less than 2 minutes lie directly upon one another. For longer exposures, there was a fairly steady increase in sensitiveness.

A new solution was mixed in the dark, drawn into the cell as soon as it seemed likely that solution was complete (about twenty-four hours) and its sensitiveness studied. It was found that the sensitiveness increased at a gradually decreasing rate for a few days and then reached an approximately steady value.

FIGURE 9

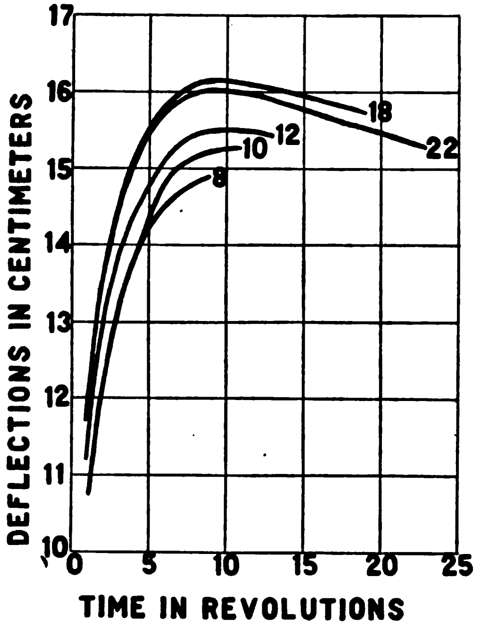


5. It was noticed that the successive curves for any solution could not be exactly superimposed upon one another, but that there was an apparent increase in sensitiveness of the solution with illumination. This was found true even when the solution had not been recently exposed to light.

Such an apparent increase in sensitiveness would occur if the molecules of the solution,—changed in some way by the action of the light,—had not had time to return to their original condition before the next exposure. In several cases this increase was gradual at first and then the sensitiveness jumped to a higher value, after which, the remaining curves were pretty well bunched together (Fig. 10). This scattering effect became less with age, both when the solution had stood "in the light" and when it was "in the dark."

When the first set of curves were taken, the cell was left in the dark only long enough for the photoelectric current to come back to its zero (dark current) value. This took from about three minutes for short exposures, to five or six minutes for longer ones, with the result that, compared to the length of exposure, the solution was in the dark much longer after short exposures than it was after the longer ones. In taking the later curves, it was made the rule to wait at least four times the length of the preceding exposure, counting from the time the shutter was closed, before illuminating the cell again. Even with this precaution, the sudden increase in sensitiveness sometimes occurred, but, on the whole, the curves tended to be more uniformly distributed.

FIGURE 10



If, for any reason, a considerable length of time, *e.g.*, three fourths hour, elapsed between two consecutive curves, the sensitiveness was found to be somewhat less.

According to Goldmann, there are at least three effects involved in producing such time-current curves:

1. The sensitiveness of the solution to light.
2. Polarization of the dark electrode, which causes a diminution of the current obtained.
3. Something called "fatigue" which causes the loss of sensitiveness with continued illumination. To these might be added whatever causes the tendency to increased sensitiveness during moderately strong illumination.

Goldmann also obtained a curve which passed through a maximum similar to curve 2, Fig. 8, but he obtained it by subjecting a saturated solution to very intense illumination. For less intense light, this same solution gave a curve of the same general shape as curve *E*, Fig. 3. He kept the concentration constant and varied the illumination, whereas I have varied the former and kept the latter constant.

Goldmann explained[1] the passing of the current through such a maximum by saying that after the potential of the illuminated electrode has reached a certain value, the losses, such as:—diffusion of negative charges through the solution to the positive electrode, the recombination of positive and negative charges, and anything which causes a spontaneous discharge of the electrode, and which increases in size with the potential of the electrode, have now become so large that they can no longer be neglected and that they cause the falling off of the current.

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[1] *Loc. cit.*, p. 510.

Until some means is found for eliminating some of these components while the others are studied, it seems rather hopeless to try to estimate the relative value of any one, in the composite curve obtained.

Before any solution was taken from the cell, its electrolytic resistance R_e was measured by the high frequency alternating current and telephone method. The reciprocal of the resistance of the cell filled with alcohol alone, R_a, was subtracted from the reciprocals of the values so obtained and the result divided by the concentration in per cent. This quantity, which is proportional to the per cent. ionization of the solutions, was plotted against the concentration in per cent. The form of the curve (Fig. 11) shows that Rhodamine-B in alcohol is a strong electrolyte.

FIGURE 11

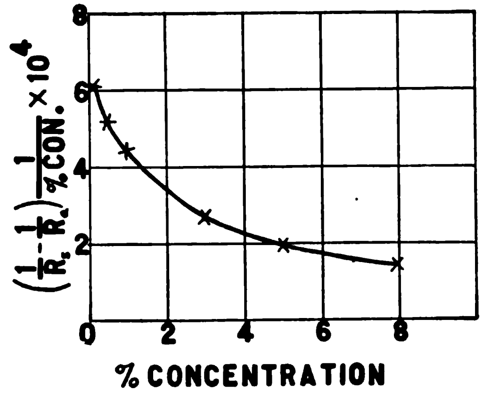

TABLE I.

Concentration in Per Cent. by Weight.	R_e in Ohms.	$1/R_e$	$(1/R_e - 1/R_a)1$/Per Cent. Concentration.
0.000	170,430	0.0574×10^{-4}	
0.224	7,735	1.292 "	6.17×10^{-4}
0.483	3,762	2.658 "	5.19 "
0.980	2,247	4.450 "	4.39 "
2.880	1,224	8.17 "	2.70 "
4.984	994	10.06 "	2.01 "
7.890	863	11.59 "	1.45 "

THE EFFECT OF FLOW OF LIQUID AGAINST ELECTRODES.

A slight variation in the regular procedure was made in an attempt to determine the effect on the photoelectric current when a constant supply of fresh electrolyte was kept moving past the electrodes. For this purpose, the cell was connected to two reservoirs in such a way that the liquid could be forced through it in either direction and at any desired speed.

It was found that, when the liquid was forced through the cell from the upper tank to the lower one, the galvanometer was consistently deflected to the left, whereas, when the direction of flow was reversed, the deflection also reversed. In both cases the magnitude of the deflection was roughly proportional to the rate of flow, although it was always larger when the liquid was drawn up through the cell than when it was forced down. In all cases the deflection lasted only as long as the liquid was in motion. When the flow stopped, the current fell rapidly to zero.

A further study of the liquid currents through the cell in the two cases, led to the conclusion that there was a selective flow of electrolyte against the electrodes in each case, the "in" current flowing more strongly against the one than the "out" current against the other. This might account for the consistent difference in the size of the electrical effects observed.

Apparently, then, a stream of electrolyte against an electrode causes it temporarily to become more negative, irrespective of its previous charge. The negativeness is a function of the force with which the liquid strikes the electrode. The stream has the same effect upon the dark current that substituting a thinner electrode film for the bombarded electrode would have. The effect was the same when superimposed upon a small but steady photoelectric current.

PHYSICAL LABORATORY,
CORNELL UNIVERSITY.

TEMPERATURE EFFECT IN BARIUM AND STRONTIUM PHOTO-ELECTRIC CELLS.

By Theodore W. Case.

Synopsis.

Photo-electric Current from Treated Barium and Strontium Surfaces.—A metal plate mounted in an evacuated cell was outgassed by heating inductively and was then coated with barium or strontium. The large temperature effect previously reported was confirmed. At dull red heat the current from the two metals was found to be respectively 1,200 and 25 times as great as at room temperature.

IN the Physical Review for March, 1921, in a brief: "New Strontium and Barium Photo-Electric Cells" the author mentioned the fact that the photo-electric current was increased about 100 times when the metal plate upon which the metallic barium was deposited was raised in temperature to just under a red heat. This was accomplished by passing a current through the metallic plate, but previously to coating with the active material the plate had not been entirely freed of gases, with the result that the barium was oxidized and the action spoiled, but not before the one hundred time increase had been observed. The resulting contamination and gas present made it doubtful as to whether the observed increase was due to a true temperature effect or not. Since then Merritt has also observed this effect on the oxide-coated filaments of audion bulbs.[1]

Recently the construction of these cells has been refined and extremely careful precautions taken in freeing all the metallic parts of gas by raising them to a bright red heat until all traces of gas have disappeared. This has been accomplished by the induction furnace method which is ideal for this work. The metal plate is then coated with either barium or strontium as desired. It is now very simple to observe the temperature effect on the photo-electric current by inductively heating the plate from the outside of the vacuum bottle, and the cell does not spoil unless the plate is heated to such a bright red that the material is actually volatilized off the plate, in which case most of it goes over and is deposited on the opposite electrode. This occurs, however, only when the plate has been heated to a bright red.

With these arrangements the author has checked his previous finding of a large temperature effect in the case of barium and a much smaller one in the case of the strontium. For strontium, the increase is only about 25 times at a dull red heat. The action is not spoiled as it previously was, because no gas is given off the plates. When these cells are giving their maximum photo-electric current under high temperature, there is not over .4 per cent. thermo-electric current if the light be turned off the cells.

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[1] Ernest A. Merritt, Photoelectric Phenomena in Coated Filament Audion Bulbs, Phys. Rev., April, 1921.

The increase of photo-electric current is gradual with increase or decrease in temperature, as illustrated by observing the galvanometer deflection slowly decrease as the plate cools from a red heat. There are no sudden jumps in the photo-electric current curve, although it is impossible at present to tell whether the increase is directly proportional to the temperature or not until a cell has been constructed with a thermo junction on the plate so as to get an accurate temperature curve.

Case Research Laboratory,
Auburn, New York.

Second Series *December, 1921* *Vol. XVIII, No. 6*

THE

PHYSICAL REVIEW.

POSITIVE RAY ANALYSIS OF LITHIUM AND MAGNESIUM.

By A. J. Dempster.

Synopsis.

Isotopes of lithium and magnesium have been *determined by positive ray analysis*, using the apparatus and method previously described in which the positive ray spectrum is determined by measuring the charge passing through a fixed slit into a Faraday cylinder as a function of the accelerating difference of potential keeping the deflecting magnetic field constant. The source of the rays was a small cylindrical anode containing the metal to be studied, which was heated by a concentric coil and bombarded by electrons. It was found that there are two isotopes of lithium with atomic weights 6 and 7, while magnesium has three isotopes with atomic weights 24, 25 and 26. The relative proportions of the isotopes of lithium varied with the conditions. For magnesium the relative numbers were 7 : 1 : 1.

Compensation Method of Measuring Small Currents.—The current to be measured is balanced by the current through an ionization chamber, which is adjusted to equality by varying the width of the slit through which the ionizing beta rays pass.

Introduction.

THIS paper discusses further experiments with the apparatus for positive ray analysis which was described in the Physical Review for April, 1918. In that paper the method of analysis was found to have a comparatively high power of separating elements with slight differences in their atomic weights, provided a steady source of positive atoms of the elements could be devised.

The positively charged atoms are allowed to fall through a definite potential difference; a narrow bundle is separated out and bent into a semicircle by a strong magnetic field. By varying the magnetic field or the accelerating potential, the beam of rays may be made to fall on a second slit and give up their charge to an electroscope. From the equation for the velocity v acquired by a particle of mass m and charge e, in falling through a potential difference V, the equation and for the radius of curvature r in which the rays are bent by a magnetic field H;

$$\tfrac{1}{2}mv^2 = eV \quad \text{and} \quad \frac{mv^2}{r} = Hev,$$

the mass of the particle is found to be given by $m = eH^2r^2/2V$, and the molecular weight may be calculated if H, r, and V are known.

The ratio of different atomic weights may be found more simply by keeping the magnetic field constant and finding the ratio of the potentials for which the rays have the same radius of curvature. This potential is inversely proportional to the atomic weight, and having identified any element such as sodium, nitrogen, or hydrogen, other atomic weights may be found by comparing the potentials necessary to bring the different atoms on to the second slit. In the previous paper this relation was shown to still hold when the deviations in the stray magnetic field before the slit are taken into account.

SOURCE OF THE RAYS.

The chief difficulty in applying the method to new elements is in obtaining steady sources of the rays desired. The arrangement illustrated in Fig. 1 has been found to work well with lithium and magnesium. The shaded portion to the left represents the iron plates between which the strong magnetic field is produced, S is the first slit, and P is a cylindrical iron shield. The main variable accelerating field, usually 800 to 1,000 volts, acts between P and S. The charged atoms come from the anode A which is heated by a coil of wire and bombarded with electrons that have fallen through 30 to 160 volts from the coated platinum cathode C. In general P and C are at the same potential. The anode is a small iron cylinder filled with the metal; in several cases it was 6 mm. long, 3 mm. in diameter, and closed by a cap with a hole which was varied in size in different experiments. The cylinder was coated with an insulating cement, the heating coil of iron-alloy wire was slipped on, and the outside again coated. The four leads to the hot cathode and anode are brought through seals in a glass inner tube as shown. The inner tube is sealed to the outer with sealing wax and can be easily removed for renewing the anode and cathode. The figure is about one third the actual size, and in the experiments the rays travelled vertically downward into the magnetic field. Even under the best conditions it has not been found possible to keep the strength of the rays absolutely constant over more than two or three minutes.

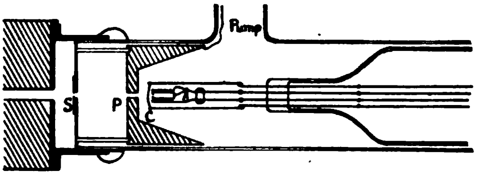

Fig. 1.

Many preliminary experiments using ribbons and rods of magnesium showed that if the anode is too much exposed to the electron bombardment, localized arcs form which are very irregular. It was also found to be unwise to depend on electron bombardment to heat the anode, as the discharge tends to become unstable. The converging electron current and the diverging vapor stream would concentrate the ionization at the anode, but it may be that under electron bombardment the positive atoms are detached from the hot surface itself in the ionized condition. The tube was exhausted with a mercury vapor pump and the pressures were lower than could be read on a McLeod gauge. It was found that no special precautions were necessary to keep mercury vapor away from the apparatus when the vapor pump was not in use, the difficulties previously attributed to mercury vapor probably being due to the warming of wax joints.

Current Measurements.

The quadrant electrometer used in the previous experiments to detect the rays was replaced by a Wilson tilted electroscope and the compensating arrangement illustrated in Fig. 2. After being deflected magnetically, the positive rays fall on the electrode through the second slit *S*, and their charge is balanced by an equal negative ionization current driven to the electrode in the ionization chamber at the right. The electroscope indicates quickly when the two currents exactly balance. *K* is a grounding key. The novel feature of the arrangement is the method used for adjusting the ionization current to balance the positive ray current. The ionizing agent is a group of a dozen old radium emanation tubes that had been discarded for medical work and emit strong β rays from RaE. They are placed in a side tube as shown, and cut off from the ionization chamber by an adjustable graduated slit, which was made from a micrometer eyepiece by replacing the cross hairs by the two halves of a slit. A calibration curve was drawn connecting the ionization current and the slit width and was found to be approximately a straight line over a considerable range. The slit scale is graduated in half millimeter divisions up to 10 mm. width and one hundredth of a division can be read on a drum attached to the screw. A slit width of one millimeter gives an ionization current of approximately 6×10^{-12} amperes. Settings for balance can be made in about 5 seconds to within 1/40 mm. Apart from the great range, convenience and rapidity of the measurement, the arrangement has the advantage of an almost continuous control of the steadiness of the positive ray current during the measurements.

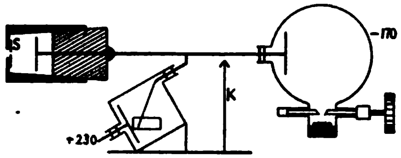

Fig. 2.

POTENTIAL MEASUREMENTS.

The potential through which the charged atoms fall was obtained from small lead cells. It was measured for groups of the cells with a high resistance voltmeter and occasionally checked by a standard voltmeter connected in a potentiometer arrangement as shown in Fig. 3. *A* and *B* are two slide wire resistances connected in series to a large storage battery of about 180 volts. The double pole switch *S* is thrown to the left and the sliders adjusted for zero voltmeter reading. The switch is then thrown to the right and the voltmeter indicates the potential of the cells on open circuit. *R* is a resistance equal to that of the voltmeter; it is connected in parallel with the main current when no current flows in the voltmeter. It was found that the readings with the high resistance voltmeter were low by about 1.3 per cent. even when the cells were fully charged.

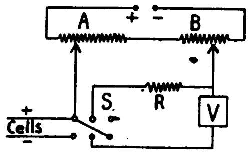

Fig. 3.

As discussed in the previous paper the potential through which the charged atoms fall is, with high vacua, equal to the total potential applied between the anode and the slit. While the rays all fall through the main accelerating field between *P* and *S*, it might be thought that the charged atoms could be formed at various points of the auxiliary field *CA*. When the pressure was not very low broad gas lines were observed, possibly partly due to this cause, but when the vacuum was good the sharpness of the curves obtained shows that the atoms have all fallen through the same potential. That this potential is the total applied, indicating that the rays are formed at the surface of the anode, is shown by the fact that the total potential required for the maximum in the curves is independent of the part used as auxiliary potential between cathode and anode, and also by the fact that the ratio of the atomic weights of known lines agrees with the ratio of the total potentials applied to make the bundles fall on the second slit.

EXPERIMENTS WITH LITHIUM.

Strong lithium rays appeared when the anode was heated to a dull red temperature. On all occasions two atomic weights appeared simultaneously, the lighter much weaker than the other. The values of the atomic weights were calculated from the magnetic field determinations to be in the neighborhood of 6 and 7, and these values were verified by comparison with hydrogen atoms, the voltage ratios for the maximum currents being, $968/138 = 7.02$, and $968/162 = 5.98$, with a possible error of 1 per cent. It is possible that this ratio may be obtained accurately enough to distinguish, as already done by Aston, between an atomic weight of 1,000 and 1,008 for hydrogen on the assumption of 6 and 7 for lithium. In a letter to Nature of Feb. 24, 1921, G. P. Thomson and F. W. Aston state that they obtained two isotopes using anodes of heated lithium salts, with the method of analysis developed by F. W. Aston. In the Proceedings of the Cambridge Philosophical Society, Vol. 20, p. 210, 1920, G. W. Thomson describes experiments with the method of Sir J. J. Thomson designed to detect any isotopes of lithium.

A curve giving the positive ray current in terms of atomic weights is given in Fig. 4. Keeping the magnetic field constant at a certain value, a potential of 969 volts between the anode and the slit gave a maximum for the element of atomic weight 6, and the abscissæ in the curve plotted as atomic weights are obtained by dividing 6 x 969 by the potential through which the rays fall in each case. The potential between the cathode and the anode was 96 volts and between the iron screen and the anode was 128 volts. The ordinates of the element at 6 are multiplied by 5 in plotting, so that in this case the ratio of the amount of the two was about 7 to 1. The width of the curves may be attributed entirely to the slits (which were .58 mms. and .75 mms. wide). In the previous paper it was shown that (with slits of equal width) the curve to be expected as the rays are moved over the second slit by increasing the potential is a linear increase to a maximum and then a linear decrease. The width of the curve half way to the maximum is given by $(M \cdot S)/r$ where M is the atomic weight, S the slit width and r the radius of curvature (= 5 cm.). A slit width of 0.6 mm. would thus account for the width found. The difference in the two slits would lead to a slight broadening of the maximum.

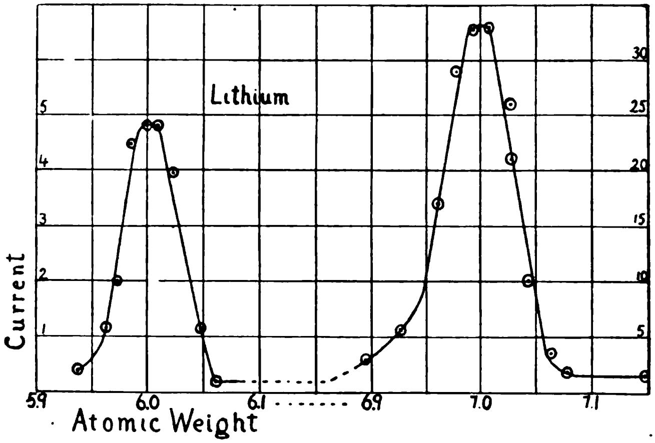

Fig. 4.

That the two components both belong to lithium is shown by the fact that they always appear together and change in intensity in the same way. No other components were found, and at 5, 8 and 9, the intensity was certainly less than 2 per cent. of that at 7. No line at 12 was observed that might suggest that 6 was due to doubly charged carbon.

Many observations were made of the voltages required for the maxima of the two lithium lines. These could be obtained within one or two volts without measuring the complete curves. Assuming the maximum of the curve to represent the position of the atomic weight, and the lighter component to have an atomic weight of exactly 6, the atomic weight of the heavier component was 7.00 within two, or under good conditions, within one unit in the second decimal place. These observations show that to this accuracy the two elements are one unit apart in atomic weight.

Proportion of the Two Components of Lithium.

If we assume the chemical atomic weight of lithium 6.94 to be the mean of the atomic weights of the two components, we should expect the number of atoms of the heavier component to be 16 times as great as the number of the lighter, and it is of interest to see if the ratio of the strengths of the positive rays agrees with the ratio of the two components in the metal. It has been found that this is in general not the case. Observations were made by comparing the intensities of the maxima of the two components under steady conditions. With one anode a ratio was found at first of 37 to 1, much greater than that to be expected. (In a letter to Science for April 15, 1921, it was overlooked that in a few cases a ratio greater than that to be expected had been obtained.) Later the ratio was 18 to 1, with a variation in several readings of less than 10 per cent. Three days later the ratio was 7.5 to 1 with a variation in six readings of less than 10 per cent. With a new anode, the ratio was at first 4.8 to 1 much less than with the other anode, and increased after a few minutes to 7 and 8 to 1. The next day the ratio was between 7 and 10 to 1. No dependence of the ratio on the strength of the rays could be observed over a small range of variation. A possible explanation is that the rates of evaporation of the two isotopes vary differently with the surface conditions of the metal. An assumption on theoretical grounds[1] of a difference in the vapor pressure curve for the two isotopes would still require an explanation of the relative variations in the amounts evaporating on different occasions. Surface films on the molten metal are possible causes of these differences; also the evaporation in these experiments occurred under electron bombardment, and it may be that this influenced the conditions of evaporation. This variability makes impossible at present any determination of the chemical atomic weight of lithium from positive ray analysis.

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[1] F. A. Lindemann and F. W. Aston, Phil. Mag., 37, p. 23, 1919. F. A. Lindemann. Phil. Mag., 38, p. 173, 1919.

It was observed that strong hydrogen rays were obtained from the lithium while it was being heated. It is of interest that the hydrogen all appeared in the form of atoms. No H_2, H_3 or helium rays as strong as 1 per cent. of the H_1 rays could be detected. The hydrogen atoms were obtained from new anodes on their first heating, and also from anodes, from which all the occluded gases had been driven off, if they had been allowed to stand in air for a day, the fresh hydrogen probably having been formed from traces of moisture.

Experiments with Magnesium.

Magnesium rays were obtained with anodes of two types. In the first experiments the heating coil was wound directly on a rod of magnesium, and the whole covered with a glass tube having a hole 3 mm. in diameter between the magnesium and the cathode to restrict the electron bombardment. Later an anode as shown in Fig. 1 was used. With both types three components with atomic weights 24, 25 and 26 were found. They appeared together as the magnesium was heated so as to vaporize slightly, and their atomic weights could be checked by com-

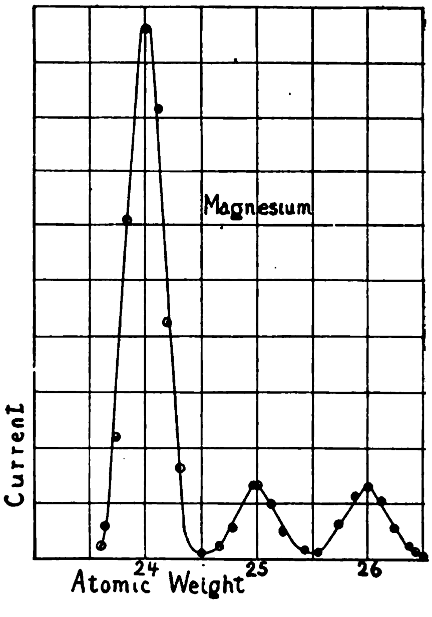

Fig. 5.

parison with rays at 28, probably due to occluded nitrogen, which is driven off before the magnesium rays appear. Two curves made with the first type of anode are given in a paper in the Proceedings of the National Academy of Sciences for February, 1921. They show that the lighter of the three components has an atomic weight 24, within one tenth of a unit, and that the other components have atomic weights 25 and 26. The curve in Fig. 5 was made from observations with the second type of anode and shows the ionization current corresponding to different atomic weights. It is drawn with the maximum of the lighter component exactly at 24, although its position was not compared in this case with the gas line at 28 with the accuracy indicated in the curve. The auxiliary potential between cathode and anode was 46 volts and between the screen and cathode 6 volts. The atomic weight abscissae are obtained by dividing 22440 by the total potential in each case. A slit width of 0.8 mm. would account for the width of the curves.

The ratios of the components shown in the curve were observed in several measurements under steady conditions. The component at 24 is about 6.7 times as strong as the one at 25, while the latter is about 1.04 as strong as the one at 26. These ratios give a mean atomic weight of times 24.336 which agrees well with the chemical atomic weight of 24.36. Although no such variations as in the case of lithium have been observed in the proportions of the magnesium components, yet several reliable comparisons have given slightly different ratios from that shown; in some cases the component at 24 was even ten times as strong as that at 25. Thus no great reliance can be placed as yet on determinations of the ratios of the various kinds of magnesium present in the metal, from observations of the ratios of the evaporating ionized atoms.

In preliminary experiments, rays of boron, aluminium and silicon have been obtained, and it is hoped that they may be made suitable for measurement. The bea-ing of these and other experimental results on general theories and speculations as to the atomatic structure is not entered into here, as the matter has been discussed at length by other authors in several recent papers.[1]

The writer wishes to express his thanks to Miss A. Hepburn for the radium emanation tubes used in the measurements, and to Mr. N. Y. Priessman for assistance in calibrating the ionization chamber.

RYERSON PHYSICAL LABORATORY,
June 2, 1921.

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[1] Especially by W. D. Harkins, PHYSICAL REVIEW, Feb., 1920, Phil. Mag., Sept., 1921.

DURATION OF IMPACT OF BARS.

By Erwin W. Tschudi.

Synopsis.

Duration of Impact of Two Spheres.—The Hertz theory was verified by *experiments* with brass and cast-iron spheres of 3.8 and 7.6 cm. diameter respectively, using a condenser discharge method of measuring short time intervals which gave accurate results for intervals of the order of 10^{-4} second.

Duration of Impact of Bars.—*Experiments* with hot-rolled steel bars, 2.86 cm. in diameter and 16 to 62 cm. long, prove that the compressional wave theory is inadequate since the duration of impact was found to be a function of the initial velocity and to be proportional not to the first power of the length of the shorter bar but more nearly to the two-fifths power. Moreover, the fundamental assumption of the compressional wave theory was shown to be wrong by means of a model in which the wave velocity was slow enough to be observed. *A theoretical expression* was then derived on the basis of certain assumptions as to the distribution of pressure and its variation with the displacement of the center of mass of the colliding body, which agrees in general with the experimental results.

Introduction.

IMPACT is usually treated by considerations based upon the momenta of the two bodies before and after collision. The force and duration of the impact, however, have been determined by two theories, much contrasted, which are not based on momentum.

The first of these two theories states that the colliding bodies, in the shape of bars, are separated by a wave of compression which travels throughout both bodies during the first half of the impact, is reflected as a wave of tension, and pulls them apart.[1] This theory assumes that the "pressure wave" is propagated approximately with uniform intensity, and is therefore unable to explain the fact that a steel bar hammered upon on one end becomes bruised at this point and spreads out, while the remainder of the bar stays unaffected.

The second theory, due to Hertz,[2] considers the effect of collision of bodies bounded by curved surfaces to be purely local, so that the greatest compression occurs in the neighborhood of the surface common to the two colliding bodies. Here the pressure gradually increases until the bodies are brought to rest and then decreases until they are separated again.[3] For the case of equal spheres of the same material, this theory gives for the duration of impact

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[1] Thomson and Tait's Treatise on Natural Philosophy, part 2, pp. 228–229 clearly describes this action.

[2] Miscellaneous Papers, p. 146, 1896.

$$\tau = 2.9432 \left[\frac{25\pi^2}{8}(1 - \sigma^2)\left(\frac{\rho}{E}\right)^2\right]^{1/5} \frac{r}{v^{1/5}}, \tag{1}$$

where σ is Poisson's ratio, ρ the density, E Young's modulus and r the radius of the spheres. The initial velocity when impact occurs is v. This theory cannot be adapted to the collision of bars.

Equation (1) has been verified[4] by Hamburg in 1886 with brass and steel spheres. His experimental results agreed very closely with Hertz's theoretical conclusions.

APPARATUS.

In order to measure the duration of impact, and thus show that the results predicted by the compressional wave theory are erroneous, an apparatus was devised in which the colliding bodies were two spheres hung by bifilar suspensions—replaced later in the experiment by cylinders with machined ends suspended horizontally by two bifilars each—from knife edge supports. Electrical connections were made so that the circuit of a standard condenser with a capacity of one micro-farad was completed by contact of the two spheres. (See Fig. 1.)

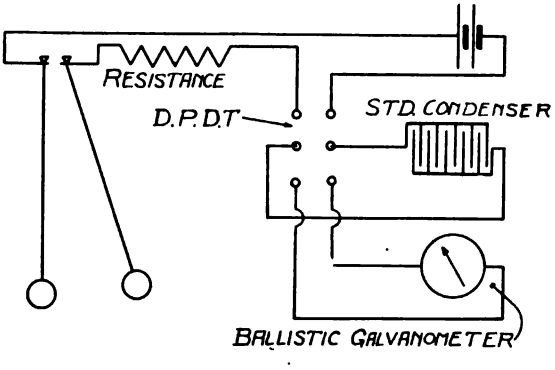

Fig. 1.

One of the spheres was drawn aside to a measured height and held in place by a stop which could be released by an electromagnet. After the falling sphere had collided once with the stationary one, the charging circuit of the condenser was broken and joined to a galvanometer circuit with the D.P.D.T. switch. This discharge of the condenser gave a galvanometer throw which afforded a measure of the duration of impact.

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[3] Love's Treatise on the Theory of Elasticity, v. 2, p. 148, ed. 1892, gives a complete mathematical treatment of this theory.

[4] Winkelmann's Handbuch der Physik, v. 1, p. 304.

The charging curve of the condenser was given by the usual equation

$$t = RC \log_e \frac{1}{(1 - q/CV)}.$$

From this it is seen that the time t necessary to give to the condenser a charge q with a constant capacity and voltage varies directly as the resistance, *i.e.*,

$$t_1 = \frac{R_1}{R_2} t_2. \tag{2}$$

A curve was obtained with a resistance $R_2 = 4{,}133{,}000$ ohms in the circuit of the condenser. This curve when used in conjunction with (2), where R_1 ranged from 400 to 2,400 ohms, gave the duration of impact.

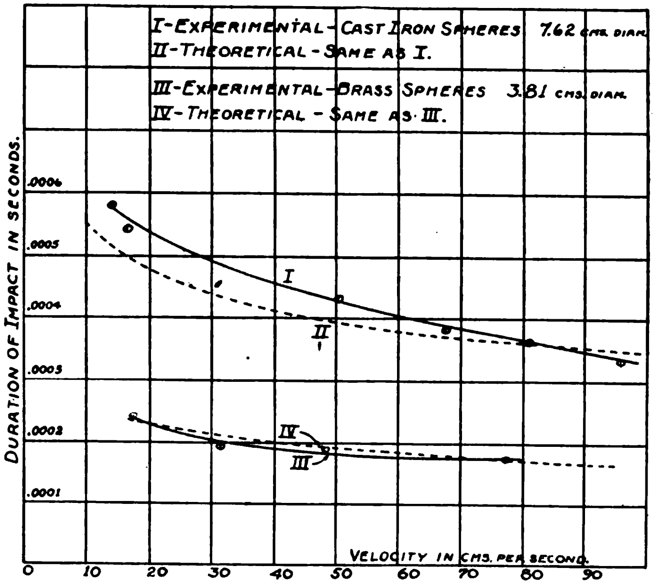

Fig. 2.

The experimental results for brass spheres of 3.81 cm. diam. and for cast iron spheres of 7.62 cm. diam. are shown by the solid curves of Fig. 2. The broken curves are the loci of (1) for these same cases. The close agreement of these experimental results with the theoretical equation which Hamburg had previously verified served to determine the accuracy of this method of measuring small periods of time.

The experimental results for three sets of cylinders of equal lengths, made of hot-rolled machinery steel, 2.86 cm. diam. and 15.90, 31.30, 61.55 cm. in length, respectively, are shown by the solid curves in Fig. 3.

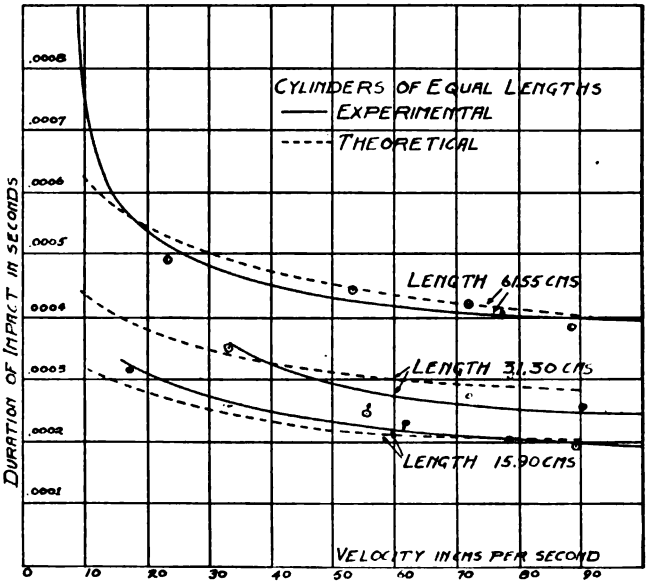

Fig. 3.

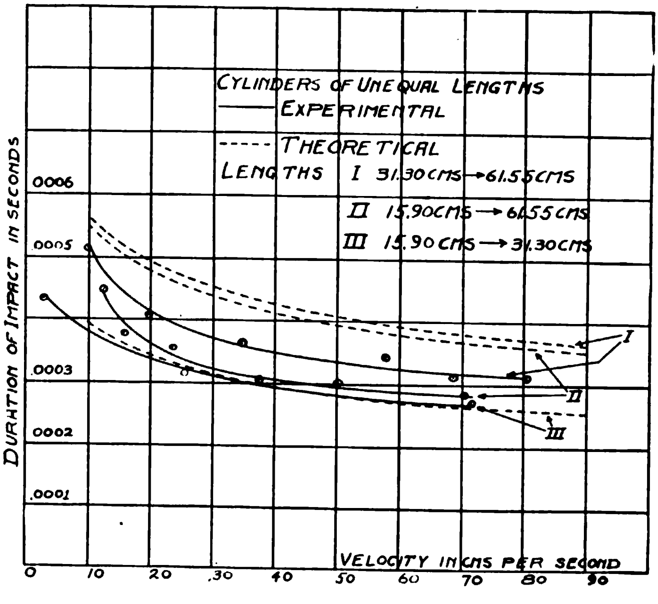

Fig. 4.

For cylinders of unequal lengths, the results are shown by the solid curves in Fig. 4. The notation 31.30 cm. → 61.55 cm. indicates that the bar of length 61.55 cm. was at rest while the one of length 31.30 cm. collided with the first bar.

If l is the length of the shorter of two colliding bars of the same material, then the duration of impact according to the compressional wave theory is

$$\tau = 2l\sqrt{\left(\frac{\rho}{E}\right)}. \tag{3}$$

The results of (3) are given in the following table:

Length in centimeters	15.90	31.30	61.55
τ in seconds	6.34×10^{-5}	12.5×10^{-6}	24.6×10^{-6}

Obviously there is no semblance of agreement between experimental results and values given by the compressional wave theory.

A low speed model of colliding bars was made with two helical springs, about a meter long and 5 cm. in diam., of no. 21 piano wire, Brown and Sharpe standard gauge. A brass disc was soldered to one end of each of the two springs in order that they could be made to collide end on. Each coil was weighted with a lead shot to lower the velocity of the wave of compression. Linen threads were attached to every third or fourth coil so that the springs could be suspended horizontally. In order that one spring might be drawn aside without distorting it, a thread was tied horizontally to its suspensions. An arrangement consisting of a buzzer circuit, which was closed by the contact of the two springs, facilitated an approximate estimate of the duration of the impact, while the progress of the wave of compression was viewed directly. It was found that only about one third of the length of each spring was traversed by the wave between the instants of collision and separation, a result quite contrary to the predictions of the wave theory.

Theory of Impact of Bars.

If a moving bar, consisting of a series of infinitesimal cylinders, were suddenly brought to rest by an impact at one end, the first cylinder to receive the blow would suffer a pressure resulting from the retardation of all the other elementary cylinders, while the last of these elements would not experience any pressure since it would have nothing to stop. With half the impact over, the elementary cylinders would be in a state similar to that of a set of weights arranged in a vertical pile upon a table. Obviously there must be a rapid dissipation of pressure along bars during impact.

Let us assume, therefore, that the pressure at a distance x, measured from the end of the bar which receives the impact, is

$$P = P_0 e^{-x}, \tag{4}$$

since dissipation functions usually contain an exponential factor. P_0 is the pressure at the origin.

Hertz has obtained the relation $P_0 = k_2 \alpha^{3/2}$, where α is the displacement of the center of mass of the colliding body toward the origin at a given instant and k_2 a constant depending upon the mass and size of the body.

For the impact of bars, a similar relation is found to be suitable of the form

$$\alpha^{5/4} = \frac{e^{nl} P_0}{ME}, \tag{5}$$

where l is the length of the bar, M and n constants to be determined empirically.

An amount of work equal to $pS/2$ is done when a bar is compressed a distance S under a uniform pressure p. If we consider λ the contraction per unit length at a distance x from the origin due to a pressure P at that point, then by Hooke's Law $P/E = \lambda$. The contraction for a length dx is then λdx and the work done in producing this contraction is

$$dW = \tfrac{1}{2}\lambda P dx = \tfrac{1}{2}\frac{P^2}{E}dx$$

$$= \tfrac{1}{2}\frac{P_0^2}{E}e^{-2x}dx,$$

and the total work done throughout the bar at any instant is

$$W = \tfrac{1}{2}\frac{P_0^2}{E}\int^{l} e^{-2x}dx = \frac{P_0^2}{4E}(1 - e^{-2l}). \tag{6}$$

The quantity e^{-2l} is very small for any appreciable value of l and can be neglected.

Let a and b be two bars approaching one another with velocities v_a, v_b and masses m_a, m_b respectively. The system of two bars will have kinetic energy of amount $\tfrac{1}{2}(m_a v_a^2 + m_b v_b^2)$ which will be gradually changed into potential energy during the first half of the impact.

If α and β are the displacements of the centers of mass of the bars a and b respectively, then from (5) we have

$$\alpha^{5/4} = \frac{e^{nl_a} P_0}{ME_a} \quad \text{and} \quad \beta^{5/4} = \frac{e^{nl_b} P_0}{ME_b}. \tag{7}$$

Substituting in (6) we have

$$W_a = \frac{\alpha^{5/2}M^2E_a}{4e^{2nl_a}} = K_a\alpha^{5/2},$$

$$W_b = \frac{\beta^{5/2}M^2E_b}{4e^{2nl_b}} = K_b\beta^{5/2}.$$

At any instant during the first half of the impact, the transformation of energy is given by

$$\frac{m_a}{2}\left[v_a^2 - \left(\frac{d\alpha}{dt}\right)^2\right] + \frac{m_b}{2}\left[v_b^2 - \left(\frac{d\beta}{dt}\right)^2\right] = K_a\alpha^{5/2} + K_b\beta^{5/2}. \qquad (8)$$

In the experiment $v_b = 0$. Eliminating P_0 from (7) we have

$$\frac{d\beta}{dt} = G^{4/5}e^{4n\,(l_b-l_a)/5}\cdot\frac{d\alpha}{dt} \quad \text{where} \quad G = \frac{E_a}{E_b}.$$

Hence (8) becomes

$$\frac{m_a v_a^2}{2} - \left[\frac{m_a}{2} + \frac{m_b}{2}G^{8/5}e^{8(l_b-l_a)/5}\right]\left[\frac{d\alpha}{dt}\right]^2 = [K_a + K_bG^2e^{2n(l_a-l_b)}]\alpha^{5/2}.$$

Substituting

$$A = \frac{m_a}{2} \qquad B = \frac{m_a}{2} + \frac{m_b}{2}G^{8/5}e^{8n(l_a-l_a)})$$

and

$$C = K_a + K_bG^2e^{2n(l_b-l_a)}$$

we have the simpler equation

$$Av^2 - B\left[\frac{d\alpha}{dt}\right]^2 = C\alpha^{5/2}. \qquad (9)$$

When the two bars have been brought to rest, *i.e.*, when $d\alpha/dt = 0$, the displacement of the center of mass of either bar will be a maximum. Let this maximum value of α be α_1. Then from

(9)
$$\alpha_1 = \left[\frac{A}{C}\right]^{2/5}v^{4/5}.$$

Integrating (9) we have

$$t = B^{1/2}\int_0^{\alpha_1}\frac{d\alpha}{[Av^2 - C\alpha^{5/2}]^{1/2}}$$

$$= \left[\frac{B}{A}\right]^{1/2}_{1/2} \frac{\alpha_1}{v} \int_0^1 \frac{d\theta}{[1 - \theta^{5/2}]^{1/2}}$$

$$= \frac{2}{5} \sqrt{(\pi)} \frac{\Gamma(^{2/5})}{\Gamma(^{9/10})} \left[\frac{B}{A}\right]^{1/2} \left[\frac{A}{C}\right]^{2/5} \frac{1}{v^{1/5}}.$$

The duration of impact is twice this value.

For cylinders of the same material, with a common radius r but different lengths l_a and l_b, the duration of impact is

$$\tau \doteq \tfrac{4}{5}\pi^{9/10} \frac{\Gamma(^{2/5})}{\Gamma(^{9/10})} \left[\frac{r^2\rho}{M^2E}\right]^{2/5} [l_a + l_b e^{8n(l_b - {}_a l)/5}]^{1/2} \frac{e^{4nla/5}}{l_a{}^{1/10}} \cdot \frac{1}{v^{1/5}}. \tag{10}$$

When $l_a = l_b$; we have

$$\tau = 4.6527 \sqrt{(2)} \left[\frac{r^2\rho}{M^2E}\right]^{2/5} l^{2/5} e^{4nl/5} \frac{1}{v^{1/5}}. \tag{11}$$

M and n were determined by substituting in (11) values for τ read from the curves in Fig. 3, for specific values of v. For example, the values of τ from the curves for $l = 61.55$ cm. and $l = 15.90$ cm., $v = 40$, were substituted. The equations thus obtained were solved simultaneously. In this manner values of M varying from 1.45 to 2.20 and of n from .0007 to .0082 were obtained. The averages of these values, namely $M = \sqrt{3}$ and $n = .0038$, were chosen.

The broken curves in Figs. 3 and 4 are the loci of (11) and (10) respectively for these values of M and n.

Conclusions.

It has been shown that the compressional wave theory is incapable of explaining the nature of reactions in bars during impact. The present experiments have proved that the duration of impact *is not a linear function of the length of the colliding bars*, a fact directly contrary to this theory. Furthermore, this time interval is a *function of the initial velocity of impact and of the dimensions of the bars*, factors not contained in the wave theory.

It gives me pleasure to acknowledge my indebtedness to Dr. R. C. Gowdy for his assistance in devising the apparatus and for his many helpful suggestions made during the course of the experiments; to Mr. P. B. Evens for his painstaking efforts in constructing the apparatus; and to Dr. H. F. Richards for his critical reading of this manuscript.

University of Cincinnati,
April, 1921.

THE BINAURAL LOCATION OF PURE TONES.

By R. V. L. Hartley and Thornton C. Fry.

Synopsis.

Theory of the Binaural Location of Pure Tones.—It is pointed out that to determine two coördinates of the sound source, angular position and distance, the simultaneous observation of both phase difference and intensity ratio for the two ears is necessary. Thus, contrary to the conclusions of certain observers, intensity ratio plays an important part in sound location. Using a method of analysis due to Stokes, the authors have prepared a series of *charts showing the intensity ratio and phase difference* for sources at various distances and angular positions emitting sound with a frequency from 310 to 1,860 cycles and for two angular separations of the ears, 165° and 180°, assumed to be placed on a spherical head. These charts show the theoretical position of the source for any combination of phase difference and relative intensity which might occur with actual sources. Experiments to test this theory are suggested. When unnatural combinations occur as a result of distortion of the sound in transmission or are produced artificially in the laboratory, the interpretation of the stimuli will depend on psychological factors and cannot be explained by any purely physical theory alone.

Psychological Readjustment of Unnatural Combinations of Binaural Stimuli.—Suggestions as to the nature of these adjustments are advanced and the results of laboratory experiments with such unnatural combinations are interpreted with the assistance of the charts and are shown to be in agreement with this theory.

1. Introduction.

In a paper on "The Function of Intensity and Phase in the Binaural Location of Pure Tones" Stewart [1] compares with theory the results of experiments which he performed to determine the effect of intensity when acting alone. He concludes that "clearly there exists neither quantitative nor qualitative agreement between the two curves (experimental and theoretical) for similar frequencies. In fact, the theory shows the existence of two values of θ for one value of relative intensity, whereas experiment shows only single values. The disagreement, quantitative and qualitative, is so very great that intensity cannot be an important factor in the location of a pure tone." The same general conclusion was also arrived at by Stewart and Hovda,[2] on the basis of less comprehensive data.

This conclusion seemed to the writers to be quite the opposite of what was to be expected in the light of their earlier work,[3] and further consideration showed that the apparent disagreement could be attributed to the current practice of considering the effects of phase and intensity separately and that the results are not inconsistent with a theory based on the combined effect of the two.

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[1] G. W. Stewart, Phys. Rev., May, 1920, p. 425.

[2] Stewart and Hovda, Psych. Rev., XXV., No. 3, May, 1918, p. 242.

[3] R. V. L. Hartley, Phys. Rev., June, 1919, p. 373.

It is the purpose of this paper to present such a theory, to apply it to published experiments, to suggest further lines of experiment and to furnish charts to facilitate such work.

2. General Theory.

We must note, first of all, that two coordinates are needed to define the position of a sound image in the horizontal plane; as, for instance, the polar coordinates angle and distance. To determine these coordinates two properties of the sound are needed. That is, the correspondence between location and stimulus is necessarily a *two to two* correspondence, and not a one to one correspondence, as most investigators in the binaural field have tacitly assumed. The assumption that any particular pair of properties is capable of determining the location of the image may be verified experimentally if it can be shown that when a stimulus is artificially set up in the laboratory, having values of these properties such as would result from a given source the observer obtains as sharp and accurate an image as if the tone actually came from that source.

A pair of properties of the stimulus which are apparently capable of determining the two coordinates of the image, are the difference in phase and the relative intensities at the two ears. To each position taken by a physical source of sound there corresponds a pair of values of these properties. No other pair can correspond to that position and only one or two positions at most can correspond to that pair.[1] There are, however, pairs of values of intensity and phase which cannot possibly arise by undistorted transmission from a single physical source. If, when such stimuli are set up by laboratory means, an image is obtained at all, it must be the result of an attempt upon the part of the observer to form a compromise between discordant data furnished by his sense organs. It follows, therefore, that experiments in which pure tones of arbitrary intensity and phase are applied to the two ears should be divided into two distinct classes according as the particular combination of intensity ratio and phase difference does or does not correspond to an actual source. The first of these, when compared with experiments on the location of actual sources, makes possible a direct test of the soundness of the above assumption, while the second can contribute to the final answer only indirectly.

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[1] There are exceptions to this statement when the frequencies are high but they are unimportant for the purposes of this argument.

In order, then, to predict theoretically the result to be expected from a laboratory experiment, we must first of all be able to determine whether or not the combination of phase and intensity relations corresponds to an actual source, and, if so, where that source is located. This may be done if we know the relations which exist between the position of an actual source and the phase difference and intensity ratio which result from it. By assuming the head to be a rigid sphere, it is possible[1] to compute the relative intensities and phases of the pressure produced at any two points on its surface, by simple harmonic sound from a point source. Conversely it is possible to find the relative position of source and obstacle when the phase and intensity relations are known.

The location obtained from any pair of phase and intensity relations depends, of course, upon the frequency of the sound and the size of the sphere. This dependence reveals itself in the final formula of Stokes's analysis by the presence of the two quantities, c, the radius of the sphere, and k, which is 2π divided by the wave-length of the sound, so that kc is the ratio of the circumference of a great circle on the sphere to the length of the sound wave. By virtue of dynamic similarity these occur in the combinations kc and r/c only, r being the distance of the source from the center of the sphere. It is, therefore, convenient to construct charts in terms of these parameters.

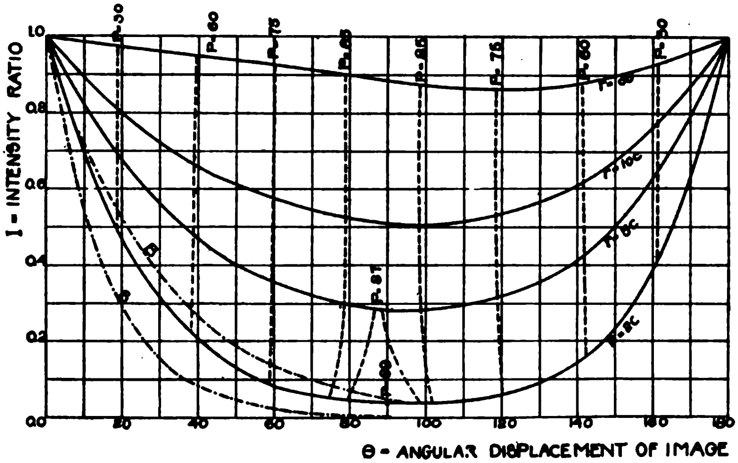

Fig. 1.

Ears 165° apart. $kc = 0.5$. Frequency 310 cycles.

Fig. 1 is such a chart drawn for the ratio $kc = 1/2$. If c is taken as 8.75 cm., which is the average obtained by measuring the heads of a number of individuals, this corresponds to a frequency of 310 cycles.

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[1] Rayleigh, Theory of Sound, Volume 2, Chapter 17. G. W. Stewart, Phys. Rev., 33, p. 467, 1911.

As measurements have also shown the ears of the average person to be about 165° apart, this separation has been used in making the computations for the chart.

The scale marked θ indicates the angular displacement of the sound source from the median plane, measured from front to back; the hammock-shaped curves correspond to various distances from source to observer; the series of approximately vertical lines indicate phase differences of various magitudes, and the scale at the left margin gives intensity ratios. Having any two of these quantities the other two may be determined. Thus it is possible to study theoretically the variation of the position of the image as the phase and intensity relations are altered. It will be observed that, as stated above, while there is an intensity ratio and phase difference corresponding to any source, there is not always a source corresponding to a pair of values of these variables.

For those experiments in which combinations are used which do correspond to actual sources the chart gives the theoretical location of the image directly.[1] Since, however, most experiments which have been reported fall into one or the other of two classes, in each of which the stimuli are of a type which could never arise from an actual source, we shall consider these two classes in some detail.

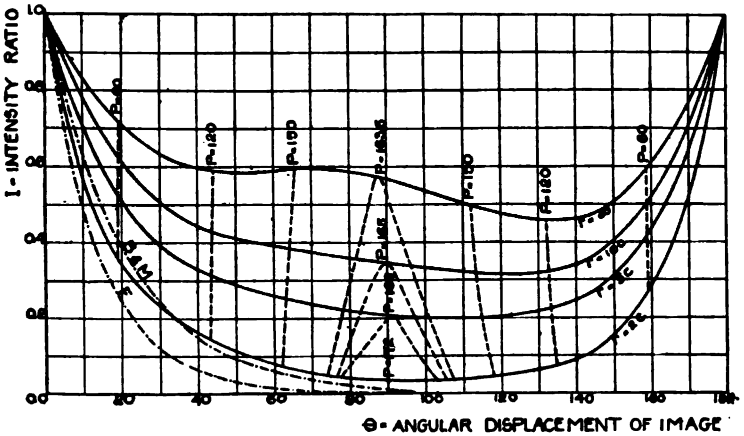

Fig. 2.

Ears 165° apart. $kc = 1.0$. Frequency 620 cycles.

Suppose that the intensity ratio is maintained at the constant value unity while the phase difference is adjusted to some value other than zero, say 60°. Since these do not correspond to any real source, it follows that purely physical considerations do not define the position of the image and that some assumption of a psychological nature is required. It seems reasonable in this connection to assume that the observer subconsciously judges either the intensity ratio or phase difference, or both, to be in error, and effects a readjustment of their values in such a manner as to obtain a combination which does correspond to a possible source. It seems logical to assume also that the observer will localize the image in that position for which the necessary readjustment is the least.

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[1] The chart shows two dotted lines for each value of P. One of these corresponds to an image in front of the observer, the other to an image behind him. Experience shows that while some observers localize the sound in one of these positions, and some in the other, it rarely, if ever, happens that both images occur simultaneously.

In the example under discussion, if the observer interprets the intensity ratio as 0.9 instead of 1.0 the two data become accordant and correspond to a *distant* image a little less than 40° from the median plane. In this case no readjustment whatever of the phase relation is necessary. On the other hand if the intensity ratio were not readjusted at all it would be

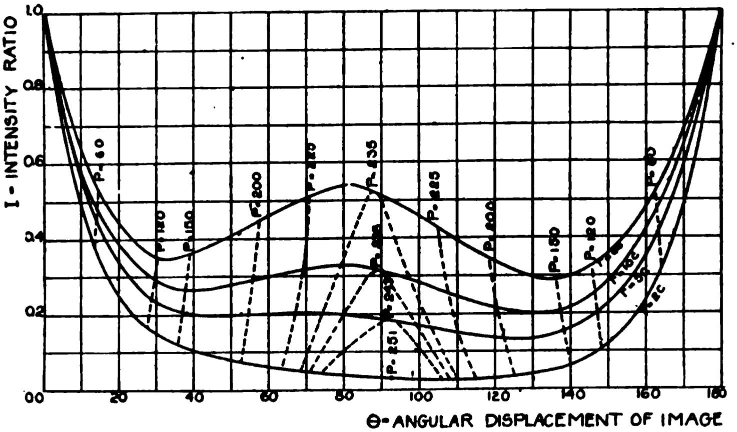

Fig. 3.

Ears 165° apart. $kc = 1.5$. Frequency 930 cycles.

necessary to conceive of the phase difference as zero instead of 60°. This is an enormous misinterpretation and it is scarcely conceivable that the observer would consent to it. Furthermore a moderate change in the phase difference does not greatly reduce the amount of readjustment of the intensity ratio which is necessary before physically possible conditions are obtained. We are, therefore, forced to the conclusion that the average observer will believe the sound to originate at a very great distance and at an angle from the median plane corresponding to the intersection of the phase curve with the curve of infinite distance. In our illustration this angle would be about 39°.

It may happen, of course, that for some other reason, such as the absolute intensity of the sound, the observer finds it difficult to believe that the source is more than a moderate distance away. He may then make a still further mental correction of the intensity ratio so as to bring the image closer to him. This will not have any appreciable effect on the direction of the image, since the line for $P = 60°$ is practically vertical.

Before discussing the results of such experiments, let us consider the other impossible condition which has been the subject of laboratory investigation, namely, the condition of equal phases but adjustable intensity ratio.

Equal phases can correspond only to the median plane—that is, to the vertical lines at the borders of the chart—and in this plane no intensity ratio other than unity can occur. As before, the observer may arrive at a combination of intensity ratio and phase difference that corresponds to an actual source in either of three ways. He may judge the phase difference to be correct and readjust the observed value of intensity ratio, or he may readjust the phase difference to conform to the intensity ratio, or he may readjust both. If he readjusts the intensity only, he must make the intensity ratio unity, obtaining thereby an image in the median plane. If he readjusts the phase difference only, the magnitude of the necessary readjustment depends upon how close to the head he is willing to place the image. The phase readjustment necessary to agree with an intensity ratio of 0.6 is obviously less if r is taken as $2c$ than $5c$. Moreover, it may be made smaller yet by placing the image still closer to the head or even within the head.[1] However, as such sources are rather uncommon in everyday experience the observer may prefer to make a larger readjustment in phase and secure an image in a more usual position. Here as in the case of the experiment with equal intensities, the distance of the image depends upon the psychological prejudices of the individual, and we may expect rather wide variations between the results of different observers. Unlike the former case, however, the direction of the image is here dependent on the distance chosen, and so it too should be subject to similar variations.

The third possibility, that of readjusting both intensity and phase, is more likely to occur here than in the experiment with equal intensities, for, as the figure shows, a small readjustment in intensity reduces materially the necessary readjustment in phase and *vice versa*. As different observers may be expected to readjust the two by different relative amounts, there are obviously a wide variety of possible positions for the image in this case.

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[1] Images within the head are not uncommon in the literature of binaural experiments. See Stewart, PHYS. REV., IX., No. 6, June, 1917, p. 502.

3. Comparison with Experiment.

While a large amount of experimental work has been done and Stewart in particular has published quantitative data taken under widely varying conditions, when this data is analyzed in the light of the above considerations it is found to be lacking in certain important features. In particular while the effect of intensity and phase on the *direction* of the image was studied their effect on its *distance* was not reported. In the absence of this data it is obviously impossible to check the theoretical deductions given above. The best that can be done is to determine whether or not such results as are available are consistent with the theory.

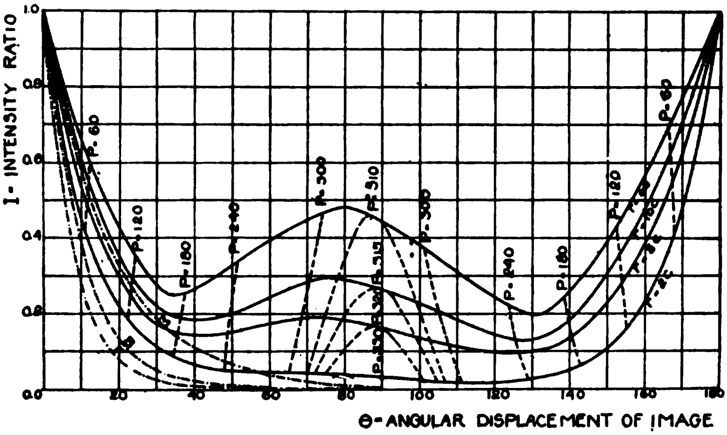

Fig. 4.

Ears 165° apart. *kc* = 2.0. Frequency 1,240 cycles.

As already indicated, almost all of the experiments were conducted with equal intensities or with equal phases, and so involve mental readjustments. For the experiments with equal intensities and varying phase differences a very good agreement has been established[1] between the direction of the image and that of a source which would produce the applied phase difference. The above theory indicates that although such an agreement is in a sense accidental, it is to be expected, since, as explained above, the observer will readjust the intensity ratio from unity to some value consistent with the phase difference observed. Also since the charts indicate that in most cases the direction of the image is practically independent of the readjustment made in the intensity ratio, the results of different observers should be in good agreement, as is found to be the case.

Of the experiments with equal phases and adjustable intensity ratios those of Stewart referred to at the beginning of the paper are by far the most comprehensive. An outstanding feature of these experiments is the wide variation in the results obtained by different observers both as regards the direction of the image and as regards its sharpness or "fusion." The variation in the direction of the image is to be expected from the theory. The observations in connection with the sharpness of the image are of interest for the light which they throw on the mental readjustments involved.

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[1] Hartley, *loc. cit.*, and Stewart, Phys. Rev., XV., No. 5, May, 1920, p. 432.

The observers were asked whether there was complete fusion and a rotation of the image about the head and, if there was partial fusion, where the other images were. It is stated[1] that "when there was only partial fusion, one image did not rotate but remained directly in front, or, by trial, in the position determined by the difference of phase. It could, therefore, be ascribed to the phase difference effect. Thus the terms 'partial' or 'incomplete' fusion really mean fusion in more than one image." This would indicate that in certain cases the observer obtained one image, that in the median plane, by making the entire readjustment in the intensity ratio and another by readjusting the phase. There is, of course, no evidence to determine whether for the second image the entire readjustment was made in phase or whether both were readjusted.

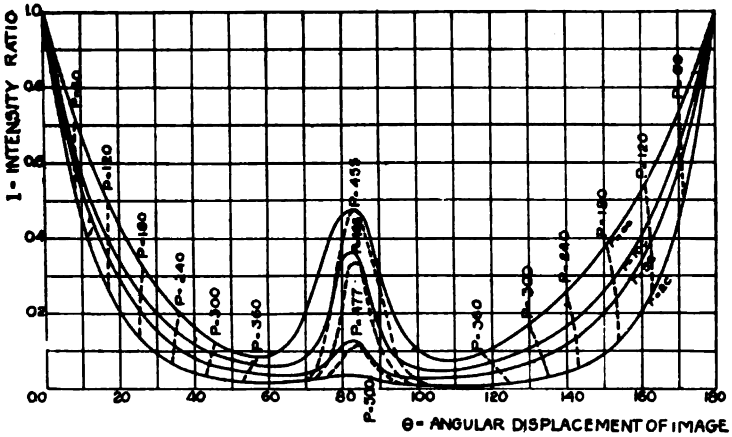

Fig. 5.

Ears 165° apart. $kc = 3.0$. Frequency 1,860 cycles.

It is stated further that "in the case where there was no fusion, and the tube at the left ear was being pinched the single image remained in front until the ratio of intensities was very large, *i.e.*, of the order of 200 with observer *S*, and then there appeared directly at the right, or approximately so, a second image which increased in clearness with further pinching, the image in front simultaneously disappearing, giving in the limit, as would be anticipated, but the one image to the right." Here the observer very definitely appears to have readjusted the intensity ratio to unity to conform with equality of phase even when it involved quite large changes. When this readjustment became too great he changed over abruptly to a large readjustment of phase, giving the image at the side.

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[1] Stewart, Phys. Rev., XV., No. 5, May, 1920, p. 428. The exact meanings of the terms "fusion" and "partial fusion" as used by Stewart are not made entirely clear. At times the latter seems to mean that the observer had difficulty in forming any judgment at all, while in other cases such as that quoted above, it refers to the formation of multiple images.

While the degree of fusion varied widely for different observers, the results of all four showed the same general variation of the completeness of fusion with frequency. As the frequency is increased the completeness of fusion falls off to a minimum around a thousand cycles per second and increases again up to 1,792 cycles, the highest frequency used. A very general qualitative explanation of this result may be obtained from the charts. Since one of the criteria given by Stewart for a "fused" image is that it move around the head as the intensity ratio is varied, its formation must involve a readjustment of phase difference; consequently the difficulty of forming a fused image should be greater the larger the readjustment of phase necessary to conform to the small intensity ratio. From Fig. 1 it will be seen that for an intensity ratio of 0.09, for example, an image may be obtained at $r = 2c$ by readjusting the phase difference from 0° to about 75°. In Fig. 3, which is drawn for a frequency of about 930 cycles, and therefore corresponds roughly to that of least fusion, the corresponding phase adjustment is nearer 200°. At the higher frequency, 1,860 cycles, represented by Fig. 5, a phase difference of 360°, which is indistinguishable from zero, is found to be consistent with an intensity ratio of 0.09 and no adjustment at all is required. Hence it should be easy to obtain a well fused image at the side for this frequency.

There remains the problem of analyzing the experimental relations between the direction of the image and the intensity ratio, the consideration of which led Stewart to the conclusion quoted in the first paragraph. Since the distance of the image was not recorded, we are not able to plot upon the chart of Fig. 1 curves which will accurately display the locations of the image as experimentally observed. However, if we arbitrarily assume no readjustment of intensity ratio, as was done implicitly by Stewart, and plot observed directions against applied intensity ratios, we get the curves *B* and *S* of Fig. 1. These curves correspond to his results, taken with observers *B* and *S*, using a frequency of 256 cycles, which is not widely different from the chart frequency of 310 cycles.

These curves indicate very considerable phase readjustments at the smaller values of intensity ratio. As drawn, they also indicate that the image should be quite close to the head in all cases. If, however, in the experiments this was not the case, it means only that intensity readjustment *did* occur and that each point on the curve should be moved upward far enough to give the observed value of r keeping θ fixed at the observed direction. In Figs. 2 and 4 are shown similar curves plotted from results given by Stewart for frequencies of 512 and 1,024 cycles respectively.

Stewart's conclusion that theory and experiment are in hopeless disagreement was arrived at by comparing curves such as B and S, Fig. 1, with the theoretical curve connecting intensity ratio and direction for a source at a distance of 477 cm., which was selected,[1] "as a common one, though at distances of this magnitude or greater there is but little difference in relative intensities." Thus the curve he used is practically identical with that for a source at infinity and differs from the $r = \infty$ curve of Fig. 1 only in having been drawn for a different ear separation. The disagreement between his theoretical and experimental curves is, of course, obvious, but in view of the above considerations it is of no particular significance. For if the images actually were close to the head the experimental curves show excellent agreement with the theoretical ones *for sources in similar positions*. If they were not, all that is necessary to secure agreement is to assume that part of the inevitable readjustment was made in the intensity ratio instead of all being made in the phase difference. In general then the experimental results are entirely consistent with the theory outlined above.

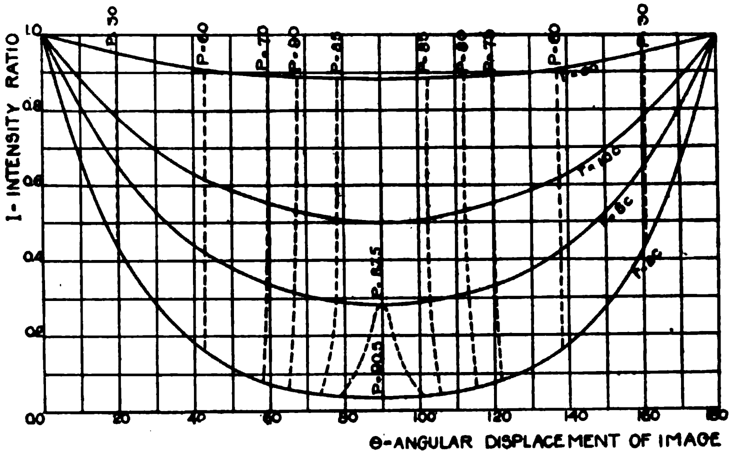

Fig. 6.

Ears 180° apart. $kc = 0.5$. Frequency 310 cycles.

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[1] Stewart, PHYS. REV., XV., No. 5, May, 1920, p. 430.

4. Suggested Experiments.

In the light of these results it appears highly desirable that laboratory experiments be performed with both phase and intensity adjustable; and that the image be located not only with respect to angular displacement but with respect to distance as well, and that the ability to locate actual sources be investigated more fully. If it is found that with stimuli corresponding to an actual source the image is as accurate and sharp as for the source itself, it will show that phase and intensity are the controlling factors. If not other properties should be investigated. The results of experiments with other stimuli might also be of interest particularly to psychologists because of the light shed upon the prejudices of the observer and the dependence of these on his previous experience. The writers are not themselves in a position to carry out such experiments, but it is hoped that the charts here presented will prove useful to others who may be working in this field.

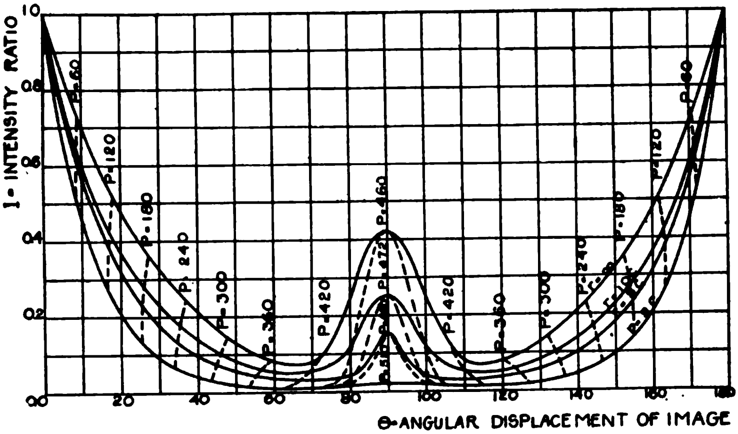

Fig. 7.

Ears 180° apart. $kc = 3.0$. Frequency 1,860 cycles.

Of these charts Figs. 1 to 5 correspond to an ear separation of 165°, while Figs. 6 and 7 correspond to a separation of 180°. Each chart is for a particular value of kc, which, as explained above, is equivalent to a particular frequency. The exact frequency depends, of course, on the values assumed for the radius of the head and the velocity of sound. The values given on the charts were computed for a sphere of 55 cm. circumference and a sound velocity of 340 meters per second.

With regard to the accuracy of these charts it is probably wise not to claim too much. The computations were carried out with considerable care and at the expense of a great deal of time, but the convergence of the series expansions used was so slow[1] for small values of r that after the arithmetical results had been obtained a few obvious discrepancies remained which had to be removed by no more exact means than the judgment of the writers. Even this did not appear to suffice for values of r less than $2c$, which accounts for the omission of such values from the chart. It is at least safe, however, to assert that the final accuracy is as great as that of the binaural sense itself, so that whatever residual errors remain are probably not of serious consequence.

RESEARCH LABORATORIES OF THE
AMERICAN TELEPHONE & TELEGRAPH CO.,
AND THE WESTERN ELECTRIC CO., INC.,
April 5, 1921.

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[1] In this connection see Rayleigh, Scientific Papers, Vol. 5, No. 292, p. 149.

A COMPARISON OF THE THERMIONIC AND PHOTO-ELECTRIC WORK FUNCTION FOR PLATINUM.

By Otto Koppius.

Synopsis.

Variation of Photo-electric Effect with Temperature, to 420°.—(1) *Pt filaments coated with oxides of Ba and Sr* were mounted in an evacuated tube with a quartz window and could be heated electrically to any desired temperature. The *photo-electric current* received by a copper oxide coated Faraday cylinder when a strip was illuminated with light from a quartz arc was measured with a sensitive electrometer, and was found to increase rapidly with the temperature reaching a value, on first heating to 420°, about 68 times as great as at 20°. The *long wave-length limit* was found to shift with increasing temperature from 2860 to about 3800 Å, the corresponding *work function* changing from 4.31 to 3.24 volts. On cooling, however, the sensitiveness and wave-length limit did not return to their original values, except very gradually, showing marked *time-lag effects,* due probably to changes in the oxide surface. (2) *Pt strips,* uncoated, were tested in the same manner. After preliminary treatment by repeated heating to 900°, the *current* was found to be independent of the temperature to 500°, except for a slight decrease due to the magnetic effect of the heating current, and the *long wave-length limit* remained constant within one per cent. at 2570 Å. Hence the *work function* or work necessary to detach an electron photoelectrically from Pt comes out 4.80 volts, which is close to the values found for the corresponding thermionic work function.

Relative Energy of Spectrum Lines of Hg, 2301 to 2804 Å from an arc in quartz, after dispersion by a quartz spectrograph, was determined with a Coblentz thermopile.

Variation in Resistance of Pt with Temperature to 600° was measured (Table I.).

I. Introduction.

According to Richardson,[1] the equation governing the emission of electrons from incandescent solids in a vacuum is

$$i = AT^{\frac{1}{2}}e^{-b/T},$$

where i is the saturation current at a temperature T, from the glowing metal to an auxiliary electrode, A is a constant of the glowing substance which may be associated with the number of free electrons per cubic centimeter of the body in question, T is the absolute temperature, and b is a characteristic constant of the body which is identified with the work necessary to bring one electron out of the body. This work may be expressed in volts through the relation $\varphi e = bK$, where e is the electronic charge, K is the gas constant for one molecule, and φ is the work expressed in equivalent volts. Practically all investigations on thermionic emission have been based on this expression, the validity of which may be considered as well established.

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[1] Richardson, Phil. Trans. Roy. Soc., A, 201, 497, 1903.

The relation governing the emission of electrons from metals in a vacuum under the influence of light was stated by Einstein[1] in 1905 in the form

$$\tfrac{1}{2}mv^2 = Ve = h\nu - p,$$

where ν is the frequency of the exciting light, h is the Planck constant, $h\nu$ is the energy absorbed by the electron from the radiation, p is the work necessary to get an electron out of the interior of the metal, and v is the maximum velocity with which the electron leaves the surface, which can, of course, be determined by the retarding potential V necessary to just prevent such an escape. Though the earlier experiments did not conclusively prove the existence of the necessary relations which follow if Einstein's equation is correct,[2] the more recent experiments[3] all bear out the validity of this equation.

THE WORK FUNCTIONS IN THE THERMIONIC AND IN THE PHOTOELECTRIC EFFECTS.

A. *Thermionic Effect.*—Several methods have been developed to measure the work function in the thermionic effect as represented by the constant b in Richardson's equation. The original method was to take the logarithms of the quantities involved in the equation, *i.e.*,

$$\log i - \tfrac{1}{2} \log T = \log A - \frac{b}{T},$$

and then to plot ($\log i - \frac{1}{2}\log T$) against $1/T$. Then if Richardson's equation is correct, we should get a straight line relation whose slope would give the value of b. In all of the work by this method an accurately linear relation has been found.

Another method which has recently been used is to measure the cooling effect due to the evaporation of electrons from the surface of the metal. The loss of energy under consideration is analogous to the heat lost during the evaporation of liquids, and it may, in fact, be regarded as the latent heat of evaporation of electricity from the substance in question. The theoretical relations involved have been worked out by Richardson,[4] and some very accurate and consistent results by this method have been obtained by Lester[1] for the case of tungsten, and by Wilson[2] for oxide coated filaments.

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[1] Einstein, Ann. d. Physik, 17, 132, 1905; 20, 199, 1905.

[2] Millikan, PHYS. REV., 7, 355, 1916.

[3] Millikan, PHYS. REV., 7, 18, 1916, and PHYS. REV., 7, 355, 1916. Hennings and Kadesh, PHYS. REV., 8, 209, 1916, and PHYS. REV., 8, 221, 1916. Sabine, PHYS. REV., 9, 210, 1917.

[4] Richardson, Phil. Trans. A., 201, 497, 1903.

The agreement between the two methods is very good as may be seen from the results for tungsten. Using the first method, Langmuir[3] and Smith[4] get for the value expressed in equivalent volts of the work necessary to extract an electron from tungsten 4.25 and 4.46, respectively, while Lester,[5] using the second method gets 4.478 volts. In these experiments, the emission is thought to be a pure electron emission, uninfluenced by gases or surface films. If we are to associate this work with that necessary to get an electron out of the metal, it is necessary that all secondary effects due to occluded gases, surface films, etc., be eliminated. It was believed that this had been accomplished in the experiments cited above, so that these values represent the work function characteristic of tungsten.

B. Photoelectric Effect.—The energy relations in the photoelectric effect are given by Einstein's equation

$$\tfrac{1}{2}mv^2 = Ve = h\nu - p.$$

According to this equation, the maximum velocity of the emitted electrons increases with an increase in the frequency of the incident light, and decreases as we go to longer wave-lengths of the incident light. This is borne out in all of the experimental work on the subject. Further, there will exist for each photoactive material a critical frequency, or wave-length, beyond which the target is no further photoactive. At this critical stage, the equation reduces to

$$h\nu_0 = p,$$

where ν_0 is the lowest frequency to which the target is just photoactive, or the longest wave-length λ_0; and p, as before, represents the work necessary to just get an electron out of the target. Hence, by determining this threshold value of the frequency, or longest wave-length, we should be able to calculate the work, p, as defined above.

In order, however, to be able to interpret this work function as a characteristic constant of the metal itself, it is obviously necessary to eliminate all secondary effects due to surface films or other contaminations.

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[1] Lester, Phil. Mag., 31, 197, 1916.

[2] W. Wilson, Proc. Nat. Ac. Sci., 3, 426, 1917.

[3] Langmuir, Phys. Zeits., 15, 525, 1914.

[4] Smith, Phil. Mag., 29, 811, 1915.

[5] Lester, Phil. Mag., 31, 197, 1916.

II. OBJECT OF EXPERIMENT, AND PREVIOUS COMPARISONS BETWEEN p AND b.

The purpose of the experiment was:

1. To study the variation of the photoelectric current from oxide-coated and pure platinum filaments with a change in the temperature of the filaments.

2. To see if a constant long wave-length limit for a substance can be determined which is characteristic of the substance; in particular to test its constancy with respect to changes in temperature.

3. To compare the work necessary to get an electron out of the substance photoelectrically with that required to get it out of the same substance thermionically.

At first it was thought possible to use the oxide-coated filaments used in audion bulbs of the Western Electric Co. as the source, for Wilson[1] had shown that there could be associated with them a definite b. However, in the determination of the long wave-length limit of this material, it was found that this limit was not constant, but changed considerably with temperature. Consequently pure platinum was chosen for this comparison work, since previous experiments[2] had shown that a definite b could be associated with it, as well as a definite long wave-length limit, and hence a definite p. The choice seems to have been a proper one, for a definite long wave-length limit for this specimen has been found which remains constant through a wide range in temperature.

COMPARISONS BETWEEN b AND p.

Considerable work has been done to find the work necessary to get an electron out of platinum by thermionic means. Richardson, in reviewing these researches, discusses[3] these various determinations critically, and concludes that the most probable value of b, expressed in equivalent volts, is about 5.00 volts for platinum. Hughes[4] taking the photoelectric data on platinum from the Richardson and Compton[5] paper compares the value of p in this case with that of the then accepted value of b in thermionics for platinum, both expressed in equivalent volts. But in this comparison, he used for the value of b in Richardson's equation 5.34 volts, while the most probable value for platinum according to Richardson is very closely 5.00 volts. Moreover, he used for the long wave-length limit for platinum $\lambda_0 = 2910$Å, from Richardson and Compton's paper, while the experiments described in this paper yield a value of λ_0 about 300Å shorter than this. If these newer values are used in the comparison, the photoelectric work function comes out very close to the thermionic, namely 4.80 volts at ordinary temperatures against 5.00 volts at the mean temperature of the thermionic emittor which may be taken as about 1100° C.

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[1] W. Wilson, Proc. Nat. Ac. Sci., 3, 426, 1917.

[2] For a summary of these expts. see Richardson, "Emission of Electricity from Hot Bodies." p. 69, etc.

[3] Richardson, "Emission of Electricity," p. 69, etc.

[4] Hughes, "Photo Electricity," p. 44.

[5] Richardson and Compton, Phil. Mag., 24, 575, 1912.

Hagenow[1] made a similar calculation for the case of tungsten. He concluded from his experiments that the long wave-length limit for tungsten must lie between the values 2100 and 2300Å, which are equivalent to 5.87 and 5.37 volts respectively. These values are much higher than that given by Lester's[2] work, namely 4.478 equivalent volts.

III. Apparatus and Procedure.

A. *The Oxide-Coated Filaments.*—The filaments were made by the Western Electric Company and consisted of platinum strips 1 mm. wide coated with the oxides of Ba and Sr in equal amounts. Four of these were welded in parallel to stout platinum leads in such a manner that the oxide of the center portions of the strips, upon which the ultraviolet light fell, were unimpaired. This target was mounted in a glass tube about 3 cm. in diameter, closed by a thin quartz plate cemented to the tube by DeKhotinsky cement. A Faraday cylinder of oxidized copper served to catch the emitted electrons, and the quantity given off was measured by a Dolezalek quadrant electrometer, which could be brought to a sensitiveness of 5,000 divisions per volt, although this high sensitivity was usually not necessary in the case of these filaments, and a capacity was frequently put in parallel with the electrometer. In the first portion of the work, a Toepler mercury pump was used to obtain the vacuum, and in the latter part a mercury diffusion pump. Liquid air kept mercury vapor out of the experimental chamber. The pressures were as low as 3×10^{-8} mm. of mercury. The temperature of the target was obtained by observing its change in resistance with temperature. In the earlier portion of this work on the oxides, an ordinary quartz spectrometer was used, in the latter work a Hilger monochromatic illuminator. The energy content of the individual lines in the spectrum was greater in the former than with the Hilger instrument, but the relative results were the same. A quartz mercury arc lamp of the Haraeus type served as the source of illumination in all of these experiments, the lamp always being run at a certain constant energy consumption.

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[1] Hagenow, Phys. Rev., 13, 416, 1919.

[2] Lester, Phil. Mag., 31, 197, 1916.

B. *The Platinum Emittor.*—A thin strip of platinum foil, 6 × 20 mm. and .0005 inch thick was riveted to stout platinum leads, two fine platinum leads were welded to these leads close to the strip in order to measure the potential drop across it, and the whole was then mounted in the center of a glass tube of about 3 cm. in diameter. A Faraday cylinder enclosed the strip except for small openings for the heating current and potential leads and the opening for the incident light. Figs. 1 and 2 will give an idea of the arrangement within the tube, and of the experimental arrangements.

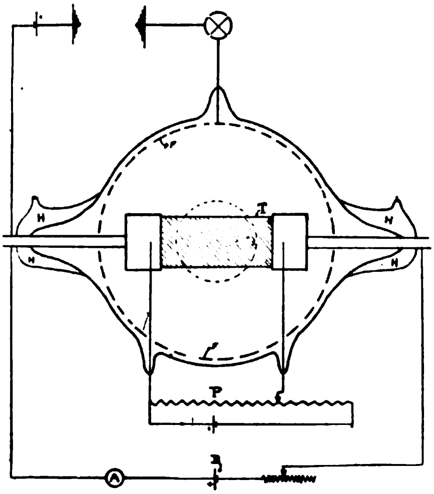

Fig. 1.

It was found that at higher temperatures, the lead-in wires became very hot, and to prevent cracking of the tube, two glass cups *H* were fitted to the tube and lead-in terminals and filled with mercury. This arrangement was found to be very satisfactory. The quartz window in this tube was cemented to the glass by a special material kindly furnished by the Mantle Lamp Company of America. This cement can be heated to 200° C. without danger of getting soft or impairing the vacuum.

The temperatures of the platinum strip were measured by its change in resistance with temperature. A temperature-resistance calibration curve was first taken, in which the strip was placed in an electric furnace and its resistance determined directly up to 650° C. In this calibration, the lower temperatures were measured by a standardized high temperature mercury in glass thermometer and the higher by a platinum resis-

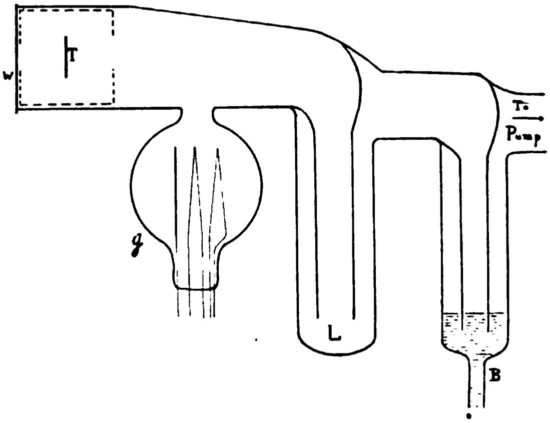

Fig. 2.

tance thermometer. According to Chappuis and Harker[1] the resistance of platinum changes with the temperature in the range from 0–500° C. according to the formula

$$R_t = R_0(1 + \alpha t + \beta t^2)$$

where R_t is the resistance at the temperature t, R_0 is the resistance at 0° C., and

$$\begin{aligned} \alpha &= .003922, \\ \beta &= -.0_6585. \\ &= -.000000585 \end{aligned}$$

Table I. shows the resistance of the strip calculated from this formula and compared with the observed resistance taken from the resistance-temperature curve. The resistance at 0° C. was 1.010×10^{-2} ohms.

TABLE I.

	Observed.	Calculated.
Resistance at 100° C.	1.42×10^{-2}	1.41×10^{-2}
200° C.	1.82	1.80
200° C.	2.21	2.20
400° C.	2.58	2.57
500° C.	2.95	2.99
600° C.	3.29	3.38

The potential drop across the strip being small, a potentiometer arrangement P was used to measure it. The current necessary to heat the filament was furnished by a set of storage batteries B.

The apparatus (Fig. 2) was exhausted by a mercury diffusion pump working against a fore-vacuum of about .001 mm. of mercury produced by a Gaede rotary mercury pump. A barometer tube, B, served as a valve to shut off the left-hand side of the apparatus, if desirable. L is the liquid air trap. The pressures were measured by means of a Buckley[2] ionization gauge, G. The electron current used with this gauge was measured with a Paul single pivot micro-ammeter, the plate current by means of a sensitive galvanometer. The pressures used varied from 1×10^{-5} to 1×10^{-6} mm. of mercury. T is the platinum strip, and W the quartz window.

The source of light for the illumination of the strip in all of these determinations was the quartz mercury arc lamp mentioned before. It was enclosed in a box, so that it could be run at a high temperature. Souder[3] found that under these conditions the energy output of this lamp remained very constant for many hours. The energy distribution in the various prominent lines of the mercury spectrum emitted by the lamp was determined by means of a Coblentz thermopile and a shielded galvanometer. The values obtained in this work of the relative energies of the mercury lines used most frequently are given in Table II.

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[1] Land. and Bornst. Tables, 1912, p. 1083.

[2] Buckley, Proc. Nat. Ac. Sci., 2, 683, 1916.

[3] Souder, Phys. Rev., 8, 683, 1916.

Table II.

Wave-Length.	Deflection $\propto$ Energy.
2301A°	1.13
2378	2.31
2400	3.02
2483	6.26
2537	16.38
2652	13.68
2700	5.46
2804	11.48

The method of obtaining λ_0 in these experiments was the direct method described by Millikan.[1] The relative energy content of the various lines in the spectrum used being known, their relative photoaction is measured and then the photocurrent per unit intensity of the line is plotted against the wave-lengths. A smooth curve is drawn through the points and continued so as to cut the wave-length axis. This gives the longest wave-length to which the metal is just photosensitive. The results obtained by this method after the metal had been sufficiently denuded of gases were remarkably consistent as is shown below.

IV. Results.

A. *The Oxide-Coated Filaments.*

Variation of Photoelectric Current with Temperature.—In 1906 Millikan and Winchester[2] and Lienhop[3] independently established the lack of dependence upon temperature both of the amount and of the maximum value of the velocity of emission of photoelectrons. The former used eleven metals in the same tube, and their temperature range was from 15° to 125° C. The latter used carbon and platinum, and his temperature range was from room temperature to − 190° C. Ladenburg[4] found that the velocities of photoelectrons from AU, Pt and Ir were independent of the temperatures up to glowing. All of these authors used the whole radiation from their respective sources, that is, they did not attempt to determine the long wave-length limit of their substances at these various temperatures. All of the subsequent experimental work has supported the results obtained by the above-mentioned authors as regards the independence of the maximum velocity of emission of photoelectrons and temperature. Varley and Unwin[1] found a constancy of photoelectric current from platinum at a pressure of .0035 mm. of mercury, but only after a certain heating current had been applied to the platinum foil so as to raise its temperature to at least 50° C. Beyond this point the photoelectric current remained practically constant up to a temperature of 350° C.

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[1] Millikan, Phys. Rev., 7, 355, 1916.

[2] Millikan, Phys. Rev., 24, 6, 1907, and Phil. Mag., 14, 188, 1907.

[3] Lienhop, Ann. d. Phys., 21, 284, 1906.

[4] Ladenburg, V. d. D. P. G., 9, 165, 1907.

The behavior of these oxide strips, however, was quite different from the above, the photoelectric current greatly increasing with a rise in temperature, the rate of increase always being far greater the higher the temperature. A typical set of readings follow in Table III., all readings being for line 2537Å, the time unit being 30 seconds and the incident light remaining constant:

TABLE III.

Oxide Coated Filaments. Variation of Photoelectric Current with Temperature.

Temperature.	Deflections ∝ Photoelectric Current.
20° C.	2.0 cm.
47	2.0
75	2.0
120	2.6
160	4.2
200	5.5
240	14.2
270	16.6
310	20.6
350	39.6
420	135.0
Then, ten minutes after the heating current in the last reading above was thrown off, and the target had returned to	
20° C.	141.0 cm.

It may be remarked that these deflections did not change abruptly as the higher heating current was thrown on, but that there was a gradual change toward a higher value as the heating at any particular temperature was continued, the rate of increase in all cases being most rapid when the heating current was first applied. After heating for some time at a constant temperature, the deflections become relatively constant. In the above particular set of readings, after the apparatus had stood for seven days at room temperature, the deflection had decreased to 40.0 cm., *i.e.*, they had not returned to their original value. How quickly and how nearly the original values of the photoelectric current were reached again depended altogether upon the temperature to which the target had been heated during the run, the return being quicker and more complete the lower the temperature to which the target had been heated. Even after four months the original value of the photoelectric current had not been reached although the deflections had decreased to 4.7 cm. Another peculiar behavior was always noted in that, immediately after the heating current was switched off, and the target had returned to room temperature, the photoelectric current always increased over that of its maximum value at the maximum heating current, and it was not until hours after, that it began to decrease. This behavior of the oxide strips was absolutely general, whether for high or lower vacuum condition.

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[1] Varley and Unwin, Proc. Roy. Soc. Edinburgh, 27, 117, 1907.

Variation of the Long Wave-Length Limit with Temperature.—Along with this 68-fold increase in the amount of the photoelectric current, the long wave-length limit of these strips also changed as the temperature was raised. The long wave-length limit of the filaments in the beginning of the experiments was about 2860Å. As the heating progressed, it was observed that this limit was shifting toward the longer wave-lengths. At the highest temperature used in the above data, the target was certainly photoactive at $\lambda = 3650$Å, but not when illuminated with the visible violet line 4047Å. Without appreciable error, the long wave-length limit at the highest temperature above may be taken as 3800Å. As time went on, the long wave-length limit would slowly shift again towards the shorter wave-lengths.

Using the relation $p = h\nu_0$, the work necessary to get an electron out of this target by photoelectric means amounts to 4.31 volts at 20° C., and to 3.24 volts at 420° C. This latter value is about of the same order as found by Wilson[1] for the thermionic work function for these oxide strips.

The cause for this increase in the photoelectric current, and the decrease in the work function with an increase in temperature need not be ascribed to a change in the work function of the unchanged surface with temperature, but rather to the fact that the surface of the oxides was changed, and hence changed the photoactive properties of the target. That a change in such targets actually does occur at higher temperatures was shown by Fredenhagen,[2] namely that the oxides gradually disappear and the underlying platinum shows corrosion. Assuming, then, a surface change with heating of the oxide coated filaments, it can at least be said that in this change the target is put in a condition where the work to get an electron out of it is much less at higher than at lower temperatures.

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[1] W. Wilson, Proc. Nat. Ac. Sci., 3, 426, 1917.

[2] Fredenhagen, Ber. d. Saechs. Ges. d. Wiss. Math. Phys. Kl., 65, 55, 1913.

B. *The Platinum Emittor.*

The procedure was similar to that used with the oxide emittor. The platinum was very weakly photosensitive at first, the deflection for the strong 2537 line only amounting to 0.6 cm. in 30 seconds, and an increase in temperature gave an increase in photosensitiveness similar to that observed with the oxide strips. After the platinum had been repeatedly heated to temperatures of 900° C. and more, however, the behavior was different. The photoelectric current no longer increased with a rise in temperature, but rather appeared to decrease slightly. This decrease was presumably due to the magnetic effect of the heating current upon the escaping electrons. The absolute values of the photoelectric current varied somewhat from day to day, but the value of the long wave-length limit of the platinum remained constant. This variation of the photoelectric current does not affect in any way the determination of the long wave-length limit, since the photoelectric currents due to the respective lines were affected alike.

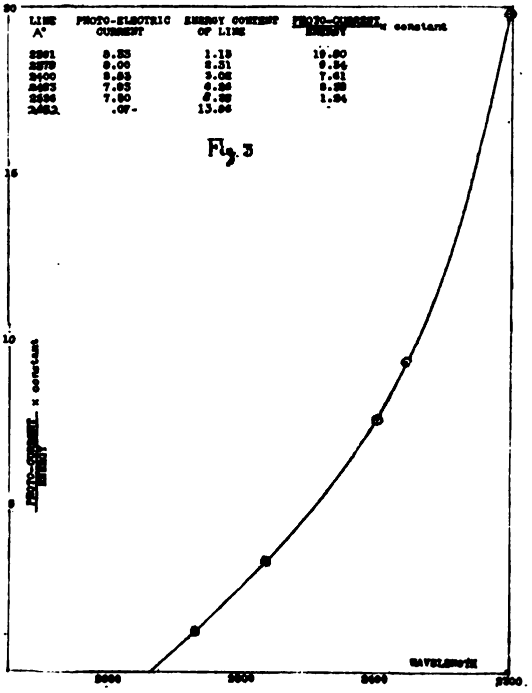

Fig. 3.

The Long Wave-Length Limit Determinations.—The method of determining the long wave-length limit of the platinum strip was the direct method described by Millikan[1] in his work on sodium and lithium. The relative energy content of the lines in the mercury spectrum of the mercury arc having been determined, the photoelectric current per unit intensity was plotted against the wave-length and the intercept of this curve with the abscissa gave the longest wave-length to which the platinum was photosensitive. At first the platinum showed a behavior somewhat similar to that of the oxide strips, though less marked. Thus the long wave-length limit shifted toward the longer waves as the temperature was raised, and then shifted slowly back toward the shorter waves when left standing for a time at room temperature. But after glowing out for several hours with a 9.0 amperes heating current, which raised the platinum foil to bright red heat, the behavior of this platinum was constant as regards its long wave-length limit. In the process of this outglowing the pressure rose only very slightly, so that if gas was emitted from the strip, the speed with which it was emitted was certainly less than the speed of evacuation. Taking then the values of the photoelectric currents at various temperatures and plotting them as shown in the sample plot of Fig. 3 we get the set of long wave-length determinations as shown in Table IV.

TABLE IV.

Temperature.	Long Wave-Length Limit = λ_0.
20° C.	2571A°
100	2560
370	2568
490	2564

It may be added that even after the original tube had to be discarded and another tube containing the same platinum strip had been constructed, the result as regards the long wave-length limit was not altered in the least after the platinum had again been brought to a steady state. Thus the threshold wave-length at which the platinum begins to be photoactive may be taken as 2570Å at room temperature, and this value varies but little if at all with a change in temperature up to 500° C. It is believed that this is a constant which is an intrinsic property of platinum. This value is about 300Å shorter than the value of the long wave-length limit for platinum found by Richardson and Compton[2] who get 2880A for this constant.

If we take then the long wave-length limit for platinum as 2570Å, we can compute the work necessary to emit an electron by photoelectric means, *i.e.*

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[1] Millikan, PHYS. REV., 7, 355, 1916.

[2] Richardson and Compton, Phil. Mag., 24, 575, 1912.

$$p = h\nu_0 = \frac{6.547 \times 10^{-27} \times 3 \times 10^{10} \times 3 \times 10^{2}}{4.774 \times 10^{-10} \times 2.570 \times 10^{-5}} = 4.80 \text{ volts.}$$

In the thermionic effect, Richardson inclines to the values of b, expressed in equivalent volts, as determined by Deininger[1] and Horton[2] who get 5.02 and 5.1 volts, respectively, for the work function in the thermionic effect of platinum.

The Variation of the Work Functions with Temperature.—The work function in the thermionic effect has been found to change very little, if any at all, with temperature in the ranges of temperatures usually employed. The same may be said for the photoelectric work function, as may be seen from Table IV., at least it does not change appreciably in the range from room temperature to 500° C. Above this temperature it becomes more difficult to determine the long wave-length limit because the thermionic effect becomes appreciable, so that the photoelectric current must be determined by difference measurements from the superposed effects of the two phenomena. This was done with temperatures up to about 700° C., and an increase in the long wave-length limit was observed, amounting to as much as 50A at these higher temperatures. Further tests will be necessary upon this point, but in view of the ease and definiteness with which the intercepts on the wave-length axis can be determined, it seems probable that the method will yield a simpler and more sensitive test of the temperature variation of the work function than can be found in measurements upon the thermionic effect.

I wish to thank Messrs. Julius and Fred Pearson, mechanicians at the Ryerson Physical Laboratory, for their work and suggestions in connection with the construction of the tubes used in these experiments.

Finally, I wish to express to Professor Millikan my appreciation for suggesting the problem and for his continued interest and assistance in the course of these experiments.

Ryerson Physical Laboratory,
The University of Chicago.

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[1] Deininger, Ann. d. Phys., 25, 285, 1908.

[2] Horton, Phil. Trans. A, 207, 149, 1907.

SOME FACTS BEARING ON THE STRUCTURE OF ATOMS, PARTICULARLY OF THE HELIUM ATOM.

BY R. A. MILLIKAN.

SYNOPSIS.

Discussion of Suggested Models for Helium and Other Atoms in the light of recent results of a study of multiple ionization by alpha rays. As will be reported more fully elsewhere it has been found that slow alpha rays detach both electrons from some of the helium atoms through which they pass, the number of doubles being one sixth the number of singles, while in the case of the other gases tested no evidence of multiple ionization has been obtained. The result is considered by the author to indicate that the electronic orbits in the helium atom must be such as to bring the two electrons into the same part of the atomic volume at least one sixth of the time, a condition which is well satisfied by the recent models of Landé and Bohr in which the orbits cross each other. In the case of other atoms the new evidence favors the Bohr models, with few electrons in the outer shells.

Multiple Ionization in Arcs and Sparks is Probably Due to successive ionization of the same atoms.

LET *p*, Fig. 1, be a minute oil drop, carrying for example one free positive electron and let the strength of a powerful electrical field between the plates *A* and *B* be adjusted until the weight of the drop is exactly balanced by the pull upon it of the electrical field. Then by opening the shutter *S* let an alpha particle from a bit of polonium at *a*, Fig. 1, be shot underneath the drop. The atoms of the gas underneath *p* have electrons knocked out of them by the passage of the alpha rays through them and the positively charged residue of such an atom as happens to be immediately underneath the drop is thrown instantly to the latter, where the charge carried by that residue is at once determined by the speed with which the drop begins to move upward. In the actual experiments which have been carried out by this method at the Ryerson Laboratory,[1] the catching of an atom from which one electron had been removed by an alpha or beta particle

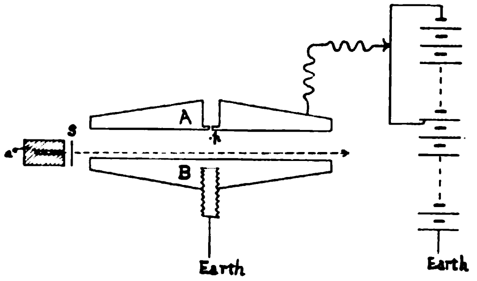

Fig. 1.

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[1] Phil. Mag. (6), 21, 753, 1911. PHYS. REV., 15, 157, 1920.

has in general caused the drop to move with a speed of 1 scale division in about 14 seconds. If the residue of the atom caught by the drop has had 2 electrons removed from it, the drop moves over one division in seven seconds, 3 electrons causes it to move over one division in 4.7 seconds, etc.

By observing, then, the speed corresponding to each catch, it is possible to determine without the slightest ambiguity exactly how many electrons have been removed from an atom by the passage of an alpha particle through it. As soon as a catch is observed, the shutter S is thrown across the path of the alpha rays so as to prevent further ionization until the measurements upon the last catch have been completed.

Results by this method have already been published [1] which reveal the interesting fact that a swift moving alpha particle practically never succeeds in detaching more than one electron from a given atom through which it passes whether that atom is one of the lighter ones like carbon, nitrogen, or oxygen, which possess but from 6 to 8 electrons, or whether it is one of the heavier ones like iodine or mercury, the latter of which possesses as many as 82 electrons. *This, in itself, throws a certain amount of light on atomic structure, for it shows conclusively that the electrons within an atom act quite independently of one another. They are certainly not in rings, of say 4 or 8 or any other number which become unstable when one of their number is removed, or which can in general be shattered as a whole.*

In view, however, of the fact that the ionizing power of an alpha particle is an inverse function of its speed, and also of the fact that J. J. Thomson's positive ray photographs show unambiguous evidence of multiple-ionization of certain types of atoms, I have recently started, as promised in a former paper,[2] some new experiments of the type described above save that the range of the alpha particle within the chamber was so adjusted, by introducing sheets of aluminium foil into its path, as well as varying amounts of gas, that it could be brought underneath the drop at the very end of its range where its ionizing power reaches as high a value as it can ever attain. These observations have been carried out in the Ryerson Laboratory chiefly in collaboration with Mr. T. R. Wilkins, who has carried the number of catches up into the thousands, but I have myself made sufficient observations to obtain an independent check upon the correctness of the main results obtained. The details of the observations will be reported by him in a following article, but the results may be summarized and discussed as follows:

1. In the case of helium, the alpha particle, when it is in that part of its range which corresponds to its maximum ionizing power, detaches

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[1] Phys. Rev., 15, 157, 1920.

[2] Phys. Rev., 15, 177, 1920.

both of the two electrons that the atom of helium contains 16 per cent.[1] of the time that it detaches either of them. Otherwise stated, *the alpha particle shooting at random through the helium atom at its maximum ionizing speed, gets both electrons every sixth shot in which it gets anything.*

2. The percentage of doubles produced in helium by the alpha particle increases from about 8 per cent. at the middle of the range up to a maximum of 16 per cent. which is reached at exactly the portion of the range which corresponds to the maximum ionizing power. In other words, *the curve of doubles plotted against range follows, just as it ought to do, the curve of ionization plotted against range.*

3. *No atom thus far tested except that of helium shows any distinct evidence of multiple ionization by a single alpha particle, even when that particle is moving at the speed corresponding to its maximum ionizing power.* The other atoms thus far tested, all in molecular combination, are those of oxygen, nitrogen, carbon, hydrogen and mercury.

Although the foregoing facts are not sufficient to determine in any way the structure of any atom, they are amply sufficient to render impossible certain sorts of suggested structures for the helium atom.

Thus, the original Bohr theory which placed the two electrons of the helium atom at the poles of a diameter, cannot be reconciled I think with these observations; for, since both electrons go out independently, otherwise we should not get one of them alone five times out of six, the alpha particle can only rob them both from the parent nucleus (which is also an alpha particle) if, in going through the atom, it comes closer to both electrons than the normal distance between each of them and the nucleus. Now, the speed with which the alpha particle is moving when it produces its maximum ionization comes out very nearly the same as the speed of rotation of the electrons within the helium atom, as computed with the aid of the Bohr theory. If, therefore, the alpha particle came close enough to one electron to catch it at one pole of a diameter, it would have very little chance, even when moving in the plane of the orbit, of catching the other electron at the other pole, since by the time it got across the orbit, the second electron would be about an orbital diameter away from it. If the alpha particle is moving at right angles to the plane of the orbit, it has no chance at all of catching both electrons. From whatever point of view this structure is approached, I cannot see that it permits both electrons to be detached by a single alpha particle one-sixth as frequently as either one of them is detached.

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[1] The observations themselves give about 15 per cent., but the helium was but about 96 per cent. pure, and since the impurity was inert in this particular, it is computed that pure helium would show about 16 per cent. of doubles.

Secondly, the facts herewith presented are also irreconcilable with a structure at one time suggested by Sommerfeld in which one electron rotates in an orbit close to the nucleus, while the other is in an orbit of very much greater diameter, for here again the chance that the alpha particle can come close enough to an electron to pull it away should vary at least as rapidly as the radius of the orbit, which would mean that the inner electron could not be detached one sixth as frequently as the outer unless its orbit were more than one sixth the diameter of the outer's orbit. The chance, then, that with any such ratio of orbits both electrons would be detached at once, would be exceedingly minute.

It is to be noted too that since the chance of an impact with the nucleus itself in such a way as to cause it to be jerked away from both its electrons is not a hundred-millionth $(10^{-12}/10^{-8})^2$ of the chance of a passage of the alpha particle through the outer regions of the atom it is at once obvious that this way of accounting for doubles is altogether excluded.

In a word, then, the fact that every sixth shot which detaches any electron at all from the atom, gets them both, demands that both electrons find themselves in the same part of the atomic volume a very appreciable fraction of the time. This condition is well satisfied by the recent theories of Landé and of Bohr, according to which the two electrons are in orbits of much the same diameter but inclined to each other at angles of 60° or 90°.

It is at first sight a matter of no little surprise that the helium atom, with but two electrons to lose, loses them both one-sixth of the time while the atoms with many electrons, in the case of mercury, 82, apparently have in general but one detached at a time by the passage of an alpha particle through them. This fact is of some significance for atomic structure. The evidence of the K series of x-rays is to the effect that the structure of the helium atom repeats itself in the K ring or shell of all atoms. That the two electrons in this ring are not detached by alpha particles except in the case of helium is not surprising, since the ring shrinks rapidly in diameter as the atomic number increases, and since the forces holding these electrons to the nucleus also increase proportionally to the atomic number. That the larger number of electrons in the outer shells appear in practically all cases to be detached by alpha particles one at a time, indicates, first, a different structure in these shells from that existing in the inmost one, *i. e.*, existing in the helium atom—a difference quite in accord with the multiplicity of orbits indicated by the theories of Bohr and Sommerfeld. In the second place, it shows, quite independently of the theory of Bohr, but also altogether in accord with it, that in an atom like mercury the great majority of its 82 electrons are in intense fields of force close to the nucleus where the chance of the alpha particle coming close enough to them to rob them from that highly charged nucleus is exceedingly remote. It will be remembered that the elementary form of the Bohr theory requires that the radii of the K, L, M, etc., rings or shells bear the ratios 1, 4, 9, etc., and that these radii all shrink in the inverse ratio of the atomic number, while the observed frequencies in the K, L, M series indicate actual ratios of radii, computed from Bohr's equations, which are somewhat larger than the foregoing values.

The present group of experiments, then, constitute new evidence in favor of an atom of the general type postulated by Bohr in which the chance that an alpha particle could detach electrons from any save the outer shell is very remote. That it does not detach two at a time, *even from the outer shells* of the N, O, or Hg atoms, while it does frequently get them both in the case of helium, is perhaps to be expected from the larger diameters, and the more powerful nuclei of N, O, and Hg in comparison with He, and from the imperfect shielding of these outer shells by the electrons of the inner shells. Bohr may also find in it support for his recent contention[1] that the outer shell of heavy atoms possesses few electrons instead of many as postulated in most of the discussions of the "Static Atom."

In view of the failure to obtain multiple ionization in these experiments with oxygen, nitrogen, mercury and other of the heavier atoms, even when the alpha particles are moving with the slowest speeds, it appears not improbable that the *multiple ionization observed by J. J. Thomson and others in connection with positive rays, and also the double ionization quite conclusively indicated by the differences between arc and spark spectra, may be due to the successive ionizations of the same atom by electrons or by radiations from adjoining atoms, or by both.* These multiple ionizations are revealed where very dense electron currents exist, but with such dense electron currents there is no doubt about the existence of such successive ionizations both by the absorption of radiation and by multiple electronic impacts. Such an assumption is required for the explanation of the phenomena of *low* voltage arcs. Further, the higher the atomic number, the more chance there is for such multiple ionization, and it is to be noticed that a study of the multiple ionization obtained with positive rays, shows that the number of multiples increases with atomic number much as it should if that hypothesis is correct.

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[1] Nature, March 24, p. 104.

THE EXTENSION OF THE X-RAY SPECTRUM TO THE ULTRAVIOLET.

By E. H. Kurth.

Synopsis.

Characteristic X-Radiation Due to Slow Electrons, 1,000 to 12 Volts.—By using the methods of modern high-vacuum technique, the difficulties experienced by previous investigators have been largely overcome. The radiation was measured in terms of the photo-electric current excited from a Pt dish. The effect was made large by utilizing a hot tungsten helix as a source of electrons, and all disturbances due to gas ions formed by the electrons were eliminated by maintaining a very high vacuum and by interposing an electrostatic field across the path of the radiation. The deflections thus obtained were large and perfectly reproducible. When the deflections *per unit* thermionic current were plotted as a function of the accelerating potential, sharp breaks appeared which each indicate the minimum energy necessary to excite the corresponding characteristic radiation. From these energy determinations the corresponding wave-lengths were computed using the quantum relation. Thus the following values have been obtained for the *convergence wave-lenghts* in Angstroms: *K-series of carbon* 42.6, *oxygen* 23.8; *L-series of carbon* 375, *oxygen* 248, *aluminum* 100, *silicon* 82.5, *titanium* 24.5, *iron* 16.3, *copper* 12.3; *M-series of aluminium* 326, *titanium* 85.3, *iron* 54.3, *copper* 41.6; *N-series of iron* 247, *copper* 116. The relation of these results to those obtained by crystal analysis and by spectrum analysis is discussed. It is suggested that the radiation from solid targets may differ from the radiation from gaseous atoms, especially for the lighter elements.

It was probably first suggested by Sir J. J. Thomson that it might be practicable to investigate by means of x-radiation produced at relatively low voltages that region of the spectrum falling between the wave-length of the longest measured x-rays and that of the shortest ultraviolet radiation studied spectroscopically. Difficulties in ruling suitable gratings and their low reflecting power have limited the exploration of this region from the ultraviolet side, while the close grating space of crystals and their strong absorption of the radiation have prevented the application of the methods of x-ray analysis. Now from the quantum relation $eV = h\nu$, it is evident that the upper frequency limit of this region corresponds to electronic impacts of approximately 1,000 volts. Investigators thus far, however, have been chiefly concerned with the question of the actual production of radiation by impacts of these slow-moving electrons against a solid anticathode, and experiments have indicated its presence for potentials down to values below 100 volts. Convincing evidence, however, of the presence of radiation in this region really characteristic of the particular anticathode material employed has not yet been secured.

A brief summary of the work done on soft x-radiation prior to 1919 was given by Dadourian in a paper published in that year.[1] Early in the following year a paper by Misses Laird and Barton appeared.[2] Still later a preliminary report relative to some work by Holweck was made before the French Academy of Sciences.[3]

The experimental arrangements used by the several observers who have endeavored to secure quantitative results have been of an essentially similar type. A hot cathode as a source of thermions and an anode, together with a device for detecting and measuring the effect of the radiation produced, are sealed into a common container which is then highly evacuated. The detecting arrangement consists of either an ionization chamber, separated from the compartment in which the radiation is excited by a very thin membrane, or simply of a metal disk connected to an electrometer, which will thus measure the photoelectric effect of the radiation falling upon the plate. It has been generally customary in any case to interpose a window of collodion or other suitable material between the source of the radiation and the arrangement employed for detecting it. The purpose of this window is to prevent the passage through to the detector of ions formed in the x-ray compartment.

This method of ensuring freedom from the effects of the ions is open to a number of objections. It is well known, for instance, that radiation of this range of wave-lengths is considerably absorbed by even the thinnest of membranes. If collodion is used for the window, furthermore, it is difficult to give the apparatus the requisite heat treatment during the exhausting process. With an apparatus in which she employed a window, Miss Laird was able to observe no radiation effects with anode potentials below 200 volts. Holweck secured no effects below 70 volts. The discrepancy between these results and those obtained by other investigators who have observed effects attributed to radiation at much lower potentials, would be best explained, perhaps, by assuming that radiation of longer wave-length was to a considerable extent absorbed by the window employed, and that the lowest voltage at which radiation is detected depends on its intensity.

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[1] A. Wehnelt and W. Treuble, Litz. Phys.-Med. Soc. Erlangen, 37, 1905.
H. Dember, Verh. Deutsch. Phys. Ges., 13, p. 601, 1911.
H. Dember, Verh. Deutsch. Phys. Ges., 15, p. 560, 1913.
R. Whiddington, Camb. Phil. Soc., 17, p. 144, 1913.
J. J. Thomson, Phil. Mag., 28, p. 620, 1914.
Elizabeth R. Laird, Ann. der Phys., 46, p. 605, 1915.
H. M. Dadourian, PHYS. REV., 14, p. 234, 1919.

[2] E. R. Laird and V. P. Barton, PHYS. REV., 15, p. 297, 1920.

[3] Holweck, Comptes Rendus, 171, p. 849, 1920.

Now it seemed probable that if recourse were taken to the methods of modern high-vacuum technique, it might be possible practically to eliminate those ion effects which have led to the use of a window. The comparatively few ions which would then be formed could be removed by a system of electrified plates without introducing serious secondary effects. The employment of a high vacuum, furthermore, would permit the use of a tungsten filament as a cathode, which, under these conditions might be placed very close to the target without danger of serious contamination of the target surface. The high thermionic current at low voltages made possible by the increased electric intensity resulting from this close spacing of the cathode and the target would be conducive to increased radiation effects at the lower potentials. The problem of freeing the target of gas would likewise be facilitated, because the bombarding process could be effectively carried on at the moderate voltages available. In arrangements which have been used up to the present, an exceedingly small proportion of the radiation produced at the target could actually contribute to the measured effect, due to the fact that the detecting device has subtended such a relatively small portion of the active solid angle about the target. It seemed that it might be comparatively easy to render a much larger proportion of this radiation effective by suitable design of the apparatus.

Thus an attempt has been made to effect improvements over previous experimental arrangements with two aims in view: (1) the practical elimination of impeditive effects; (2) the production of a large increase in the effect to be measured. The first consideration requires the complete elimination of the possibility of gas ions reaching the detecting device, and the reduction to a minimum of the effect of all radiation not arising as a result of impacts of electrons against the atoms of the target element. With regard to the second consideration, the radiation effect has been increased to such a magnitude that the readings may be taken upon a shunted and comparatively insensitive electrometer. With this measuring arrangement one experiences an almost complete freedom from insulation troubles and from electrical disturbances coming from without the system. As a result the deflections are found to be remarkably consistent, and the resulting curves are entirely reproducible.

APPARATUS.

The form of the apparatus which was used in obtaining the results given in this paper is represented diagramatically by Fig. 1. A hot cathode, C, consisting of a short length of helically wound, 10-mil tungsten wire was mounted in the small end of a large conical glass tube. The target, *T*, normally about 6 by 12 mm. in section, was arranged to be changed at will through the agency of a small ground glass joint, *J*, which was made tight with a little De Kotinsky cement. The long glass stem on which the target is mounted was designed to fit the side tube quite closely, and originally a short glass appendix which could be immersed in liquid air was attached at the base of the seal. This device, it was hoped, would make it improbable that any trouble might be encountered due to the leakage into the apparatus of water vapor coming off the glass near the joint. In actual operation, however, it was found that the escape of gas from this joint was negligibly small. In the body of the tube a system of seven diverging plates was arranged, between which the radiation from the target must pass before reaching the further compartment of the tube. Alternate plates of the set were connected together forming two groups, *P* and *P′*, one of which was then joined to the gauze *G* and the other to the gauze *G′*. Leads from these two gauzes as well as from the third gauze, *G″*, were brought to the outside of the tube as indicated in the figure. These gauzes are all of exceptionally coarse mesh and they do not intercept an appreciable proportion of the radiation. The radiation detecting plate, *D*, is about 5 cm. in diameter and is made of platinum, as are all of the other metal parts. Appropriate guard ring devices, not shown in the drawing, were provided to take care of volume and surface leakage of electricity to the detecting disk.

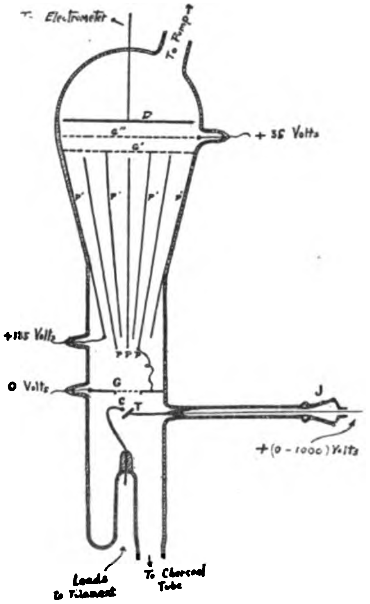

Fig. 1.

A large two-stage condensation pump backed up by an oil pump is used to evacuate the apparatus. Large bore tubing is employed for connections and two liquid air traps, one of which contained charcoal, are located between the pump and the apparatus for the purpose of freezing out mercury vapor. A reasonably sensitive McLeod gauge and a mercury cut-off are provided close to the pump for use in detecting the presence of possible leaks. An appendix containing a small quantity of charcoal is also connected directly to the apparatus. Electric heaters are arranged to bake out the apparatus, charcoal and traps at about 400° for several hours before every run. Simultaneously the connecting tube is well heated with a Bunsen flame. Finally at the conclusion of this heating process the charcoal and vapor traps are immersed in liquid air.

As long as the charcoal is being heated, a pressure of a small fraction of a bar is always indicated by the gauge. When this heating is discontinued, however, the pressure at once becomes immeasurably small. The pump is always kept in operation during the runs.

The tungsten cathode is heated by a set of high-capacity storage cells. Its resistance is approximately 0.7 ohm and it requires from 4 to 6 amperes to light it. The negative end of the cathode is joined to the gauze *G* and the connection is earthed. The anode voltage is provided by a small 1,500-volt direct-current generator. This potential is regulated by slide resistances and is measured on a 150-volt Weston voltmeter which is fitted with an adjustable series resistance to give suitable range to the scale. The thermionic current in this circuit is read upon a Paul Universal Testing Set. The set of plates which is joined to the gauze G' is connected to a group of dry cells which provide an adjustable source of potential up to 600 volts. The gauze G'', which receives the photo-electrons from the detecting disk, *D*, is raised to a potential of 35 volts furnished by a small battery. A Dolezalek electrometer with a sensitivity of about 1,700 mm. per volt is connected to the detecting disk, and the instrument is fitted with a series of India ink shunts of different resistances. Thus definite scale deflections rather than rate of deflection are observed when radiation falls upon the detecting disk.

In the original set up a large electro-magnet was arranged so that a strong magnetic field could be applied perpendicularly to the plates. Under the influence of this field the normal, direct path of an ion passing between the plates would become converted into a series of loops, and the corresponding length of time that the ion would remain between the plates would be considerably increased. One might expect to remove the ions under these conditions with an electric field of very moderate strength between the plates. When the arrangement was actually tried, however, serious complications were introduced. The stray magnetic field greatly reduced the thermionic bombarding current and the photoelectric current from the disk. The tube was therefore carefully shielded from the effects of the stray lines by means of a series of soft steel frames. When this was done, and when an electric field of about 100 volts was applied to the plates, it was found that variations in the strength of the magnetic field did not affect the magnitude of the electrometer deflections. As a result of this test the magnetic field was proved unnecessary and this feature was completely eliminated after a few of the preliminary runs on the apparatus.

In fact it is easy to show by a simple calculation that it ought to be possible to remove ions, moving with the maximum velocity that one might reasonably expect under the conditions, by the application of a very moderate potential to such a system of plates. The calculation shows that a hydrogen ion, for instance, with a velocity corresponding to a fall through 1,000 volts, would be removed by a plate potential of the order of 100 volts. Secondary effects arising from the impacts of the ions against the plates in this apparatus, furthermore, are not likely to be serious if the number of impacts is not excessive. With a view, therefore, to reducing the number of ions present to a minimum particular care has been exercised to secure the very best vacuum conditions, and, as far as possible, to free the target from occluded gas.

Thus, preliminary to all runs, the target is given a thorough heat treatment, which consists in raising it to as high a temperature, by electronic bombardment from the cathode, as the particular element in use will safely withstand. A current of about 30 milliamperes at 300 volts is sufficient to bring the anode to a bright red heat. This preliminary heat treatment of the target is always carried out at a higher temperature than will be reached in the subsequent run.

EXPERIMENTAL TESTS.

When the cathode is heated, a small deflection of the electrometer regularly takes place before the anode voltage is applied and it is in a direction corresponding to a positive charging up of the detecting disk. This deflection results from a photoelectric action upon the disk produced by intercepted light from the glowing cathode. Its magnitude depends upon the temperature of the cathode, and when this is very high, it may amount to 25 millimeters with a shunt permitting moderate electrometer sensitivity. This small deflection is, of course, constant for any particular set of readings, and its effect is completely eliminated by resetting the zero of the scale.

If now the anode voltage be gradually applied—assuming a potential of, say, 100 volts between the plates—the electrometer will remain unaffected until a potential of from 12 to 25 volts is attained depending on the material of the target. Then it will begin to deflect slowly in the same direction as before, and the magnitude of this deflection increases rapidly with further increase of the anode voltage.

If the anode voltage be now adjusted to some value which will give a fair scale deflection, an increase in the potential across the plates will not change its magnitude. If, however, the plate potential be reduced, the deflection will not change until a comparatively low critical potential difference is attained. Then a sudden increase is observed, and a further decrease in the plate potential will result in an off-scale deflection. The setting in of this effect is certain indication that positive ions formed in the radiation compartment are beginning to pass between the plates to the detecting disk. The value of this critical plate potential seems to be somewhat characteristic of the target element which is being used. With the carbon target it was found to be as high as 30 volts but with titanium it was below 12 volts. Since, during most of the actual runs, the plates have been charged to 135 volts, these experiments indicate unquestionably that the system of plates as used is perfectly effective in preventing the passage of charged bodies through to the detecting disk.

There is still, however, the question of the radiation associated with the formation of these ions to be considered. No direct tests with a view to differentiating between the true target radiation and gas radiation have yet been made. However, there are several indications that the latter effect may be safely neglected in the present investigation. First, the results secured are characteristic, in a recognizable fashion, of the different elements used as targets. There is no reason to believe that radiation from residual gas would behave in this manner. Second, the only gas presumedly present which might cause trouble in this work is oxygen. No characteristic radiation effects from this gas similar to those which were later secured when the element itself was used in an oxide as a target were ever observed. Third, the gas pressure was known to be too low to permit a sufficient number of impacts against gas atoms to give a detectable effect.

Results.

When the electrometer deflection is plotted as a function of the anode potential, curves of the type shown in Fig. 2 are obtained. In some cases marked discontinuities or "breaks" in the curvature are discernible, as in the curve for iron referred to, while in others it is practically impossible to determine with any degree of precision the position of a definite break in the curve. If, however, the deflection per unit thermionic current is plotted against the anode potential, the resulting relation is linear. Any change in the rate of increase of the effect with the voltage will now be very evident, and to determine the position of the break point with considerable precision, one has only to draw the two best straight lines through the plotted points on each side of the break, and the position of the break will be indicated by their point of intersection.

The radiation produced by impacts of electrons against a solid consists of two distinct types: general radiation and characteristic radiation. Both types are generally present and both evidently increase in intensity linearly with the voltage. A break point in the curve indicates the setting in of a new type of radiation, which is found to be characteristic of the target element. From *a priori* considerations the break may be either an upward or downward inflection, depending on whether or not the new characteristic radiation produces a larger photoelectric effect than would be produced by the additional general radiation which would be emitted if this characteristic radiation did not set in. It has been found that in general the setting in of characteristic radiation is indicated by an increase in the total emission. However, one exception has been thus far noted in these experiments. The L series of silicon is evident as an actual falling off in the total radiation. Evidently, in this case, the increase in characteristic radiation is insufficient in amount to balance the falling off in general radiation. This phenomenon has been previously noted in an investigation of the variation of total x-ray intensity with voltage in the case of silver, for radiations of the ordinary x-ray type.[1] However, while the presence of characteristic radiation is usually shown by an upward inflection in the curve, the sharpness of the break varies considerably with the different elements and among the several x-ray series of the same element. It is hoped that later, when more elements have been studied, it will be possible to establish some sort of a periodic relationship relative to the intensity of the characteristic radiation from the different elements.

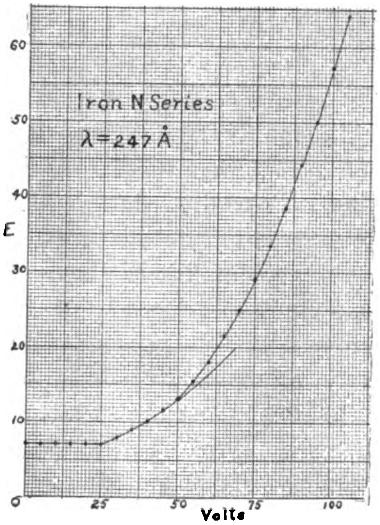

Fig. 2.

CARBON.

The radiation curves for carbon are given in Fig. 3. The upper curve shows the break corresponding to the K series of the element and the lower one indicates the L series. Both curves are the results of single runs over the respective ranges. The target in this case was cut from a piece of graphite, and during the procedure of evacuating the apparatus, it was brought to a white heat by thermionic bombardment.

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[1] Brainin, PHYS. REV., 10, p. 461, 1917.

Despite this exceptionally favorable heat treatment, however, the carbon seemed to require, in order to eliminate the direct effect of positive ions, a little higher minimum voltage across the plate than any other element thus far studied. Perhaps this fact may account for the apparent slight departure which is to be noted in the case of the carbon curves from a fair linear relationship between break points. The L series break indicated for this element is of special interest because it presumedly arises as a result of electrons falling into the very outer shell of the atom. There was considerable reason to expect, in fact, that one might actually not observe a sharp break corresponding to this series for carbon in the solid form because of the proximity of the neighboring atoms. As it is, there is probably little question but that the potential energies of the electrons in the outer shell of the carbon atom vary somewhat, depending upon whether the atom is free as in a vapor or whether it is combined with other atoms as in a solid. Thus one might expect to find that the L series of the solid carbon, for instance, is somewhat different from the L series, if it might be obtained, of the vapor.

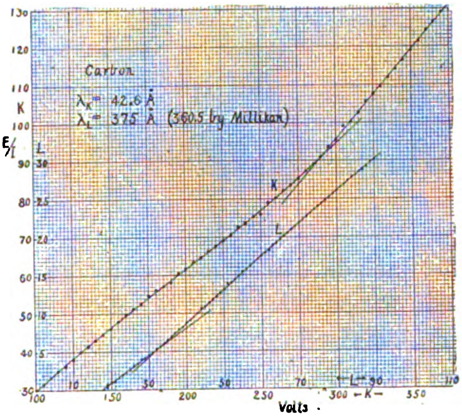

Fig. 3.

Copper.

The copper target used in these experiments was heated for a considerable time to a temperature close to its melting point. Finally, just prior to making the runs, the temperature was raised until the target melted a little at one end, and during this process, a considerable quantity of copper was distilled upon the inner surface of the glass tube. In the case of one of the preliminary runs which was made with an unusually high thermionic current, a strong break in the curve was obtained at about 500 volts. The position of this break was, however, found to depend upon the temperature of the target and it was unquestionably due to vaporization of the copper at these voltages, the new radiation arising from the copper vapor. But the actual runs, for which the curves are given, were made with the target at relatively low temperatures and in no case did this exceed that of a dull red heat.

The results for copper are given by the curves of Figs. 4, 5, and 6. A range up to 600 volts is shown in Fig. 4. This curve shows a break corresponding to the M series and the presence of a somewhat less sharply defined but intense effect beginning a little above 100 volts. This lower effect is shown to better advantage by the curve of Fig. 5, taken with larger currents. The absence of the customary sharpness in the case of this break may be accounted for by assuming that the group of lines in the spectral series which it represents may actually include two or more convergence limits. This is found to be the case in the portion of the M series already studied and presumedly it would likewise be true of a still more complicated N series. The L series break for the element is given in the curve of Fig. 6.

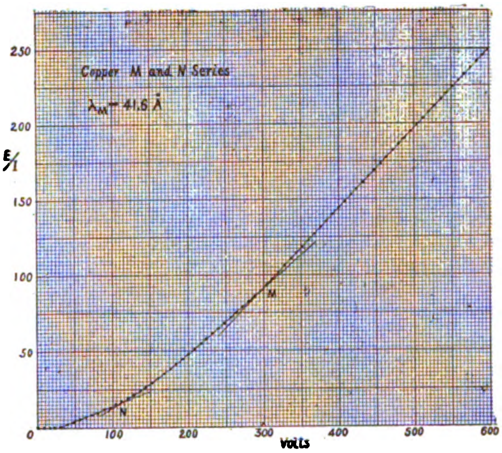

Fig. 4.

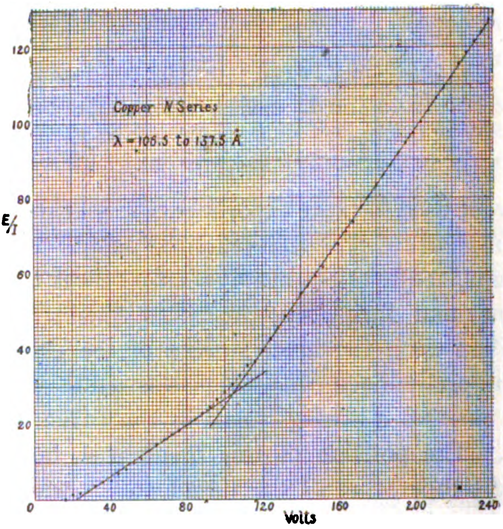

Fig. 5.

OXYGEN.

When the experiments with copper were concluded, the target was removed, and the surface of the metal was carefully oxidized in a Bunsen flame. It was immediately reinserted and the apparatus re-exhausted. It was found that the oxide on the copper would safely withstand a temperature corresponding to a low red heat. The curves of Fig. 7 give the positions of the K and L series breaks obtained for this element.

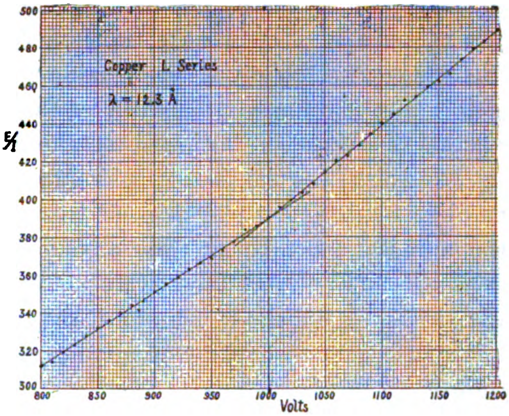

Fig. 6.

The breaks for copper could also be obtained with this target, but their positions were so far removed from the oxygen breaks as to cause no uncertainty in the interpretation of the results.

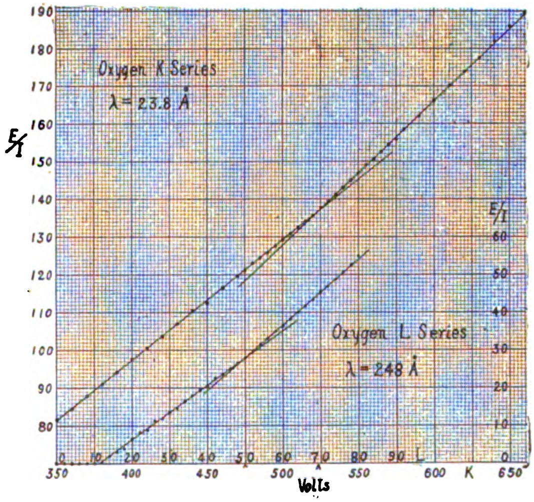

Fig. 7.

Aluminium.

Some unique difficulties were experienced in the work with this element. In the first place, the bombardment process had to be very carefully carried out because of the comparatively low melting point of 657°.

Secondly, it was found that the break points would gradually decrease in sharpness during a series of runs on successive days and, eventually, one would obtain practically a straight-line relationship over the entire region of the breaks. If the target were now removed from the apparatus, resurfaced, and again replaced, the break points would not be observed, or at best, perhaps, very weakly, upon making a run. In order to obtain the break as originally it was necessary to insert a new target. It is possible that the unusual behavior of this element is due to some peculiar action of mercury vapor upon the aluminium. Since it was not ordinarily convenient nor considered desirable to keep liquid air on the traps over night or during the baking-out treatment, the target was exposed to the action of the vapor at three or four microns' pressure between the experiments. However, there were no visible indications of any effect of the mercury upon the target and if the tendency were to form an alloy, it is a little difficult to see why a temperature of red heat did not break it down.

IRON.

The L and M series breaks for iron are given in the two curves of Fig. 8. The lower break, corresponding to an N series, is shown in the curve of Fig. 2. This break is likewise evident in the lower curve of Fig. 8. The lack of sharpness of the M series break which is to be noted is evidence that the M series at this point contains more than one convergence limit.

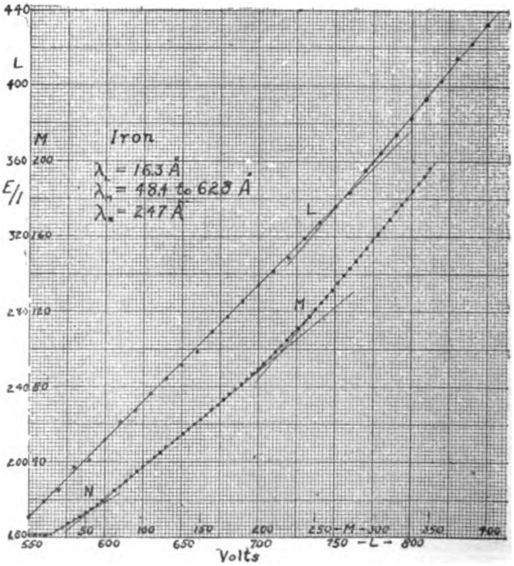

Fig. 8.

TITANIUM.

There is little of special interest evident as yet with regard to the radiation of this element except that the break corresponding to the L series is exceptionally strong. Both the L and M series are observed.

SILICON.

The work with silicon has not yet been completed, but the tendency, first noted with this element, for the intensity of the effect to fall off coincidently with the setting in of the characteristic radiation has been previously discussed.

INTERPRETATION.

Fig. 9 shows the Moseley curves extended to include the types of characteristic radiation discovered in the investigation described above.

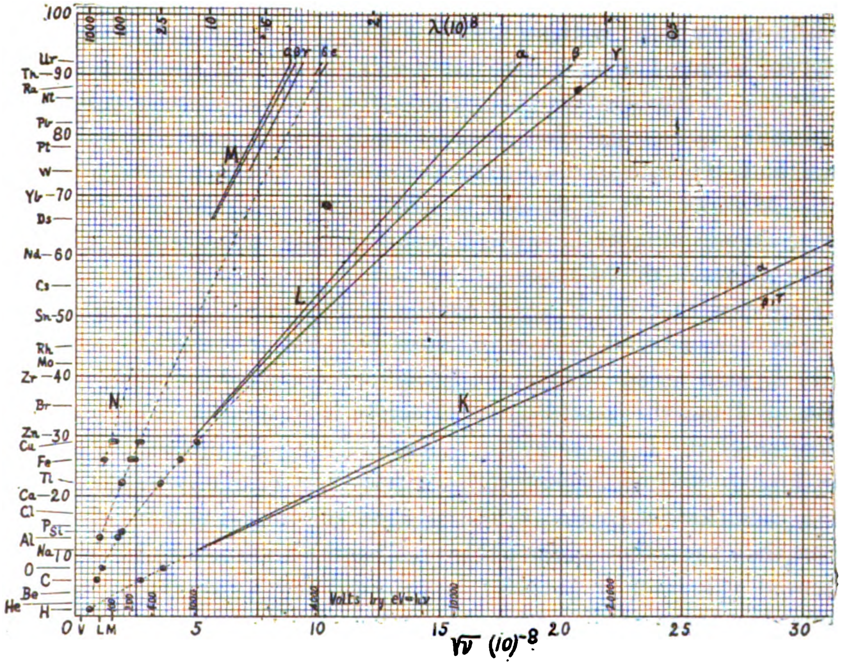

Fig. 9.

The square roots of the frequencies of the characteristic radiations are plotted against the atomic numbers of the chemical elements, showing the K, L and M series of radiations. The solid curves refer to radiations investigated by the ordinary crystal method of x-ray analysis and show how far toward the region of longer wave-lengths each type of radiation has been detected. The plotted points represent the results of the present investigation and the dotted lines through them indicate their probable relationship with the ordinary types of x-radiation.

The abscissæ marked V, L and M indicate the short wave-length limits reached by spectroscopic methods in the regions designated as the ultraviolet region, the Lyman region and the Millikan region, respectively. There remains a large gap from about 100 Å. to about 10 Å. in which no previous method of spectrosopy has been applicable and in which the first definite results are given by the present work.

In conformity with the results of Webster's work on the excitation of x-rays,[1] it is evident, by extrapolation from the x-ray curves, that the characteristic radiations in the present experiments are excited only when the bombarding electrons possess energy corresponding to the *convergence* frequencies of the series. These convergence frequencies are almost identical with the γ lines of the K and L series, but lie very slightly to the right of these lines in Fig. 9. In the K and L series the observed points fall on a reasonable extrapolation of the known lines. In the case of the M series the extrapolation is too great to be considered as anything but a suggestion, whereas the N series is purely hypothetical, since the two points ascribed to it may prove to belong to a second group in the M series. It is planned to investigate a number of elements of higher atomic number in order to connect these observations with the known points of the M series.

It appears that the K series continues uniformly down to hydrogen, for which the plotted value is obtained from the convergence frequency of the Lyman series. The curve is slightly concave downwards.

By applying the combination principle it is possible to predict the extension of the L α line as far as sodium, atomic number 11, and the L γ line to calcium, atomic number 20. The observed frequencies of the L radiations of copper, iron, titanium, silicon and aluminium are all larger than the values predicted by the combination principle. But the observed values of frequencies in the ordinary x-ray region are also larger than those predicted by the combination principle, and by about the same amount. Thus there is no reason for believing that the curve through the observed points is not an accurate continuation of the curve through points representing the L convergence frequencies in the ordinary x-ray region. It is planned to test some elements of atomic numbers slightly higher than that of copper in order to secure an actual overlapping of the two methods in this part of the spectrum.

The lower part of the dotted curve for the L series departs considerably from the extension of the L series curves predicted by the combination principle, which principle suggests that the L α line should continue straight into the "Y" axis at about atomic number 7, while the L γ line should lie below it and be somewhat concave downwards. Some recent unpublished results, kindly communicated to us by Dr. Foote and Dr. Mohler, on soft x-rays from gases, fall consistently on, or to the *left* of, the extension of the L α line predicted by the combination principle. But the present results *should* lie to the right of this line because they give *convergence* frequencies, and because this line is known to be to the left of the actual values where they are known in the x-ray region. Furthermore, Millikan has observed spectroscopically the L series of carbon, and Sommerfeld points out strong reasons for believing that hydrogen possesses an L series in its Balmer series. Thus the L curves must actually curve downward toward the origin, and there is no obvious reason for believing that the L curve in Fig. 9 is not correct. It is likely that the difference between the results shown in Fig. 9, which applies throughout to radiation from solid targets, and those obtained by Foote and Mohler for gases and vapors is due to an actual modification of the characteristic frequencies in atoms of solids, arising from the influence of neighboring atoms. This modification would be expected to be less important at the higher frequencies, but would probably be very important in the case of radiation from electrons in the outer shells, or orbits. It may be that this influence accounts for the inexactness of Kossel's relations when applied to x-radiation from solid targets.

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[1] Webster, Phys. Rev., 7, p. 599, 1916; Webster and Clark, Nat. Acad. Proc., 3, p. 185, 1917.

The M α curve can be predicted by Kossel's relation down as far as calcium. It shows a strong curvature downward, occurring at about cobalt, atomic number 27, and by analogy, leading us to expect a similar downward inflection near the foot of the L curves. There are no data whereby the convergence frequencies of the M series can be predicted in the region in which we are interested. The observed characteristic radiations, ascribed to the M series, are of considerably higher frequencies than the predicted M α radiation, the frequency difference being about the same as that found in the L series.

The lowest voltage at which detectable radiation was produced is 12.5 volts, in the case of oxygen. This corresponds to a wave-length of 990 Å. In most of the other cases the radiation was first detected at about 20 volts. The relatively large importance of velocity distribution corrections and of slight zero shifts on the accuracy in this region of low voltages probably renders these results of little interest, except in that they prove the production of radiation by impacts at these small voltages. It is hoped that a subsequent re-design of the apparatus may enable better accuracy to be secured in this region of weak effects.

The following table gives the averaged results obtained thus far in this investigation.

TABLE I.

Convergence Wave-lengths.

Atomic Number.	Element.	K Series.	L Series.	M Series.	N Series
6.......	Carbon	42.6 Å.	375 Å.	——	——
8.......	Oxygen	23.8	248	——	——
	Aluminium	——	100	326 Å.	——
	Silicon	——	82.5	——	——
	Titanium	——	24.5	85.3	——
	Iron	——	16.3	54.3	247 Å.
	Copper	——	12.3	41.6	116

There is overlapping of these results with those obtained by Professor Millikan[1] in the extreme ultraviolet. He has definitely placed the convergence wave-length of the L series of carbon at 360.5 Å. He finds a strong iron line at 271.6 Å., aluminium lines at 136.5 Å. and 144.0 Å. and an oxygen line at 231 Å. These are, presumably, the M α iron line, the L α aluminium lines and the L α oxygen line. He does not, however, find an aluminium line near or slightly longer than 326 Å. Remembering that our values refer to convergence wave-lengths, the agreement seems to be good. It must be remembered that the accuracy of the present method is relatively poor at the longest wave-lengths, owing to the weakness of the radiation and the uncertainties introduced by the distribution of velocities of the bombarding electrons. This correction, which could not amount to more than two or three volts, was entirely neglected since the potential drop across the filament was about sufficient to balance the average kinetic energy of emission of the electrons. At the higher voltages used, this correction would be entirely negligible, but it may have introduced small errors at the lowest voltages.

It is proposed to continue this investigation with other metals. A comparison of complete data given by this method with those being obtained for radiation from gases by Foote and Mohler should prove extremely interesting and might lead to some explanation of the condition of electrons in atoms of solids and of the failure of Kossel's relation when applied to characteristic radiation from solids.

I take this occasion to express my indebtedness to Professor Karl T. Compton for his inspiring interest in this work and for the kindly help which he has given all through the experiments.

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[1] Astrophys. Jour., 52, p. 47, 1920, and private correspondence.

ADDENDA AND ERRATA.

Vol. XVIII., August, 1921, abstract of paper read before American Physical Society, April 22, 1921, by S. J. Mauchly, entitled "Note on the Diurnal Variation of the Atmospheric-Electric Potential Gradient," p. 162, reference is made to the following figures which were omitted from the text:

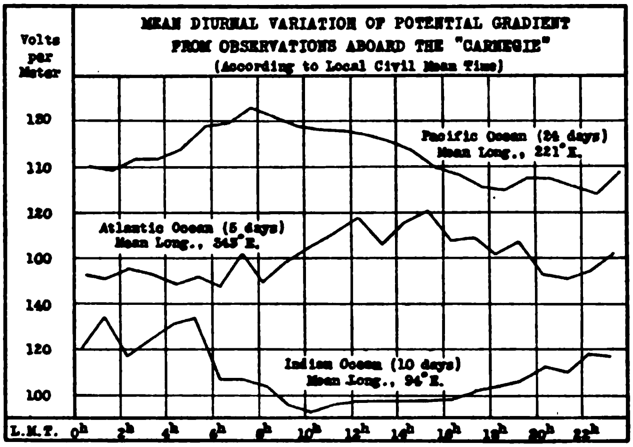


Fig. 1.

Mean Diurnal Variation of Potential Gradient from observations aboard the *Carnegie*, according to local Civil Mean Time.

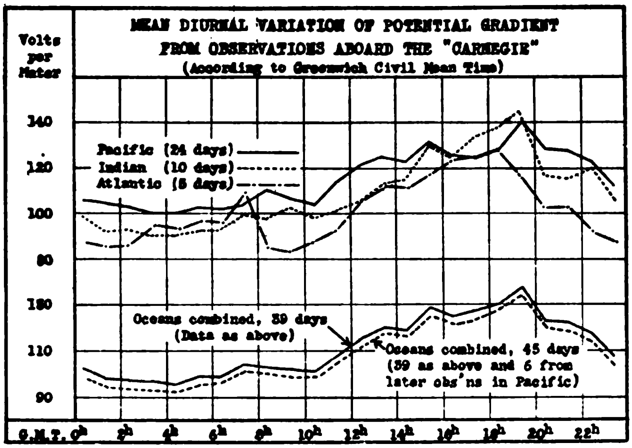


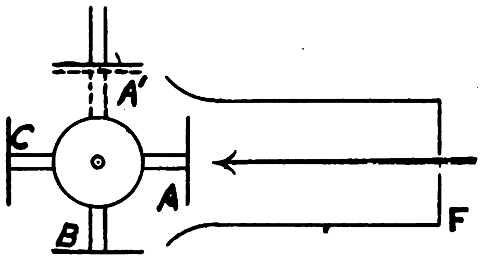
Fig. 2.

Mean Diurnal Variation of Potential Gradient from *Carnegie* observations according to Greenwich Civil Mean Time.

Vol. XVIII., September, 1921, in article by Willard Gardner entitled "Note on the Dynamics of Capillary Flow," page 208, in equations (9), (10) and (11) for the letter *l* read *e*, the Naperian base.

ADDENDA AND ERRATA *(Continued.)*

Vol. XVIII., September, 1921, in article by R. A. Millikan entitled "The Distinction Between Intrinsic and Spurious Contact E. M. F. S. and the Question of the Absorption of Radiation by Metals in Quanta," on page 239 for Fig. 1 substitute the following figure. It is suggested that the reader cut this out and paste it in its proper place.



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